Cochlear Compression and Voltage Mapping Explained

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Consider a basic feedback compressor with a threshold of compression of $T_{db}$ and an instantaneous ratio of $k$ [dB/dB]. Let $X_{db}$ be the input level and $Y_{db}$ by the output. Compression is performed via a nonlinear gain $G_{db}$ applied to the input signal. The equations describing this system are

$$Y_{db} = T_{db} + k(X_{db} - T_{db})$$  \hspace{1cm} (1)

and

$$Y_{db} = X_{db} + G_{db}.$$  \hspace{1cm} (2)

In general, there are also time constants associated, but we will consider only the steady state. We seek $G_{db}$ as a function of only $Y_{db}$ and the known parameters $T_{db}$ and $k$. Because the system is a feedback compressor, $G_{db}$ cannot be a function of $X_{db}$. We can substitute Eq. 1 into Eq. 2 to remove $X_{db}$:

$$G_{db} = Y_{db} - T_{db}(k - 1) + Y_{db} = (Y_{db} - T_{db})(1 - \frac{1}{k}).$$  \hspace{1cm} (3)

Equation 3 gives us a form for the gain of the feedback compressor for a given value of $Y_{db}$ with the system parameters.

In the specific case of the cochlea, $k$ and $T_{db}$ are somewhat functions of place, but they are generally constant with respect to $X_{db}$. The exception is that $k \to 1$ when $X_{db} < T_{db}$, but we are less interested in the low level, linear range of the cochlea. We are instead interested in why the voltage on the hair cell is a log-linear mapping of the input level $X_{db}$ for levels where compression is active.

By the Sewell effect,

$$G_{db} \approx -m(v - v_0)$$  \hspace{1cm} (4)

where $m$ is about 1 [mV/dB] at most cochlear places, $v$ is the voltage in [mV] on the hair cell (here the OHC), and $v_0$ is some threshold voltage in [mV], below which compression is inactive. If we set Eq. 3 and Eq. 4 equal to each other and simplify, we find

$$Y_{db} - T_{db} = \left(\frac{m}{k - 1}\right)(v - v_0),$$  \hspace{1cm} (5)

and if we then substitute $X_{db}$ back in for $Y_{db}$, we find the final relationship

$$X_{db} - T_{db} = \left(\frac{m}{1 - k}\right)(v - v_0).$$  \hspace{1cm} (6)

Equation 6 shows that $X_{db}$ and $v$ are linearly related. For every change in $X_{db}$ by $\Delta X_{db}$ there must be a corresponding change in $v$ by $(m/(1 - k))\Delta X_{db}$. 


This satisfying relationship is only log-linear for approximately constant parameters, and $k < 1$ (such as the $\approx 1/3$-law empirically predicted for the human compression curve). If $k$, $m$, or $T_{db}$ were strong functions of $X_{db}$, the relationship would no longer be log-linear. However, $T_{db}$ is guaranteed to be constant, $m$ is assumed to be nearly a constant by the Sewell effect, and $k$ is only a strong function of $X_{db}$ around the breakpoint $T_{db}$ (and possibly when the cochlea saturates at very high levels). This means that, as expected, the log-linear mapping holds only while compression is active: where $X_{db} > T_{db}$.