Measurement of distortion product phase in the ear canal of the cat

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(Received 11 October 1996; revised 25 June 1997; accepted 26 June 1997)

Amplitudes of odd order distortion products (DPs) that are detected in animal ear canals have been used to probe cochlear health, to search for cochlear amplification, and to measure aspects of cochlear mechanical frequency response. Like the DP amplitude, DP phase is also an important measure of the cochlear mechanical response. Reported here are measurements of DP phase in the ear canal of the cat. The phase data show frequency-dependent time delays. One of these delays is a function of $f_2$, the frequency of the higher-frequency primary. Hence the DP phase $\phi_d$ is of the form $\phi_d = \phi_0 + \omega_d \tau$, where $\omega_d$ is the DP angular frequency and $\tau$ is a fixed time delay. Our results show that $\phi_d$ is independent of input level $a_1$, as long as the ratio $a_2/a_1 \leq 2$, where $a_2$ and $a_1$ are the amplitudes of the input tones. As $a_2/a_1$ becomes greater than two, the fixed time delays increase for DPs whose frequencies are less than the frequencies of the input tones. When both levels are varied together the delay increases as the levels decrease. There can be phase changes as large as $\pi$ associated with deep nulls in the DP magnitude for the two lower-frequency DPs. Features of the nulls may be modeled assuming that there is partial reflection of the DP wave from the DP place. The assumption of energy reemitted from the DP place also explains amplitude-ratio-dependent time delays and $2\pi$ level-dependent bifurcations in phase. The DP phase shows different dependencies for $f_2 < 1$ kHz compared to $f_2 > 2$ kHz. © 1997 Acoustical Society of America. [S0001-4966(97)02111-5]
PACS numbers: 43.64.Kc, 43.64.Bt, 43.64.Jb [RDF]

INTRODUCTION

Intermodulation distortion products (DPs) that are generated by the nonlinear mechanical response of the inner ear to an input of two primary tones are detectable in the ear canal. Since mammals, birds, and reptiles have nonlinear inner ears and, since DPs generally show good signal to noise ratios, DPs can be a useful noninvasive probe of the mechanical response of the cochlea. Indeed, attempts have been made to use DPs as an objective measure of hearing integrity in humans (measurement issues summarized in Whitehead et al., 1994); as a probe of cochlear amplification (Allen and Fahey, 1992); and as a probe of the cochlear frequency response (Brown and Gaskill, 1990; Brown et al., 1992; Allen and Fahey, 1993). Reported here are measurements in cat of the phase of the odd order DPs at frequency $f_d$ equal to $2f_1-f_2$, $3f_1-2f_2$, $2f_2-f_1$, and $3f_2-2f_1$ ($f_1$ is the frequency of the lower-frequency primary whose respective amplitude is $a_1$, and $f_2$ is the frequency of the higher-frequency primary whose respective amplitude is $a_2$).

Frequencies of the odd order DPs are given by the general expression $f_d = f_1 + n(f_2-f_1) = f_1 + n\Delta f$, where $n$ is either a positive or a negative integer. If $n \geq 2$, the DPs have frequencies greater than either of the primary frequencies and if $n \leq -1$ the DP frequency is less than either of the primary frequencies. (When $n = 0$, the equation above evaluates to one of the primary frequencies.) Because, when reading the text, it is easy to confuse $f_d = 2f_1-f_2$ and $f_d = 2f_2-f_1$, we will use the following notation throughout.

I. DISTORTION PRODUCT MEASUREMENT

A. Animal preparation and sound system

Both the sound system and the animal preparations in this study were essentially the same as in Allen (1983), except that there was no surgery to expose the auditory nerve. The animals were anesthetized with sodium pentobarbital and the bulla and septum were open. Enough of the ear canal was removed to allow positioning of the probe microphone to within a few millimeters of the tympanic membrane. Input tone bursts of 40-ms duration and 50% duty cycle were digitally synthesized. The DP phase is the fast Fourier transform (FFT) phase of the time response waveform built from ten
The DP amplitude was maximum when the value of \( a \) given by

\[ P_{\text{Th}} = P_{\text{ec}} (1 + Z_{\text{in}} / Z_0) \]

was expressed as a Thevenin equivalent source pressure \( P_{\text{Th}} \). The measurement typically across more than a dozen animals.

### B. The measurement

For each animal studied, both the sound delivery system impedance \( Z_{\text{in}} \) and the input impedance of the animal preparation \( Z_0 \) were measured after Allen (1986) and Voss and Allen (1994). The DP pressure measured in the ear canal \( P_{\text{ec}} \) was expressed as a Thevenin equivalent source pressure \( P_{\text{Th}} \) given by \( P_{\text{Th}} = P_{\text{ec}} (1 + Z_{\text{in}} / Z_0) \) (Fahey and Allen, 1986, 1988). Since DP volume velocity in the ear canal due to the DP pressure source in the cochlea depends upon the value of the transducer impedance, the use of the Thevenin equivalent pressure reduces the experimental variations of the results due to the transducer impedance. For most of the data presented here for 0.8 kHz < \( f_d \) < 8 kHz, \( P_{\text{Th}} \approx 2 P_{\text{ec}} \) since \( Z_0 \approx Z_{\text{in}} \). Therefore while the measurements presented here are of \( P_{\text{Th}} \) phase, they are similar to raw \( P_{\text{ec}} \) phase. The magnitude and phase of the DPs were measured by taking the FFT of the time waveform of the ear canal pressure.

### TABLE I. Fixed time delay as a function of \( f_2 \).  

<table>
<thead>
<tr>
<th>( f_2 ) (kHz)</th>
<th>0.5</th>
<th>1.0</th>
<th>2.0</th>
<th>3.0</th>
<th>4.0</th>
<th>5.0</th>
<th>6.0</th>
<th>7.0</th>
<th>8.0</th>
<th>10.0</th>
<th>12.0</th>
<th>17.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tau ) (ms)</td>
<td>2.0</td>
<td>1.3</td>
<td>1.1</td>
<td>1.0</td>
<td>0.9</td>
<td>0.85</td>
<td>0.8</td>
<td>0.7</td>
<td>0.7</td>
<td>0.60</td>
<td>0.60</td>
<td>0.60</td>
</tr>
<tr>
<td>( \Delta \tau = \tau - 0.6 )</td>
<td>1.4</td>
<td>0.7</td>
<td>0.5</td>
<td>0.4</td>
<td>0.3</td>
<td>0.25</td>
<td>0.2</td>
<td>0.1</td>
<td>0.1</td>
<td>0.60</td>
<td>0.60</td>
<td>0.60</td>
</tr>
<tr>
<td>( f_d \Delta \tau )</td>
<td>0.7</td>
<td>0.7</td>
<td>1.0</td>
<td>1.2</td>
<td>1.2</td>
<td>1.25</td>
<td>1.2</td>
<td>1.2</td>
<td>0.7</td>
<td>0.8</td>
<td>0.8</td>
<td>0.8</td>
</tr>
</tbody>
</table>

Since the phase has an uncertainty of 2\( \pi \), when plotting phase versus frequency, the phase data was unwrapped so that change in the phase from one point to the next was minimized. By first subtracting that part of the phase that is due to a fixed time delay we eliminate much of the potential for uncertainty in the phase unwrapping. In the data that follows it will occasionally be useful to keep this uncertainty in mind when viewing some of the features in the plotted phases. Stover et al. (1996) used DP magnitude and phase to define a “filter function” and its inverse Fourier transform that was called the “IFFT waveform.” We will refer to these constructions as the FILTF and WAVEF. We constructed FILTF’s by inserting a 128 point spline fit of the data into a spectrum that was padded with zeroes both at frequencies below the lowest value of \( f_d \) and at frequencies above the highest value of \( f_d \). A 1024 point WAVEF was generated from the FILTF inverse Fourier transform. The functional shape of the WAVEF resembles a system impulse response, but the WAVEF is not a system impulse response because it is nonlinear in origin. The WAVEF is a different way of presenting the same data as the DP magnitude and phase data. The advantage of the WAVEF presentation of the data is that delays that are implicit in the raw phase data are presented weighted by the energy at the various delays.

![FIG. 1. The top left panel shows the magnitude of the \( f_d (t) = 2 f_1 - f_2 \) DP as a function of DP frequency for constant \( f_2 = 9 \) kHz. Each magnitude curve was measured at a level wherein \( a_2 \) was decreased by 3 dB from the previous curve while \( a_1 \) was held constant at 1.0 Pa (1.0 Pa is 94 dB SPL). The DP amplitude was maximum when the value of \( a_2 \) was 6 dB less than the value of \( a_1 \). The value of the phase was independent of the level of \( a_2 \) for 52 < \( a_2 \) < 85 dB SPL. The bottom left panel shows the phase of the DP as a function of DP frequency for no subtracted fixed time delay. In the top right panel the phase is plotted after a fixed time delay of 0.65 ms has been subtracted and in the bottom right panel phase is plotted after subtracting a fixed delay of 1.0 ms. It is evident that a fixed delay of 0.65 ms does the best job of making the phase independent of frequency. The frequency axis in the magnitude plot is logarithmic while the frequency axes in the phase plots are linear.](image1)

![FIG. 2. The top plot shows the values of the fixed time delays \( \tau \) plotted versus \( f_2 \), symbolized by ‘’’s; \( \tau = 0.6 \) ms = \( \Delta \tau \) is represented by the ○’s; and the curve is a plot of \( f_1 / f_2 \) vs \( f_2 \). A reasonable first approximation to \( \Delta \tau \) is \( 1 / f_2 \) as shown by the solid curve. The bottom plot shows the logarithm of \( \Delta \tau \) versus the logarithm of frequency. The ○’s are the data and the solid line is \( 1 / f_2 \).](image2)
The animal and file and the constant value of different level of the input. At the top of the figure are listed FIG. 4. The higher-frequency fifth-order DP, \( f_d \), for the lower frequency fifth-order DP, \( f_d(-2) = 3f_1 - 2f_2 \), as \( a_2 \) is decremented from \( a_2 = 0.78 \, \text{Pa} \) in 3-dB steps at a constant \( a_1 = 0.77 \, \text{Pa} \). The lowest value of \( a_2 \) is 0.14 Pa. The phase changes little with \( a_2 < 0.5a_1 \). In the right panels the higher-frequency third-order DP, \( f_d(+2) = 2f_2 - f_1 \), magnitude, and phase are plotted as functions of DP frequency. The value of \( a_1 = 0.65 \, \text{Pa} \) and \( 0.16 \leq a_2 \leq 0.65 \, \text{Pa} \).

is conceptually more mathematically simple than the method of Brown et al. (1996) and it has the added potential of giving more information about signal delays.

C. Data presentation

Figure 1 displays the effects of subtracting a fixed time delay from the phase data and Fig. 2 is a plot of Table I. Then Figs. 3–8 have the same presentation format. Each is a plot of the ear canal Thevenin equivalent DP pressure plotted against the DP frequency \( f_d \). Each curve is measured at a different level of the input. At the top of the figure are listed the animal and file and the constant value of \( f_2 \). At the bottom of the figure are the value of the fixed time delay \( \tau \) in ms. That part of the phase that is due to the fixed time delay \( \omega_1 \tau \) has been subtracted from the phase that is plotted. Therefore given that \( \phi_d = \phi_0 + \omega_d \tau \), the phase that is plotted is \( \phi_0 \). Also identified at the bottom of the plot is the level that was varied. The data in this study are at fixed values of \( f_2 \). Here, \( f_d \) is varied by changing the value of \( f_1 \).

The input levels used in this study were as high as 1.0 Pa (or 94 dB SPL) and were higher than levels generally used in human studies. Correspondingly, the DP levels were as high as 0.02 Pa (or 60 dB SPL).

FIG. 3. In the left panels magnitude and phase are plotted as a function of \( f_d \) for the lower frequency fifth-order DP, \( f_d(-2) = 3f_1 - 2f_2 \), as \( a_2 \) is decremented from \( a_2 = 0.78 \, \text{Pa} \) in 3-dB steps at a constant \( a_1 = 0.77 \, \text{Pa} \). The lowest value of \( a_2 \) is 0.14 Pa. The phase changes little with \( a_2 < 0.5a_1 \). In the right panels the higher-frequency third-order DP, \( f_d(+2) = 2f_2 - f_1 \), magnitude, and phase are plotted as functions of DP frequency. The value of \( a_1 = 0.65 \, \text{Pa} \) and \( 0.16 \leq a_2 \leq 0.65 \, \text{Pa} \).

FIG. 4. The higher-frequency fifth-order DP, \( f_d(+3) \), magnitude, and phase are plotted as functions of DP frequency. \( a_2 \) is decremented in 3-dB steps. In the left panels the phase is independent of \( a_2 \) for \( 0.39 < a_2 < 0.78 \, \text{Pa} \) while \( a_1 = 0.77 \, \text{Pa} \). In the last curve the data above \( f_d = 7 \, \text{kHz} \) is in the noise. In the right panel, starting at \( a_2 = 0.38 \, \text{Pa} \), \( a_2 \) is decremented in 3-dB steps to a final value of \( a_2 = 0.13 \, \text{Pa} \) while \( a_1 \) is constant at 0.37 Pa. The phase changes little with \( a_2 \) when \( 1 < f_d < 1.7 \, \text{kHz} \).

FIG. 5. In the left panels magnitude and phase are plotted as functions of \( f_d \) for the lower-frequency third-order DP, \( f_d(-1) \), as \( a_1 \) is decremented in 3-dB steps from \( a_1 = 0.63 \, \text{Pa} \) to \( a_1 = 0.15 \, \text{Pa} \) at a constant \( a_2 = 0.63 \, \text{Pa} \). The phase changes little with \( a_1 \) except in the vicinity of the nulls in the magnitude between 1.5 and 2.5 kHz. In the right panels magnitude and phase are plotted as a function of \( f_d \) for the higher-frequency third-order DP, \( f_d(+2) \), as \( a_1 \) is decremented in 3-dB steps from \( a_1 = 0.75 \, \text{Pa} \) to \( a_1 = 0.067 \, \text{Pa} \) at a constant \( a_2 = 0.78 \, \text{Pa} \). The lowest four phase curves are independent of \( a_1 \) level.

FIG. 6. In the left panels magnitude and phase are plotted as functions of \( f_d \) for the higher-frequency third-order DP, \( f_d(+3) \), as \( a_1 \) is decremented in 3-dB steps starting at \( a_1 = 0.73 \, \text{Pa} \) and ending at \( a_1 = 0.09 \, \text{Pa} \) at a constant \( a_2 = 0.74 \, \text{Pa} \). All seven phase curves superimpose. In the right panels magnitude and phase as functions of \( f_d \) are shown for the higher-frequency fifth-order DP, \( f_d(+3) \), as \( a_1 \) is varied from \( a_1 = 0.62 \, \text{Pa} \) to \( a_1 = 0.11 \, \text{Pa} \) and \( a_2 \) is constant at 0.64 Pa. Notice that the phase does not depend on \( a_1 \) at this frequency of \( f_2 \). For \( f_d > 2.3 \, \text{kHz} \) the magnitude is in the noise at the lower values of \( a_1 \); hence, the phase is noisy also. When \( f_2 \approx 1.0 \, \text{kHz} \) the phases of the higher-frequency DPs seem to show less dependence on \( a_1 \) than at the higher \( f_2 \).
FIG. 7. Magnitude and phase are plotted as a function of \( f_d \) at the lower-frequency third-order DP, \( f_d(1) \), as \( a_1 \) and \( a_2 \) are decremented in 3-dB steps beginning at \( a_1 = a_2 = 0.95 \) Pa and ending at \( a_1 = a_2 = 0.06 \) Pa in the left panel and from 1.0 to 0.26 Pa in the right panel. In the left panels at a value of \( f_d \) just less than 2 kHz the phase seems to be independent of level (except for the phase curves measured at the two lowest levels). For higher frequencies than this fixed phase point the appearance of the phase varies slightly with decreasing level. For the two phase curves at the lowest input levels a \( \pi \) phase shift occurs at about this frequency. Comparing the two lowest input level phase curves with the higher level curves shows that there is a level-dependent phase bifurcation of \( 2\pi \) for \( f_d < 2 \) kHz. In the right panels, while there is not a fixed phase point as in the previous figure, the phase varies little around \( f_d = 0.7 \) kHz. In this figure, at the lowest level of the primaries a sharp null is evident in the magnitude at \( f_d = 2 \) kHz. There is also a \( \pi \) phase shift at this frequency.

II. RESULTS

A. Time delays

In Fig. 1 the ear canal Thevenin equivalent pressure magnitude of a \( f_d(1) = 2 f_1 - f_2 \) DP is plotted versus its own frequency in the top left panel; the phase versus frequency with no fixed delay subtracted is plotted in the bottom left panel; in the top right panel a fixed time delay of 0.65 ms has been subtracted from the phase; and, in the bottom right panel a fixed delay of 1.0 ms has been subtracted. Here \( f_2 = 9 \) kHz and the subtracted time delay of 0.65 ms removes most of the phase variation at low frequencies (and, in this file, also at high frequencies).

The modeling results of Matthews and Molnar (1986) show that most of the DP energy is generated in the vicinity of the \( f_2 \) place. There are also good general theoretical reasons that DPs generated by a saturating nonlinearity would be generated near the \( f_2 \) places. The maximum DP is generated when the levels of the two inputs to the nonlinearity are approximately equal. For cochlear excitations that region is near the \( f_2 \) place. The data presented in this study are at constant \( f_2 \).

In Table I we tabulate the values of the fixed time delay as a function of \( f_2 \) under the condition that one of the primary levels was constant and the other primary level was varied. For the \( f_d(1) \) and \( f_d(2) \) DPs a further condition is that \( a_2 / a_1 = 2 \). Under these conditions the phase is independent of the level of the varied primary and the low-frequency slope of the phase as a function of \( \omega_d = 2 \pi f_d \) is constant; therefore, \( \tau \) is constant. As \( f_2 \) increases the time delay decreases. These values are a good representation of the fixed time delay across the dozen animals used in this study and across the four DPs. For higher values of \( f_2 \) the time delays come primarily from \( f_d(1) \) and \( f_d(2) \) data records. At values of \( f_2 \) on the order of 10 kHz or greater, the fixed time delay reaches an asymptote of about 0.6 ms. The third line of Table I shows the values of the time delays with the value of the asymptotic delay subtracted and the fourth line shows the values in the third line multiplied by their respective \( f_2 \). This product is of order one, suggesting that the time delays after the asymptotic delay has been subtracted is, to a first approximation, equal to \( 1/f_2 \). A graph of the results in the table is Fig. 2.

When \( a_2 / a_1 \) is greater than about two or three, the measured time delays for the \( f_d(1) \) and \( f_d(2) \) DPs can be greater than the values in Table I. The increase ranged from 1.0 ms at lower values of \( f_2 \) to 0.3 ms at the highest values. The effect of the ratio \( a_2 / a_1 \) is clearly seen in Fig. 9 and will be discussed in a following section. The longer delays could well be due to an increase in energy that arrives later because it has been reflected from the DP place.

B. Phase dependence on primary level(s)

1. \( a_1 \) constant, \( a_2 \) varied

Figures 1, 3, and 4 show both DP magnitude and phase as \( a_1 \) is held constant and \( a_2 \) is decremented in 3-dB steps. In Fig. 1 the \( f_d(1) \) DP phase data shows that, except for the data acquired at the four highest levels of \( a_2 \), the phase curves are independent of level. In Fig. 3 (left) the same general independence of phase with level is shown for the \( f_d(2) \) DP. Phase as a function of \( a_2 \) and \( f_d \) is as shown in Fig. 3 (right) for the \( f_d(2) \) DP and again there is independence of \( a_2 \). Figure 4 (left) illustrates the independence of phase with \( a_2 \) level for the \( f_d(3) \) DP. This behavior is typical for \( f_2 \approx 2.0 \) kHz. However, Fig. 4 (right) shows that
for the \( f_d(\pm 3) \) DP when \( f_2 \leq 1.0 \) kHz there is some independence of phase with changing \( a_2 \), but only for \( f_d \) within an octave or so of \( f_2 \).

2. \( a_1 \) varied, \( a_2 \) constant

The marked independence of phase across the DP frequency range with changing \( a_2 \) is not generally as pronounced with changing \( a_1 \). Figure 5 (left) shows that, as \( a_1 \) is decreased by 3 dB, the phase changes in the vicinity of nulls in the \( f_d(1) \) DP magnitude versus frequency curve. Indeed, the amplitude peak to valley ratio of the nulls seems to increase as \( a_1 \) is decremented. We will show in a later figure that the presence of nulls increases as the \( a_2/a_1 \) ratio increases.

For the \( f_d(\pm 2) \) DP when \( f_2 \geq 2.0 \) kHz we observe that the phase is independent of \( a_1 \) only at the lowest levels [as seen in Fig. 5 (right)]. This is also seen in the \( f_d(\pm 3) \) data. When \( f_2 \leq 1.0 \) kHz, the phase of the \( f_d(\pm 2) \) DP is generally independent of \( a_1 \) as is shown in Fig. 6 (left). Likewise, when the DP is \( f_d(\pm 3) \) and \( f_2 \approx 1.0 \) kHz, we also observe independence of phase with \( a_1 \) as shown in Fig. 6 (right).

Comparing the data of this and the previous section with the data of the next section, it is evident that phase changes much less when only one primary level is varied than when both primary levels are varied together.

C. Both \( a_1 \) and \( a_2 \) decreased

Figure 7 shows \( f_d(-1) \) DP phases and magnitudes as both \( a_1 \) and \( a_2 \) are each decremented by 3 dB. Qualitatively, the same phase dependence on level is observed in the \( f_d(-2) \) phase (e.g., see phase in Fig. 12). In Fig. 7, at \( f_d \approx 2 \) kHz (except for the two phase curves measured at the two lowest levels) the phase seems to be almost independent of input level and, as the levels decrease, the phase above \( f_d = 2 \) kHz decreases in a regular way. This pattern is commonly observed when changing \( a_1 \) and \( a_2 \) together. When \( f_d \) is greater than the value of \( f_d \) where the magnitude is maximum, \( f_d^{\max} \), the phase decreases when both levels are decremented together. It is also observed that, as \( a_1 \) and \( a_2 \) are decremented together, the frequencies at which the sharp nulls occur in the DP magnitude increase. For the two higher-frequency DPs the change in phase with the decrease of both levels seems to be a simple decrease in phase as seen in the panels of Fig. 8.

D. Sharp nulls

In Fig. 7 (right panel) there is an example of a sharp null in the DP magnitude at \( f_d = 2 \) kHz. In the phase a shift of \( \pi \) radians is observed at the null. When nulls begin to form as the level is decreased, the phase always shows a discontinuity at the null frequency. For the deepest nulls the change in phase is \( \pi \) radians. The sharpness of the nulls and the sharpness of the \( \pi \) phase shift are indicative of wave interference.

The appearance of nulls in the data is a function of the ratio \( a_2/a_1 \). When \( a_2/a_1 > 2 \) the density of nulls is greater. Figure 9 illustrates this effect. In each set of panels both primaries are decremented together, while the ratio \( a_2/a_1 \) changes from 0.3 in the left panels to 1.0 in the middle panels to 3.0 in the right panels. The nulling is least when the ratio is least. When the data of Fig. 9 (right) is plotted on a linear frequency scale (not shown) it is evident that the frequency spacing between the nulls is approximately constant (possibly increasing slightly as \( f_d \) decreases). When both \( a_1 \) and \( a_2 \) are varied together the nulls shift toward higher values of \( f_d \) as the primary levels decrease.

III. DISCUSSION

A. Time delays

As previously mentioned, the fixed time delays in Table I were chosen to minimize the slope of the phase for \( f_d \leq f_d^{\max} \). We also measured time delays by taking the negative of the slope of the complete phase versus linear angular frequency curve (Allen, 1983; Kimberley et al., 1993; O’Mahoney and Kemp, 1995) and, within the error of both techniques, measured the same values for delay. The correspondence of the two methods is clear from the phase plots of Fig. 1. The negative of the slope of the phase in the lower left-hand panel is equal to 0.65 ms. This is the delay that has been subtracted from the lower left panel phase in the plot of the upper right-hand panel.

Furthermore, following Stover et al. (1996) we built FILTF’s from our data that we inverse Fourier transform to get WAVEF’s. As observed by them, the maximum of the magnitude of the WAVEF can be used to define a latency. The latencies found using this technique, with \( a_2/a_1 < 1 \), gave us the same values as shown in Table I, within the error of the techniques. Of course, this is to be expected from the mathematical properties of the inverse Fourier transform if, globally, the phase slope is approximately constant. The top panels of Fig. 10 show the WAVEF magnitudes for the same data as shown in Fig. 1 and in Fig. 5 (left). The peak in the magnitude occurs at times consistent with the values of the delay for \( f_2 = 3 \) kHz and \( f_2 = 9 \) kHz, respectively. The time delays in Table I characterize all of the animals used in this
study. The values in Table I apply to every animal that we studied to within ±0.3 ms, except at the lowest value of \( f_2 \), where the range was ±0.5 ms. Kimberley et al. (1993) have shown that the time delay (also called latency) in humans decreases as the level of both primaries are increased together. This is also apparent in the cat data of Figs. 7 and 9. For example, calculating the delay from the complete phase versus linear frequency curve for Fig. 7 (left), the slope increases by a factor of about 1.5 from the highest levels to lowest.

The delays published here for a cat are much smaller than the delays in a human in the same frequency range. In a human, when \( f_2 = 4 \) kHz, the latency is on the order of 3 to 4 ms (Brown et al., 1992; Kimberley et al., 1993; Whitehead et al., 1996; Stover et al., 1996). Stover et al. have observed that the latency for some other odd order DPs is the same as for the \( f_d \left( -1 \right) \) DP. The cat data in this study also suggests that the fixed time delay is independent of DP order. This is certainly not surprising if it is assumed that the various odd order DPs are generated by the same mechanism at the same place. Whitehead et al. (1996) directly measured the time to onset of the DP signal in the ear canal and found times consistent with the measurement of latency from the slope of the phase. Stover et al. (1996) have shown that the delays derived from the WAVEF are commensurate with latency determined by the slope of the phase. Hence the four methods used to measure delay all yield consistent results. The phase data that we show in this study has the fixed delay subtracted from the total phase so that we emphasize and magnify the other features in the phase data (i.e., the nulls and what is happening to the phase above \( f_d = f_d^{\text{max}} \)).

When the DP delays are compared with neural delays (of inputs at \( f = f_d \) at the \( f_2 \) place) in the same kind of animal preparation (Allen, 1983), the DP delays match up well with minimum values for neural delays (after subtracting out approximately 0.9 ms of synaptic delay. These delays also generally correspond to the latencies of neural onset to condensation clicks in a cat of Kim and Molnar (1979, Fig. 8), although for characteristic frequencies greater than 2.0 kHz their latencies are systematically smaller than our fixed time delays. Part of neural delay is due to one-way travel time. Initially, one might think that some of the DP delay as due to a two-way travel time (for fixed \( f_2 \), i.e., the travel time of \( f_1 \) to the \( f_2 \) place and the travel time out of the cochlea of the DP. However, the travel time of \( f_1 \) to the \( f_2 \) place changes little as \( f_1 \) decreases; hence, the DP phase delay is primarily due to travel time of the DP excitation to the ear canal. That the minimum neural delay and the DP delay are approximately the same as a function of frequency again suggests that the DP is generated primarily at the \( f_2 \) place.

In a human, there seems to be some question on whether the DP delay is a measure of two-way travel time or one-way travel time (Kimberley et al., 1993; O Mahoney and Kemp, 1995; Bowman et al., 1997). Assuming that human DP biophysics is the same as a cat’s, and that the cat data show that neural travel time delay and DP delay are the same, it is clear that, for at least one DP component, the travel delay is due to one way travel time.

The relationship between fixed delay and \( f_2 \) is both what one would expect from models of cochlear macromechanics and from the theory of causal transfer functions. From Table I it is evident that there seems to be a minimum delay that is frequency independent and would be due to nonspecific delays such as measurement system delay (determined from the impulses response as derived from the wideband frequency response of the animal preparation) of between 0.2 and 0.3 ms and delay in communicating a signal from the base of the cochlea to the ear canal. This minimum delay is approximately 0.6 ms. If this value is subtracted from all of the other delay values and then the answer is multiplied by the respective \( f_2 \) for 1.0 \( \leq f_2 \leq 8.0 \) kHz, this time-frequency product is a constant \( N \) of order 1 (specifically, \( 0.7 \leq N \leq 1.2 \)), showing that the low-frequency limit is basically proportional to \( 1/\omega_2 \) (Fig. 2). The correlation of delay with \( f_2 \) can be expected if it is assumed that DP generation is at the \( f_2 \) place. For direct excitation of the cochlea, the proportionality of delay to \( 1/\omega_2 \) (which is the same as \( f_2 \) in this context) for low-frequency excitation has been derived by Allen (1983) using the WKB-approximation cochlear model. Furthermore, using one of the relations between the real part and the imaginary part of a general transfer function (or a general immittance) derived by Bode (Bode, 1945, Chap. 13) it can be shown that the time delay in the low-frequency limit is inversely proportional to the resonant frequency (or the band-edge frequency for a low-pass system). The constant of proportionality depends only on the shape of the transfer function. (This relationship is easily seen in the expression for phase of a
damped simple harmonic oscillator where the constant of proportionality is the inverse of $2\pi Q$, where $Q$ is the quality factor.) Assuming that the shape of the transfer function varies little with $f_2$; then, the proportionality of $\tau$ to $1/f_2$ would be expected.

B. The level dependence

Both the magnitude and phase curves show that there is much more regularity to the DP generation when only one input level is varied than when both levels are varied (e.g., Zwicker, 1981; Zwicker and Harris, 1990; Fahey and Allen, 1986, 1988; Whitehead et al., 1995).

For the $f_d(1)$ DP and the $f_d(2)$ DP, when $a_2 < (a_1 = \text{const})$, the phase and the shape of the magnitude curve are invariant with respect to the value of $a_2$, as seen in the example of Fig. 1 and of Fig. 3 (left). Given that basilar membrane (BM) response changes shape (Rhode, 1980; Ruggero and Rich, 1991) with level, it could be expected that the DP generation could not be invariant with level. However, the conditions of the DP generation reported here, when thought of in terms of a two-tone suppression experiment where $f_1$ is a low-frequency suppressor at the $f_2$ place, could decrease the sensitivity of the most sensitive response region (or “tip”) of the BM and/or the neural frequency tuning response at the $f_2$ place. The levels used were generally high, as the varied primary usually ranged between roughly 94 and 54 dB SPL. The primary that was not varied was usually fixed somewhere between 94 and 84 dB SPL. For the $f_d(1)$ DP with $a_2$ varied the value of the fixed $a_1$ was in the range where it would suppress the $a_2$ response at its characteristic place. Basically, the most input sensitive region of the response would have been removed (Fahey and Allen, 1985; Ruggero et al., 1992).

Given the measurements of magnitude and phase for the $a_1$ constant with $a_2/a_1 < 2$ for the two lower-frequency DPs, the data show that the DP generation process simply scales the DP magnitude with $a_2$ and that the frequency features of the DP are invariant under $a_2$ scaling.

When $a_1$ is varied with $a_2$ fixed, the phase is more dependent upon input level for the two lower-frequency DPs. In Fig. 5 (left) the phase varies around the frequencies where there are nulls in the magnitude response curve. The phase change upon stepping through the null frequency gets greater as $a_1$ decreases. The WAVEF of the data of Fig. 5 (left panel) is shown in the top left panel of Fig. 10. The energy seems greatest at a time delay of $\approx 1$ ms; however, there is also a second peak in the energy at $\approx 2$ ms. As $a_1$ decreases (and the depth of the nulls increases) the peak in the energy at $\approx 2$ ms increases relative to the peak at $\approx 1$ ms. This same phase behavior is seen in the $f_d(-2)$ phase data (not shown). When $a_1$ only is varied, the nulls in the magnitude response do not change frequency.

For both the two higher-frequency DPs and for the lower-frequency DPs as long as $a_2/a_1 < 2$ phase is generally independent of the changing of one input level while the other input level is held constant.

C. Effect of the $a_2/a_1$ ratio as both levels are varied

In Fig. 9 we show that the presence of nulls depends upon the ratio $a_2/a_1$. When this ratio is large there are more nulls and/or the nulls are more evident. Also, from the values of the subtracted time delays given at the bottom of each phase panel, the time delay is largest at the largest value of the ratio. When both of the primary levels are varied together, the nulls in the magnitude of the $f_d(-1)$ and $f_d(-2)$ DPs shift to higher values of $f_d$ as the levels decrease. The shifting of human DP nulls with level has been reported by He and Schmiedt (1993). They also show that the frequency spacing of nulls in a human is approximately 1/10 of an octave. In Fig. 9 (right), using both the magnitude and phase information it appears that there are nulls at $f_2 = 1$, 2, and 3 kHz for an $f_2 = 4$ kHz. If the magnitude data in Fig. 9 (right) are replotted on a linear frequency scale (not shown), it is evident that the frequency spacing between the adjacent nulls is, roughly, constant.

WAVEF’s of the data at $a_2/a_1 = 3$ and $a_2/a_1 = 1$ are shown in the bottom panels of Fig. 10. We find that the energy is more spread over time and peaks at later times as the $a_2/a_1$ ratio increases. Under some conditions the energy that arrives later can be greater than the energy that arrives earlier. The bottom panels of Fig. 10 also show that the time of the first peak increases as both of the primary levels decrease. This is consistent with the nulls moving to higher frequency with decreasing level and with the latency increasing as primary levels decrease (Kimberley et al., 1993; Stover et al., 1996; Whitehead et al., 1996).

The relative growth of the later peaks to the early peak with decreasing both $a_1$ and $a_2$ that is evident in the WAVEF presented here for a cat was observed by Stover et al. (1996) in a human.

The number of nulls per Hz (the null density) in a human (He and Schmiedt, 1993) is much greater than the null density in a cat (this study). Another difference between the cat data and the human data is the level of the DPs. In He and Schmiedt the DP level is generally between 15 and 0 dB SPL, while the data of our Fig. 9 is around 30–40 dB SPL. The input levels in the human are also corresponding smaller. The noise floor in our measurements was dependent on frequency, but for 1 kHz $< f_2 < 8$ kHz it was roughly a constant 10 dB SPL and we did measure DPs down to this level. At these lower levels (data not shown) we found no evidence that the null density would substantially increase as levels decrease. Even at the lowest levels, the null density was not much greater than 2 nulls per octave in a cat. Hence the null density difference between a human and a cat is not due to level differences. The ratio of the null density in the two species is roughly equal to the inverse of the ratio of the latencies. That the null spacing would be inversely proportional to the latency would suggest that propagation delays are implicated in the wave interference producing the nulls.

For a given pair of input levels, the nulls in human $f_d(-1)$ and $f_d(-2)$ do not occur at the same $f_d$ frequencies; however, the spacing between nulls (or peaks) is similar (Brown et al., 1993b, Fig. 2). In a cat, it is also generally observed that the nulls of the two DPs do not occur at
the same value of \( f_d \), nor do they occur at the same value of \( f_1 \).

The higher-frequency DPs, \( f_d(+2) \) and \( f_d(+3) \), do not show nulls like the two lower frequency DPs. At low levels of the DPs (20 dB SPL or less) we do occasionally observe nulls. However, these nulls are generally neither stable nor regular in frequency as input levels are decreased. The higher-frequency DPs do show predictable frequency-dependent features that are due to mixing with other components, such as the third harmonic of \( f_1 \) or the DPs at \( 2f_1 + f_2 \) or even \( 5f_1 - 2f_2 \).

### D. Source of the nulls

If it is assumed that the nulls are due to wave interference, then there are two or more waves mixing at the site of the microphone in the ear canal. If there are only two dominant waves, then, for a deep null the phase difference between the two waves of comparable amplitude must be \( \approx \pi \). If there are several waves, then the phasor diagram must (almost) close. Waves of differing phases could be due to: (a) a path/time differences, such as path differences due to reflections; (b) a “birefringence,” i.e., two (or more) different values for wave velocity (e.g., Hubbard, 1993); or (c) two (or more) different nonlinear mechanisms with different phase at output.

The nulls observed psychophysically (i.e., within the cochlea) in the \( f_d(-1) \) DP have been attributed by Zwicker (1981) to the interference of wavelets (i.e., waves from multiple sources).

Parallel approaches to modeling psychophysical DP nulls have been proposed to modeling ear canal DP nulls. Substantial discussion has been devoted in the literature to two possible mechanisms responsible for the microstructure observed in human DP emissions. The one proposed mechanism is the mixing of wavelets (Sun et al., 1994; He and Schmiedt, 1996) and the other mixing due to energy reflected from the DP place (Talmadge et al., 1996; Talmadge et al., 1995; Brown et al., 1996). If we assume that the nulls measured in a cat are due to the same physics as the microstructure measured in a human, then our cat data strongly support reflection from the DP place over distributed source generation of wavelets.

First, if the mixing of wavelets due to a distributed source of DP waves were the source of the nulls in the \( f_d(-1) \) and the \( f_d(-2) \) DPs, then one would also see nulls in the \( f_d(+2) \) and \( f_d(+3) \) DPs.

Second, as described below, using a simple semiempirical model, the assumption of reflection of energy from the DP place explains (a) several features of the nulls: (b) the commonly observed \( 2\pi \) phase splitting seen in DP data; and (c) the long delays in the \( f_d(-1) \) and \( f_d(-2) \) DP when \( a_2/a_1 \) is large.

There are several features of the nulls in our data that can be replicated with a simple physical picture. This picture assumes that the distortion product is generated at the \( f_2 \) place and that some of the DP wave is propagated basally (toward the stapes) and the remaining energy is propagated apically (toward the helicotrema). The time to propagate from the \( f_2 \) place to the stapes is assumed to be \( 1/f_2 \) as the data in Table I would suggest. Waves that reach the stapes may be partially reflected and partially transmitted. The reflection coefficient at the stapes is \( r_{st} \). Waves that propagate apically may be partially reflected at the DP place (Allen et al., 1995). [This is essentially the same assumption that has been used by Zweig and Shera (1995) to explain the periodicity of evoked stimulus frequency emissions that are observed in the human ear canal.] The reflection coefficient at the DP place is \( r_{dp} \). Two ways to model the phase change of the DP wave between the \( f_2 \) place and the DP place are to assume either a scaling symmetric cochlea, e.g., after Zweig and Shera (1995) or a nearly scaling symmetric cochlea, e.g., after Allen (1980). The scaling symmetric cochlea would have a phase change from the stapes to the characteristic place that scales as \( \phi \log(f_d) \). The Allen model can be shown to have a basilar membrane delay between the stapes and the characteristic place \( f_d \) that can be approximated by a constant divided by \( (f_d/f_{rel})^\alpha \) or \( a/f_d^\beta \), where \( a \) has units of time and \( f_d \) is now a relative frequency that is dimensionless.

The model basilar membrane response data of Allen yield \( a \approx 0.3 \) and \( \beta \approx 0.8 \). (\( \beta = 1 \) would be a scaling symmetric cochlea.) The time for the DP wave to travel from the \( f_2 \) place to the DP place is assumed, to a first approximation, to be the time to travel from the stapes to the DP place, minus the time to travel from the \( f_2 \) place to the stapes, \( 1/f_2 \). An interferometer with either a delay after the Allen model or a delay after the Zweig and Shera model produces interference nulls that have a frequency spacing that is similar to that of the data. For the scaling symmetric cochlea assumption, a value of \( \phi \approx 0.31 \) produces interference patterns that look like the cat data. A picture of the interference pattern produced by the scaling symmetric cochlea assumption is shown in the top panel of Fig. 11 and the pattern produced by the Allen basilar membrane model is shown in the bottom panel of Fig. 11. To a first approximation the reflection coefficients are assumed to be independent of amplitude. Details of this physical picture will be published separately. With reflections at the stapes and at the DP place there will be an infinite sum of waves that mix at the detector in the ear canal. The phase differences of the various waves that mix at the detector will depend upon the different travel times and upon any phase changes at reflection. The amplitude differences of the wavelets will depend upon the magnitudes of the reflection coefficients (we will assume that there is no significant attenuation on the basilar membrane). Letting \( r_{st} \) become zero, the above picture becomes a simple two-source model.

The major consequence of \( r_{st} \neq 0 \) is to sharpen the nulls and to broaden the maxima of the interference patterns. As \( r_{st} \) is allowed to increase, one generates the sharp interference patterns commonly pictured in optics textbooks (Born and Wolf, 1975, Sec. 7.6).

Our data are consistent with values of \( r_{st} \leq 0.3 \). The null density depends upon travel time assumptions and would be different for a human than for a cat. Indeed, as O Mahoney and Kemp (1995) have pointed out, “In humans ears tone-burst-evoked emissions commonly have ten or more waves of delay.” An examination of the data in the paper by Kimberly et al. (1993) shows a delay of between six and seven waves in human. Our data in Table I show that cats have one
wave of delay. Hence the frequency spacing of such an interference pattern as described above in a human would be somewhere around one-tenth to one-fifth of the cat frequency spacing or from 0.1 to 0.2 kHz. Values of the observed frequency spacing in a human (He and Schmiedt, 1993; Brown et al., 1993a; Piskorski et al., 1995) are in this range.

The null density will depend upon the rate that the phases of the interfering waves change as the \( f_d \) changes. Using the model described above, during a frequency sweep of \( f_d \), we find a maximum of three nulls. This maximum is only slightly dependent on the starting value of \( f_d \). The relative independence on the starting value of \( f_d \) on the maximum number of nulls per \( f_d \) sweep is also a feature of the data. The null density in both the model and the data (Piskorski et al., 1995) is the same if \( f_d \) is held constant, if \( f_1 \) is held constant, or if the ratio \( f_1/f_2 \) is held constant.

How is it possible that more energy arrives later than the energy that goes directly from the source to the detector? There are at least three possible explanations. First, if from the source one-half of the wave goes in each direction and if there is energy creation apical to the source, i.e., the effective \( |r_{DP}| > 1 \), then, independently of the value of \( r_{st} \), more energy will arrive later than earlier.

The third possibility that would allow more energy to arrive late than early assumes that, from the source, more energy is emitted apically than is emitted basally. If the impedence that the source sees in the apical direction is different than the impedence that the source sees in the basal direction, then the assumption that one-half of the energy goes in each direction would obviously be false.

Our data suggests that increasing the \( a_2/a_1 \) ratio might increase the energy emitted apically relative to that emitted basally. Both the \( f_{d}(−1) \) and the \( f_{d}(−2) \) show an increase in both delay and the presence of nulls as \( a_2/a_1 \) becomes greater than 2 or 3. Since this effect does not depend on the absolute values of the DPs [the \( f_{d}(−2) \) DP is generally 10–15 dB less than the \( f_{d}(−1) \) DP for the same level of primaries], the effect does not seem to be simply due to the nonlinearity of \( r_{DP} \).

However, because it is a common observation that a null can occur for a particular value of the input levels at constant \( f_2, f_1, f_d, \) and \( a_2/a_1 \) when both levels are varied together, the ratio of \( a_2/a_1 \) is not the only factor that determines the energy returned from the DP place. To be able to explain all of the details of the generation of nulls one would have to know the level dependence of the reflection coefficient \( r_{DP} \).

The interference pattern seen in stimulus frequency emissions disappears with increasing level, suggesting that the effective reflection coefficient at the emission site decreases as the level increases. Other studies imply that the cochlear reflection coefficient increases with decreasing input level (e.g., Allen et al., 1995). This correlates well with the observation that the measured DP delay regularly increases as the input levels (\( a_1 = a_2 \)) and, therefore, the DP levels decrease (Kimberley et al., 1993).

The features of the nulls that are replicated with this simple model are: (a) The null distribution of broad maxima with narrow minima shows a maximum null density which is approximately the same for the two lower-frequency DPs, \( f_{d}(−1) \) and \( f_{d}(−2) \). (b) The null density is the same whether \( f_2 \) is held constant, \( f_1 \) is held constant, or \( f_1/f_2 \) is held constant (Piskorski et al., 1995). (c) Since, for the two higher-frequency DPs, reflections from the DP place would not reach the ear canal microphone, this model predicts that the fixed time delays for the \( f_{d}(+2) \) and the \( f_{d}(+3) \) DPs would not depend upon \( a_2/a_1 \) and these two higher-frequency DPs would not show nulls.

Other features in the phase data that can be explained by the model are: (a) It is possible that the later wave can have more energy than the first wave, i.e., it explains the increase in the fixed time delay as \( a_2/a_1 \) exceeds a critical value. (b) It explains the commonly observed 2π bifurcations in phase upon changing level (such as shown in Fig. 7 where the phases at the two lowest input levels are different from the phases measured at the higher levels by 2π for \( f_d < 2 \) kHz). (c) Also, phase patterns like the one in Fig. 5 (left) are exactly what one would observe when a wave with more delay increases in relative amplitude to an early wave (as a function of the change in \( a_1 \)). (d) Assuming \( r_{DP} \) increases with
decreasing amplitude of the DP, the increasing phase delay with decreasing \( a_1 \) and \( a_2 \) at constant \( a_2/a_1 \) ratio is expected.

E. Base versus apex

There seem to be differences in the DP generation mechanics when \( f_2 \leq 1 \) kHz versus when \( f_2 \geq 2 \) kHz. When \( f_2 \approx 1 \) kHz, the two lower-frequency DPs, \( f_d(-1) \) and \( f_d(-2) \), do not have a distinctive \( f_d^{\max} \) that differs from \( f_2 \). For \( f_2 \geq 2 \) kHz, \( f_d^{\max} \) is defined by a second cochlear map (Allen and Fahey, 1993). The two higher-frequency DPs, \( f_d(+2) \) and \( f_d(+3) \), behave in the opposite fashion. They do show a distinct \( f_d^{\max} \) different from \( f_2 \) when \( f_2 \leq 1 \) kHz. When \( f_2 \geq 2 \) kHz, they do not. Some of this effect is probably due to the increasing of the input impedance \( Z_{in} \) as the frequency decreases below 1 kHz (Allen, 1986; Rosowski et al., 1986). This means, that for a given value of \( a_1 \) in the ear canal, relatively less \( f_1 \) energy gets to the nonlinearity as \( f_1 \) decreases.

When \( f_2 \approx 1 \) kHz, the \( f_d(+2) \) and the \( f_d(+3) \) phase varies less with respect to \( a_1 \). Since low-frequency neural frequency tuning curves are less sharply tuned, the change in the DP generation properties might be due to the same mechanisms that change the low-frequency neural tuning.

Another observation that may be relevant to the difference of generation mechanics in the base versus apex is that there is broad maximum in the phase of \( f_d(+2) \) and \( f_d(+3) \) [see Fig. 5 (right panel) and Fig. 8] that is centered about \( f_1 \approx 1 \) kHz. This is a common feature in the data of the two higher-frequency DPs and it seems to be a function of \( f_1 \).

IV. SUMMARY

A. Phase observations that are a property of saturating nonlinearities

Much of the phase data discussed in this study may be explained by the basic properties of saturating nonlinearities. In Fig. 12 the phase of the \( f_d(-2) \) DP is about \( \pi \) radians less than the phase of the \( f_d(-1) \) DP over the range where phase was measured. This is also true when the phases are plotted versus \( f_1 \). In simulations we have found a phase difference of \( \pi \) between these two DPs to be a generic property of the saturating nonlinearities commonly used in modeling auditory phenomena. This was commented upon by Schroeder (1969) for the special case of the \( f_d(-1) \) DP phase when he compared the psychophysical phase with that expected from either an expansive nonlinearity or a compressive nonlinearity and found a \( \pi \) phase shift that showed that the source of the \( f_d(-1) \) DP is most likely due to a compressive nonlinearity. The deviations from \( \pi \) for successive order DPs would be a measure of the phase difference of \( f_1 \) vs \( f_2 \) at the site of the nonlinearity. A simple way to understand this is to imagine that the input/output characteristic of a saturating nonlinearity is described in terms of a power series built of odd powers only. The successive terms alternate sign. Each successive term is the dominant term in the next order DP. The change in sign leads to a change in \( \pi \) radians in the phase. For example, the dominant term generating the \( f_d(-1) \) DP is the cubic term \( x^3 \) and the dominant term generating the \( f_d(-2) \) is \( x^5 \). In a power series expansion of the input/output characteristic of a saturating nonlinearity the coefficient of \( x^3 \) is opposite in sign to the coefficient of \( x^5 \).

The phase of a DP generated by a simple saturating nonlinearity is independent of input level(s). If it is assumed that the DP observed in the ear canal is the sum of wavelets from an ensemble of distributed saturating nonlinearities along the cochlea, and if it is assumed that the relative amplitudes of the wavelets are independent of level, then the phase of the DP measured in the ear canal would be independent of level. Observation of the magnitude and phase when \( a_2 \) is varied and is much less than \( a_1 = \text{const} \) seem to fit the above assumption.

B. Phase observations explainable by two delays

For the two higher-frequency DPs \([f_d(+2) \text{ and } f_d(+3)]\) and, under the condition that \( a_2/a_1 \ll 2 \), for the two lower-frequency DPs \([f_d(-1) \text{ and } f_d(-2)]\) the phase has a

FIG. 12. The three highest magnitude and phase curves (dashed lines) are the \( f_d(-1) \) DP and the three lowest (solid lines) are the \( f_d(-2) \) DP. The nulls in the 3 to 4 kHz range overlap. These DPs were measured simultaneously (\( f_1 \) was decreased by the same amount for each) and, therefore, the point density in the \( f_d(-2) \) DP plots is about one-half the point density in the \( f_d(-1) \) plots. \( a_1 \) and \( a_2 \) are decremented together from 1.11 to 0.54 Pa.

As the levels decrease the frequency of the nulls decrease and the changing pattern in the magnitude curve for the \( f_d(-1) \) DP blends into the pattern for the \( f_d(-2) \) DP.
level independent delay that is approximately equal to $1/f^2$.

It appears that the DPs are generated at the place of the higher-frequency input tone.

However, as $a_2/a_1$ becomes greater than 2 or 3, the delay that is measured for the two lower-frequency DPs becomes greater. This fact, combined with the observation of nulls in the amplitudes, 2 m level-dependent phase bifurcations, and phase patterns such as in Fig. 5 for the two lower-frequency DPs suggest that there is a second "source" of the DPs that has a greater delay. We have shown that the nulls can be modeled by assuming that there is reflection of the DP from the $f_d$ place.

ACKNOWLEDGMENTS

The authors would like to thank L. Stover and S. Neely for discussions on the filter function and the IFFT-waveform technique and to thank C. Shera and C. Talmadge for reading and helping to revise a late draft of this work.


