Ohm's Law

For resistors, the voltage over the current is called the resistance. In general, impedance is defined as the ratio of a force over a flow. For electrical circuits, the voltage is the force and the current is the flow. The impedance is defined as

\[ Z = \frac{F}{V} \]

where \( F \) is the magnitude of the force and \( V \) is the magnitude of the voltage. In the case of an electrical circuit, the force is the voltage over a resistor and the flow is the current through the resistor. The impedance is a complex number that includes both the resistance and the reactance of the circuit.

Möbius transforms and infinity

Problem #5: The bilinear transform is used in signal processing to design a digital (discrete-time) filter. The bilinear transform is a mapping from the complex plane to another complex plane. The mapping is defined as

\[ s = \frac{2}{\tau} \frac{u - 1}{u + 1} \]

where \( s \) is the complex frequency in the z-domain and \( u \) is the complex frequency in the s-domain. The bilinear transform is a one-to-one mapping, and it is used to convert continuous-time filters to discrete-time filters.

Problem #6: Geometrically, what is the effect of this Möbius transform? Consider your drawing of a hyperbola and indicate these using '-', '+', and blank spaces. Assign the weights to a matrix indicating the pan assignments, multiplied by a vector of the weights. Use the Matlab/Octave command 

\[ \text{Z} = \text{ztrans}(\text{z}, \text{s}, \text{t}) \]

to determine the transform of the hyperbola. If the hyperbola is to the left, provide the position of the weights. If the hyperbola is to the right, provide the position of the weights.

Problem #7: The geometric interpretation of this Möbius transform is the effect of the hyperbola on the position of points in the complex plane. The geometric interpretation is the effect of the hyperbola on the signal processing of noise and signals (i.e., not combinatorics).

Problem #8: The geometric interpretation of this Möbius transform is the effect of the hyperbola on the signal processing of noise and signals (i.e., not combinatorics).
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3.8.3 Exercises AE-3

Topic of this homework: Visualizing complex functions; Bilinear/Möbius transform; Riemann sphere.

Problem #1: Fundamental theorem of algebra (FTA)

−Q 1.1: State the fundamental theorem of algebra (FTA).

Problem #2: Order and complex numbers:
One can always say that 3<4, namely that real numbers have order. One way to view this is to take the difference, and compare to zero, as in 4 − 3 > 0. Here, we will explore how complex variables may be ordered. Define the complex variable z = x + iy ∈ C.

−Q 2.1: Explain the meaning of |z1| > |z2|.

−Q 2.2: If x1, x2 ∈ R (are real numbers), define the meaning of x1 > x2.

Hint: Take the difference.

−Q 2.3: Explain the meaning of z1 > z2.

−Q 2.4: If time were complex how might the world be different? (not graded)

Problem #3: It is sometimes necessary to consider a function w(z) = u + iv in terms of the real functions, u(x, y) and v(x, y) (e.g. separate the real and imaginary parts). Similarly, we can consider the inverse z(w) = x + iy where u(x, v) and y(u, v) are real functions.

−Q 3.1: Find u(x, y) and v(x, y) for w(z) = 1/z.

Problem #4: Find u(x, y) and v(x, y) for w(z) = ce with complex constant c ∈ C for the following cases:

−Q 4.1: c = e

−Q 4.2: c = 1 (recall that 1 = e^{2\pi\mathbb{i}k} for k = 0, 1, 2, \ldots)

−Q 4.3: c = \mathbb{j}. Hint: \mathbb{j} = e^{\pi/2}.

−Q 4.4: Find u(x, y) for w(z) = \sqrt{z}. Hint: Begin with the inverse function z = w^2.