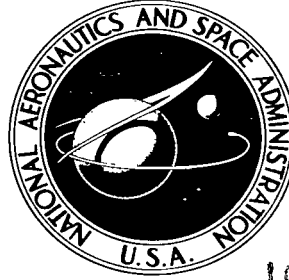


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# HYSTERESIS AND EDDY-CURRENT LOSSES OF A TRANSFORMER LAMINATION VIEWED AS AN APPLICATION OF THE POYNTING THEOREM

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LAMINATION VIEWED AS AN APPLICATION OF  
THE POYNTING THEOREM

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

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# HYSTERESIS AND EDDY-CURRENT LOSSES OF A TRANSFORMER LAMINATION VIEWED AS AN APPLICATION OF THE POYNTING THEOREM

by John Barranger  
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## SUMMARY

An energy-loss equation is developed based on the Poynting theorem. The resultant integrals provide a unified viewpoint that is shown to be the same as the classical eddy-current and hysteresis energy formulations. An expression is derived for the eddy-current loss of a thin transformer lamination by assuming that the permeability is constant, and further, that the magnetic material is isotropic and homogeneous. The hysteresis loss is found by assuming that the introduction of the hysteresis loop does not alter the fields. The results indicate that, for a given total flux and at frequencies approaching total shielding, both losses are proportional to the  $3/2$  power of frequency and the square root of the conductivity and inversely proportional to the square root of permeability.

## INTRODUCTION

As part of a study of the distribution and use of power on spacecraft, the Lewis Research Center has been evaluating power system components in unusual configurations and at temperatures and frequencies outside the ordinary ranges. Early in this study it became evident that neither classical nor modern approaches to magnetism could completely predict transformer properties and their variations with frequency.

The classical approach to the study of losses in ferromagnetic materials is the conventional engineering analysis which assumes the magnetic material to be isotropic and homogeneous and defines the losses in terms of the hysteresis loop and classical electrodynamics. The modern theory of magnetism utilizes crystal energies and magnetic domain configurations to provide an understanding of losses (refs. 1 to 3). The classically derived losses are characterized by measurements that are relatively easy to make while the modern theory relies on crystal anisotropy energies and domain wall velocities that

are very difficult to evaluate and measure in polycrystalline materials.

Traditionally, transformer core losses have been divided into two groups, hysteresis and eddy-current loss. Hysteresis loss is thought to be associated with the energy required to rotate or move the walls of the magnetic domains over a full cycle. The hysteresis loss is proportional to the area of the familiar hysteresis loop. The eddy-current loss is due to the currents generated within a real conductor subjected to a varying magnetic field.

The purpose of this report is (1) to provide a better understanding of classical theory for the readers already familiar with the basic engineering approach and (2) to derive expressions useful in predicting the dependence of core losses on frequency and easily measured properties. Hysteresis and eddy-current losses are usually treated separately with no common underlying basis. A more unified viewpoint will be provided starting with Maxwell's equations and will show that considerations of the Poynting theorem yield the classical core loss formulations. The separation of core losses into their two traditional components results as a natural consequence of the analysis.

A large part of the study is devoted to finding expressions for the hysteresis and eddy-current losses for thin transformer laminations. The magnetic material will be assumed to be isotropic and homogeneous and to have a permeability that is constant. The permeability is obtained from the mean magnetization curve by taking the ratio of the flux density to the corresponding magnetic field intensity. The formulas will then be examined to ascertain the behavior of the losses with frequency. To simplify the resultant expressions, low- and high-frequency approximations will be derived. The results will also be used to predict the important parameters of use in high-frequency transformer design. This should be of interest to designers who are seeking methods of using laminated magnetic material for high-frequency power applications. It is possible that improvements in weight and power efficiencies may be gained through the use of high-frequency power systems.

## SYMBOLS

- a lamination thickness
- B flux density
- b length of lamination
- E potential
- f frequency
- H field intensity
- J current density

$l$	height of lamination
$P$	power loss
$S$	shape factor
$t$	time variable
$V$	volume
$W$	energy
$\delta$	skin depth
$\theta$	dummy variable
$\mu$	permeability
$\rho$	free charge density
$\sigma$	conductivity
$\Phi$	total flux

## POYNTING THEOREM AND CORE LOSSES

Energy transfer from a source to a magnetic material is accomplished through electromagnetic induction, the amount of energy depending on the magnitude, distribution, and phases of the electric and magnetic fields. Maxwell's equations, written in terms of the total fields and currents within the magnetic material, describe the electromagnetic behavior of the material. The following conditions are appropriate to these equations when applied to good conductors:

- (1) The free-charge term is zero,  $\rho = 0$ .
- (2) Conduction current density is given by Ohm's law,  $\vec{J}(t) = \sigma\vec{E}(t)$ .
- (3) Displacement current is negligible in comparison with conduction current.
- (4) The material is isotropic.

The two curl equations become

$$\nabla \times \vec{E}(t) = - \frac{\partial \vec{B}(t)}{\partial t} \quad (1a)$$

$$\nabla \times \vec{H}(t) = \vec{J}(t) \quad (1b)$$

If the divergence of  $\vec{E}(t) \times \vec{H}(t)$  (the Poynting vector) is considered,

$$\nabla \cdot [\vec{E}(t) \times \vec{H}(t)] = \vec{H}(t) \cdot [\nabla \times \vec{E}(t)] - \vec{E}(t) \cdot [\nabla \times \vec{H}(t)] = -\vec{H}(t) \cdot \frac{\partial \vec{B}(t)}{\partial t} - \vec{E}(t) \cdot \vec{J}(t) \quad (2)$$

Integrating over the enclosed volume produces

$$\int_V \nabla \cdot [\vec{E}(t) \times \vec{H}(t)] dV = - \int_V \left[ \vec{H}(t) \cdot \frac{\partial \vec{B}(t)}{\partial t} + \vec{E}(t) \cdot \vec{J}(t) \right] dV \quad (3)$$

and using the divergence theorem results in

$$- \int_S [\vec{E}(t) \times \vec{H}(t)] \cdot d\vec{S} = \int_V \left[ \vec{H}(t) \cdot \frac{\partial \vec{B}(t)}{\partial t} + \vec{E}(t) \cdot \vec{J}(t) \right] dV \quad (4)$$

The total of all the terms to the right of the equal sign are identified as representing the energy stored and dissipated per unit time, the left side of equation (4) represents the energy flow into the volume per unit time. The energy flow into the volume, then, between two points of time  $t_1$  and  $t_2$  is

$$W = \int_{t_1}^{t_2} \int_V \left[ \vec{H}(t) \cdot \frac{\partial \vec{B}(t)}{\partial t} + \vec{E}(t) \cdot \vec{J}(t) \right] dV dt \quad (5)$$

The time rate of change of flux density for a stationary volume is

$$\frac{d\vec{B}(t)}{dt} = \frac{\partial \vec{B}(t)}{\partial t} \quad (6)$$

and

$$W = \int_{t_1}^{t_2} \int_V \left[ \vec{H}(t) \cdot \frac{d\vec{B}(t)}{dt} + \vec{E}(t) \cdot \vec{J}(t) \right] dV dt \quad (7)$$

Since the material was assumed isotropic,  $\vec{B}(t)$  and  $\vec{H}(t)$ ,  $\vec{E}(t)$  and  $\vec{J}(t)$  are in the same direction and the energy expression becomes

$$W = \int_V \int_{t_1}^{t_2} \left[ H(t) \frac{dB(t)}{dt} + \frac{J^2(t)}{\sigma} \right] dt dV = \int_V \left[ \int_{B_1}^{B_2} H dB + \int_{t_1}^{t_2} \frac{J^2(t)}{\sigma} dt \right] dV \quad (8)$$

where  $B_1$  and  $B_2$  are the flux densities corresponding to  $t_1$  and  $t_2$ , respectively. The second term represents the energy loss due to conduction currents or the eddy-current loss. Over a complete cycle, the first term represents the energy loss, which is called hysteresis loss. Thus, according to Maxwell's equations the total loss or core loss is

$$P_c = P_h + P_e \quad (9)$$

where  $P_h$  is the hysteresis loss and  $P_e$  is the eddy-current loss.

Consider now the nonlinearities commonly encountered in ferromagnetic substances. The two significant nonlinearities are

- (1) The hysteresis loop
- (2) The mean magnetization curve

To simplify the analysis, the mean magnetization curve will be assumed to be a straight line, that is, the permeability is a constant. The material will also be assumed to be homogeneous as well as isotropic.

## CALCULATION OF CURRENT AND FIELD

Before any calculation of core loss can be made, the current and field distribution within the volume of the material must be known. This is usually accomplished by solving a differential equation in the quantity of interest. By returning once more to Maxwell's equations, the curl of the magnetizing force  $\vec{H}(t)$  may be written

$$\nabla \times \vec{H}(t) = \sigma \vec{E}(t) \quad (10)$$

and taking the curl of both sides yields

$$-\nabla^2 \vec{H}(t) + \nabla \left[ \nabla \cdot \vec{H}(t) \right] = -\sigma \frac{\partial \vec{B}(t)}{\partial t} \quad (11)$$

Since it is desirable to solve the previous expression in terms of one variable, such as  $B(t)$ , the classic problem of relating  $B(t)$  and  $H(t)$  for ferromagnetic substances must be faced. Even though the hysteresis loop shows that this is a nonlinear functional relation, it will be assumed that the introduction of the loop alters only slightly the distribution and waveshape of the fields. Oriented silicon iron approximates this condition at low to moderate flux density levels. The functional relation then becomes

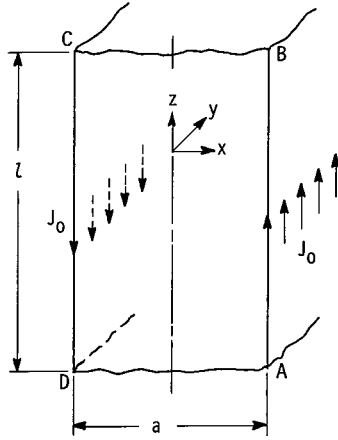


Figure 1. - Lamination of transformer.

$$\vec{B}(t) = \mu \vec{H}(t) \quad (12)$$

where the permeability  $\mu$  is the constant slope of the mean magnetization curve. Substituting, the resultant equation becomes

$$\nabla^2 \vec{B}(t) = \sigma \mu \frac{\partial \vec{B}(t)}{\partial t} \quad (13)$$

Consider the lamination of a transformer as a thin sheet of thickness  $a$  where the height of the lamination  $l$  is much greater than the thickness. Let the field be applied to the surface of the lamination in such a way that the induced eddy currents are as indicated in figure 1. It is necessary to know how these currents are distributed before an expression for power loss can be found.

The equation derived previously for the variation of  $\vec{B}(t)$  in a conductor is in the form of a standard differential equation similar to the diffusion or heat equation. By a similar derivation, the equation that gives the relation between space and time derivatives of current density at any point in a conductor is

$$\nabla^2 \vec{J}(t) = \sigma \mu \frac{\partial \vec{J}(t)}{\partial t} \quad (14)$$

Letting  $\vec{J}(t)$  be sinusoidal and introducing the complex exponential form

$$\vec{J}(t) = \text{Re} \left( \vec{J} e^{j\omega t} \right) \quad (15)$$

results in the previous expression becoming

$$\nabla^2 \vec{J} = j\omega \sigma \mu \vec{J} \quad (16)$$

Study of figure 1 will show that, since no net current can flow out of a conductor, the following symmetry and boundary conditions exist for thin sheets with an axially applied field:

$$(1) J_x = 0$$

$$(2) J_y = 0$$



$$(3) \frac{\partial J_z}{\partial y} = \frac{\partial^2 J_z}{\partial y^2} = 0$$

$$(4) \frac{\partial J_z}{\partial z} = \frac{\partial^2 J_z}{\partial z^2} = 0$$

$$(5) J_z \Big|_{x=(a/2)} = J_0$$

$$(6) J_z \Big|_{x=-(a/2)} = -J_0$$

By using the first four conditions, the current distribution equation becomes simply

$$\frac{d^2 J_z}{dx^2} = j\omega\sigma\mu J_z = T^2 J_z \quad (17)$$

where  $T$  is a constant and is equal to (taking the root with positive sign)

$$T = (1 + j) \sqrt{\pi f \mu \sigma} \quad (18)$$

It is convenient to write the solution of the differential equation in terms of hyperbolic functions

$$J_z = A \sinh Tx + C \cosh Tx \quad (19)$$

To evaluate arbitrary constants  $A$  and  $C$ , boundary conditions (5) and (6) (above) are utilized

$$J_0 = A \sinh T \frac{a}{2} + C \cosh T \frac{a}{2} \quad (20a)$$

$$-J_0 = -A \sinh T \frac{a}{2} + C \cosh T \frac{a}{2} \quad (20b)$$

Solving these results in

$$C = 0 \quad (21a)$$

$$A = \frac{J_0}{\sinh T \frac{a}{2}} \quad (21b)$$

and

$$J_z = J_0 \frac{\sinh Tx}{\sinh T \frac{a}{2}} \quad (21c)$$

Defining the quantity  $\delta$  as  $1/\sqrt{\pi f \mu \sigma}$ , the current density distribution may be written as

$$J_z = J_0 \frac{\sinh\left(\frac{1+j}{\delta}x\right)}{\sinh\left(\frac{1+j}{\delta}\frac{a}{2}\right)} \quad (22)$$

where  $\delta$  is usually called the skin depth or depth of penetration. In terms of a semi-infinite solid, it is the depth at which current density has decreased to  $1/e$  (about 36.9 percent) of its value at the surface. There are a number of important factors associated with the skin depth; they are the following:

- (1) The depth of penetration decreases as the conductivity, the permeability and the frequency increases. Thus, materials of high permeability have a smaller depth of penetration, or conversely, skin effect takes place at a lower frequency.
- (2) The current does not fail to penetrate below the depth  $\delta$ ; this is merely the point at which current densities and fields have decreased to  $1/e$  of their value at the surface.
- (3) The skin depth may be considered a constant for a given material at a frequency  $f$  and is a useful parameter in the analysis of thin sheets.

The current distribution at low frequencies may be obtained by making the ratio  $a/2\delta$  much less than one, that is, the skin depth is much greater than the thickness. Taking the absolute value of the current density gives

$$|J_z| = |J_0| \left| \frac{\frac{\sinh \frac{x}{\delta} \cos \frac{x}{\delta} + j \cosh \frac{x}{\delta} \sin \frac{x}{\delta}}{\sinh \frac{a}{2\delta} \cos \frac{a}{2\delta} + j \cosh \frac{a}{2\delta} \sin \frac{a}{2\delta}}}{\left( \frac{\sinh^2 \frac{x}{\delta} + \sin^2 \frac{x}{\delta}}{\sinh^2 \frac{a}{2\delta} + \sin^2 \frac{a}{2\delta}} \right)^{1/2}} \right| \quad (23)$$

and if  $a/2\delta \ll 1$  and  $x/\delta \ll 1$ ,

$$|J_z| = |J_0| \frac{x}{\left(\frac{a}{2}\right)} \quad (24)$$

This distribution is linear with  $x$ . There remains only  $|J_0|$  to be evaluated.

Using Faraday's line integral and integrating around the path ABCD in figure 1 yield

$$\oint \vec{E}(t)_0 \cdot d\vec{l} = - \frac{d\Phi(t)}{dt} \quad (25)$$

where  $\Phi(t)$  is the total flux within the line ABCD. Multiplying both sides by conductivity  $\sigma$  and using the relation  $\vec{J}(t)_0 = \sigma \vec{E}(t)_0$  make the following obtainable:

$$\oint \vec{J}(t)_0 \cdot d\vec{l} = -\sigma \frac{d\Phi(t)}{dt} \quad (26)$$

If the sheet has height  $l$  and the height is much greater than the thickness, the current density  $J_0$  in complex notation form is

$$J_0 = \frac{j\omega\sigma\Phi}{2l} \quad (27)$$

If the absolute value is taken

$$|J_0| = \frac{\omega\sigma|\Phi|}{2l} \quad (28)$$

then

$$|J_z| = \frac{\omega\sigma|\Phi|}{2l} \left( \frac{\sinh^2 \frac{x}{\delta} + \sin^2 \frac{x}{\delta}}{\sinh^2 \frac{a}{2\delta} + \sin^2 \frac{a}{2\delta}} \right)^{1/2} \quad (29)$$

The previous expression represents the current density distribution, which will be used to find the eddy-current loss. In the analysis of hysteresis loss, the flux density distribution must also be known. This may be found directly from the current density expression. Consider the curl equation

$$\nabla \times \vec{E}(t) = - \frac{\partial \vec{B}(t)}{\partial t} \quad (30)$$

Using the complex exponential form

$$\nabla \times \vec{J} = - j\omega\sigma\vec{B} \quad (31)$$

and noting that  $J_z$  is the only component of  $\vec{J}$  result in

$$- \frac{\partial J_z}{\partial x} = - j\omega\sigma B_y \quad (32)$$

or

$$B_y = - \frac{j}{\omega\sigma} \frac{\partial J_z}{\partial x} \quad (33)$$

If the formulas for  $J_z$  and  $J_o$  found previously are now substituted,

$$B_y = - \frac{j}{\omega\sigma} \frac{\partial}{\partial x} \left( J_o \frac{\sinh Tx}{\sinh T \frac{a}{2}} \right) = \frac{\Phi T}{2l} \left( \frac{\cosh Tx}{\sinh T \frac{a}{2}} \right) \quad (34)$$

The absolute value of the flux density becomes

$$|B_y| = \frac{\sqrt{2} |\Phi|}{2l\delta} \left( \frac{\cos^2 \frac{x}{\delta} + \sinh^2 \frac{x}{\delta}}{\sin^2 \frac{a}{2\delta} + \sinh^2 \frac{a}{2\delta}} \right)^{1/2} \quad (35)$$

## EDDY-CURRENT LOSS

The first section showed that the eddy-current loss may be expressed as

$$\int_V \int_{t_1}^{t_2} \frac{J^2(t)}{\sigma} dt dV \quad (36)$$

It can be shown that the average power loss in complex notation is

$$P_e = \int_V \frac{|J|^2}{2\sigma} dV \quad (37)$$

If the element of volume is  $b l dx$  (fig. 2), then the total power may be expressed as

$$\begin{aligned} P_e &= 2 \int_0^{a/2} \frac{|J_z|^2}{2\sigma} b l dx = \frac{b l}{\sigma} \left( \frac{\omega \sigma |\Phi|}{2l} \right)^2 \frac{1}{\sinh^2 \frac{a}{2\delta} + \sin^2 \frac{a}{2\delta}} \int_0^{a/2} \left( \sinh^2 \frac{x}{\delta} + \sin^2 \frac{x}{\delta} \right) dx \\ &= \frac{\omega^2 |\Phi|^2 \sigma b \delta}{8l} \left( \frac{\sinh \frac{a}{\delta} - \sin \frac{a}{\delta}}{\cosh \frac{a}{\delta} - \cos \frac{a}{\delta}} \right) \end{aligned} \quad (38)$$

The previous expression represents the power loss in a transformer lamination due to eddy currents. Latour (ref. 4) considered the same problem from the standpoint of a constant lag of phase angle between the magnetic induction  $\vec{B}(t)$  and magnetizing field  $\vec{H}(t)$ . His loss formula is the same as the one derived (eq. 38). For a given lamination material, thickness, and frequency, the loss varies as  $\Phi^2$ . Simpler expressions may

be obtained by examining the behavior of the equation as a function of frequency. At low frequencies or when  $a/\delta \ll 1$ , the power loss is

$$P_{e(L)} = \frac{\omega^2 \Phi_{\max}^2 \sigma b a}{24 l} \quad (39)$$

where  $\Phi_{\max}$  is the maximum value of total flux. If  $B$  is assumed to be uniform at power frequencies, the average power per unit volume yields the familiar result

$$p_{e(L)} = \frac{\pi^2 f^2 B_{\max}^2 a^2 \sigma^2}{6} \quad (40)$$

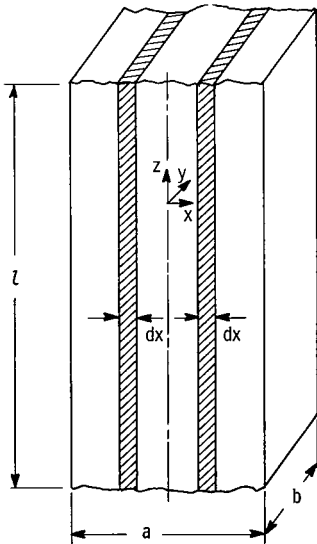


Figure 2 - Lamination of transformer showing element of volume.

At frequencies where skin effect predominates, that is,

when  $a/\delta \gg 1$ , the power loss is

$$P_{e(H)} = \frac{\omega^2 \Phi_{\max}^2 \sigma b \delta}{8l} = \frac{\pi^{3/2} f^{3/2} \Phi_{\max}^2 \sigma^{1/2} b}{2l \mu^{1/2}} \quad (41)$$

and the average power per unit volume is

$$p_{e(H)} = \frac{\pi^{3/2} f^{3/2} \Phi_{\max}^2 \sigma^{1/2}}{2l^2 a \mu^{1/2}} \quad (42)$$

Thus, if the sinusoidal field is regulated so that there is constant  $\Phi_{\max}$ , the eddy-current loss varies as the square of the frequency at power frequencies and as the 3/2 power of frequency at frequencies approaching total eddy-current shielding.

## HYSTERESIS LOSS

It was stated previously that the integral

$$\int_V \int_{B_1}^{B_2} H \, dB \, dV$$

represents the hysteresis energy loss when it is taken over a full cycle, that is,

$$W(\text{per cycle}) = \int_V \left[ \oint (H \, dB) \right] dV \quad (43)$$

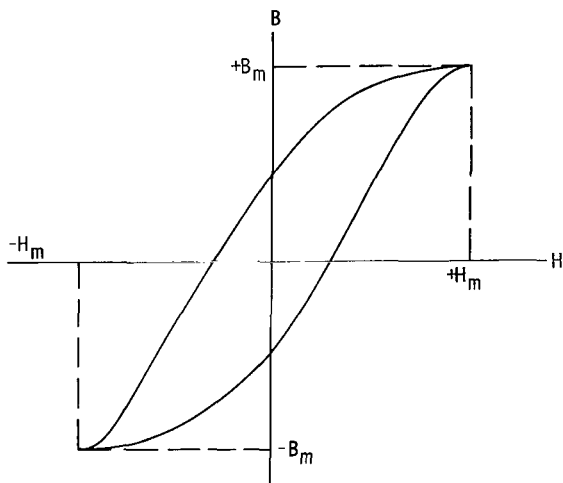


Figure 3. - Hysteresis loop.

The bracket represents the area of the hysteresis loop for the element  $dV$  under consideration. If the area is expressed as a function of the variable  $V$ , then the formula may be integrated over the entire volume to yield the total hysteresis loss per cycle. The  $H$  and  $B$  within the bracket are obtained from the static hysteresis loop of the material. The end point

of the hysteresis loop corresponds to the maximum value of  $B$ . Thus, the solution of the differential equation in  $\bar{B}$  may also be used to describe this point. The area of the hysteresis loop will be found first for the surface of the sheet, and then the results will be extended to provide an expression for the area at any point.

The hysteresis loop for the surface is given in figure 3. To find the area of this loop, a dummy variable  $\theta$  defined by the following equation must be introduced:

$$B = -B_m \cos \theta \quad (44)$$

where  $B_m$  is the maximum value of  $B$ . If  $\theta$  is limited to a half period, the magnetizing force  $H$ , may be described by the half-range Fourier expansion

$$H = a_0 H_m + a_1 H_m \cos \theta + a_2 H_m \cos 2\theta + \dots + a_n H_m \cos n\theta \quad (45)$$

where  $H_m$  is the maximum value of  $H$ . Examination of figure 3 shows that, as the flux density traverses the loop from  $-B_m$  to  $+B_m$  and back to  $-B_m$ , the area over the complete cycle is twice the area between  $-B_m$  to  $+B_m$ . Thus,

$$\oint H dB = 2 \int_{-B_m}^{+B_m} H dB \quad (46)$$

and if it is noted that  $\theta = 0$  and  $\theta = \pi$  correspond to  $-B_m$  and  $+B_m$  respectively, the area becomes

$$\begin{aligned} &= 2 \int_0^\pi H_m B_m \sin \theta (a_0 + a_1 \cos \theta + a_2 \cos 2\theta + \dots + a_n \cos n\theta) d\theta \\ &= 2H_m B_m \left( 2a_0 - \frac{2}{3} a_2 - \frac{2}{15} a_4 - \dots + \frac{2a_n}{1 - n^2} \right) \quad \text{for } n \text{ even} \\ &= 2H_m B_m \left( \sum_0^n \frac{2a_n}{1 - n^2} \right) \quad \text{for } n \text{ even} \end{aligned} \quad (47)$$

Let the summation within the parentheses be called the shape factor  $S$ . The value of the shape factor may be determined by solving the parametric equations for  $B$  and  $H$  for the hysteresis loop being investigated. The number of terms in the summation depends

on the number of data points and the closeness of fit desired. Since the permeability was assumed constant,  $B_m$  and  $H_m$  are related by

$$B_m = \mu H_m \quad (48)$$

and the area of the hysteresis loop is  $2B_m^2 S / \mu$ .

The hysteresis energy integral can now be solved

$$W(\text{per cycle}) = \int_V \left[ \oint (H \, dB) \right] dV = \int_V \frac{2B_m^2 S}{\mu} dV \quad (49)$$

According to figure 2 (p. 11),

$$W(\text{per cycle}) = 4lb \int_0^{a/2} \frac{B_m^2 S}{\mu} dx \quad (50)$$

and since  $S$  and  $\mu$  are assumed constant, they will not depend on the coordinate  $x$ . Thus,

$$W(\text{per cycle}) = \frac{4lbS}{\mu} \int_0^{a/2} B_m^2 dx \quad (51)$$

It is now necessary to find  $B_m$  in terms of  $x$  to integrate the expression. It was found previously that

$$|B_y| = \frac{\sqrt{2} |\Phi|}{2l \delta} \left( \frac{\cos^2 \frac{x}{\delta} + \sinh^2 \frac{x}{\delta}}{\sin^2 \frac{a}{2\delta} + \sinh^2 \frac{a}{2\delta}} \right)^{1/2} \quad (52)$$

and by noting that

$$|B_y| = B_m \quad (53)$$

$B_m^2$  may now be solved



$$B_m^2 = \frac{2|\Phi|^2}{(2l\delta)^2} \left( \frac{\cos^2 \frac{x}{\delta} + \sinh^2 \frac{x}{\delta}}{\sin^2 \frac{a}{2\delta} + \sinh^2 \frac{a}{2\delta}} \right) \quad (54)$$

Substituting into the energy equation yields

$$\begin{aligned} W(\text{per cycle}) &= \frac{2lbS}{\mu} \left( \frac{|\Phi|}{l\delta} \right)^2 \frac{1}{\sinh^2 \frac{a}{2\delta} + \sin^2 \frac{a}{2\delta}} \int_0^{a/2} \left( \cos^2 \frac{x}{\delta} + \sinh^2 \frac{x}{\delta} \right) dx \\ &= \frac{bS|\Phi|^2}{\mu l\delta} \left( \frac{\sinh \frac{a}{\delta} + \sin \frac{a}{\delta}}{\cosh \frac{a}{\delta} - \cos \frac{a}{\delta}} \right) \end{aligned} \quad (55)$$

The hysteresis loss is  $f$  times the energy lost per cycle. Thus,

$$P_h = fW = \frac{fbS|\Phi|^2}{\mu l\delta} \left( \frac{\sinh \frac{a}{\delta} + \sin \frac{a}{\delta}}{\cosh \frac{a}{\delta} - \cos \frac{a}{\delta}} \right) \quad (56)$$

The previous expression represents the power loss in a transformer lamination due to hysteresis effects. Latour (ref. 4) found the same formula except that this formula for hysteresis loss has the added advantage of being defined in terms of data taken from the static hysteresis loop. At power frequencies or when  $a/\delta \ll 1$ , the power loss is

$$P_{h(L)} = \frac{2fbS\Phi_{\max}^2}{\mu la} \quad (57)$$

If  $B$  is assumed uniform at power frequencies, the average power per unit volume yields the result

$$P_{h(L)} = \frac{2fSB_{\max}^2}{\mu} \quad (58)$$

which again is frequency times the area of the hysteresis loop.

At frequencies where skin effect predominates (i. e. , when  $a/\delta \gg 1$ ) the power loss is

$$P_{h(H)} = \frac{fbS\Phi_{\max}^2}{\mu l \delta} = \frac{\pi^{1/2} f^{3/2} b S \Phi_{\max}^2 \sigma^{1/2}}{l \mu^{1/2}} \quad (59)$$

and the average power per unit volume is

$$P_{h(H)} = \frac{\pi^{1/2} f^{3/2} S \Phi_{\max}^2 \sigma^{1/2}}{\mu^{1/2} l^2 a} \quad (60)$$

Thus, if the sinusoidal field is regulated so that there is constant  $\Phi_{\max}$ , the loss varies as  $f^{3/2}$ .

## TOTAL CORE LOSS

At high frequencies, the hysteresis and eddy-current losses can be combined to form the total core losses

$$P_c(H) = P_e(H) + P_h(H) = \frac{\pi^2 f^2 \bar{B}_{\max}^2 \sigma a \delta}{2} + \frac{f S \bar{B}_{\max}^2 a}{\mu \delta} \quad (61)$$

where the average flux density  $\bar{B}_{\max}$  is defined as

$$\bar{B}_{\max} = \frac{\Phi_{\max}}{l a} \quad (62)$$

After substitution, the previous equation yields

$$P_c(H) = \frac{\pi^{1/2} f^{3/2} \bar{B}_{\max}^2 \sigma^{1/2} a}{\mu^{1/2}} \left( \frac{\pi}{2} + S \right) \quad (63)$$

For a given  $\bar{B}_{\max}$  and frequency, then, the core loss at high frequencies is proportional

to the square root of conductivity and inversely proportional to the square root of the permeability. Clearly, as the frequency is increased, the eddy-current loss varies from the square to the  $3/2$  power of frequency, while the hysteresis loss varies from the first to the  $3/2$  power of frequency.

## CONCLUDING REMARKS

It has been demonstrated that the use of Maxwell's equations and the Poynting theorem yield the classical eddy-current and hysteresis loss formulations. The resultant integrals were evaluated to find expressions for each loss for a thin transformer lamination by assuming that the permeability was constant, and further, that the magnetic material was isotropic and homogeneous. The expressions were then simplified to yield low-frequency formulas. The hysteresis loss was found to be proportional to the area of the hysteresis loop, while the eddy-current loss became the conventional engineering expression.

By making a high-frequency approximation, it has been found that for a given average  $B_{\max}$  and frequency, the core loss at high frequencies is proportional to the conductivity and inversely proportional to the permeability. Thus, to minimize losses, high-frequency transformer laminations should be made of a material that has high resistivity and high permeability.

Examination of the core loss expressions also showed that for a given lamination material, thickness, and  $\Phi_{\max}$ , as the frequency is increased, the eddy-current loss varies from the square to the  $3/2$  power of frequency while the hysteresis loss varies from the first to the  $3/2$  power of frequency. This means that the eddy-current loss contribution to the total loss is less than would be expected from the power frequency expression and that the hysteresis-loss contribution to the total loss is greater than would be expected from the power-frequency considerations alone. Thus, hysteresis loss plays an important role in transformer core loss at high as well as low frequencies.

Since the expressions for the thin transformer lamination assumed that the permeability was constant, application of the loss formulas without regard to variations of permeability with the applied field and saturation effects will result in large errors. It is felt that by maintaining the same basic approach, an approximation scheme could be devised that would take the true permeability into account (refs. 5 to 7). The assumption that the material is isotropic and homogeneous would still have to be made.

Lewis Research Center,  
National Aeronautics and Space Administration,  
Cleveland, Ohio, September 1, 1965.

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3/22/57

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