The EM Algorithm

Parameter modeling and extraction

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The EM Algorithm

- This method is one of the most successful (and therefore important) methods used today.
- It is a parametric model.
- It is a nonlinear iterative method.
- It is easy to program for simple problems.

Summary: EM is a must-learn technique.
An example

- Lets take the case of a tone-burst in noise

- A simple threshold is used to classify two regions

- How do we pick the threshold?
Picking the Threshold

We start with a parametric model of the two distributions

\[ P(x|\text{tone}) = \mathcal{G}(\mu_t, \sigma_t), \quad P(x|\text{noise}) = \mathcal{G}(\mu_n, \sigma_n) \]

Find the \( \hat{\sigma}_n \approx \min(\text{std}(x(t : t + M))) \), \( M \) point blocks

Set threshold \( \Theta_n \equiv 3\hat{\sigma}_n \) (i.e., 0.13)
More generally

**Estimate:** the $k^{th}$ model parameters $k \geq 1$
(guess the first time, $k = 0$)

\[
\mu_{\text{noise|signal}}^{(k)} \equiv \text{mean} \left( x(i_{\text{noise|signal}}^{(k-1)}) \right)
\]

\[
\sigma_{\text{noise|signal}}^{(k)} \equiv \text{std} \left( x(i_{\text{noise|signal}}^{(k-1)}) \right)
\]

**Maximize:** $\Theta$ is the threshold on the likelihood ratio, given the $k^{th}$ parameter estimates:

\[
i_{\text{signal}}^{(k)} = \text{find} \left( \frac{G(x, \mu_{\text{signal}}^{(k)}, \sigma_{\text{signal}}^{(k)})}{G(x, \mu_{\text{noise}}^{(k)}, \sigma_{\text{noise}}^{(k)})} \geq \Theta \right)
\]

\[
i_{\text{noise}}^{(k)} = \text{find} \left( \frac{G(x, \mu_{\text{signal}}^{(k)}, \sigma_{\text{signal}}^{(k)})}{G(x, \mu_{\text{noise}}^{(k)}, \sigma_{\text{noise}}^{(k)})} < \Theta \right)
\]