

# Axial and transverse forces in an axisymmetric suspension using permanent magnets

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## Abstract

This paper presents the evaluation of magnetic forces in an axisymmetric spring, used to levitate a small platform. The main components of the forces are obtained with Nd–Fe–B magnets and the equilibrium force in the non-stable axial direction is provided by electromagnets. Coaxial cylinders are used to align the vertical axis. The extra force necessary to stabilize the longitudinal position is obtained with solenoids. The finite-element method is employed to determine the magnetic field configuration. To simplify the force evaluation the properties of the Nd–Fe–B magnets are taken into account. The material magnetization is replaced by equivalent surface currents and the material permeability is assumed to take the value of the vacuum permeability.

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## 1. Introduction

This paper presents the study of the axial and transverse components of the magnetic force that are obtained using permanent magnets with an axisymmetric shape. The purpose of this work is to develop a magnetic spring which will be part of a levitating mechanism. However, it is well known, that it is not possible to maintain a magnet in stable equilibrium using other magnets to levitate it, due to the conservative nature of the field originating by permanent magnets. This result was presented for the first time by Earnshaw [1] and is generally referenced to as the Earnshaw theorem. It may be shown that such systems always present at least one unstable axis. To overcome this difficulty, our system is stable in all directions except one, the vertical direction, along which the load is applied. To attain the equilibrium in this direction we utilize an electromagnet whose current is feedback controlled in order to obtain the stabilizing force.

At the present time several different examples of levitating systems can be found in the literature. Some of them use diamagnetic materials to stabilize small levitated magnets [2]. The forces which are obtained with these materials are so small that they have been used only to construct small toys. With high-temperature superconductors it is possible to construct levitating devices with practical applications, due to the Meissner effect. A good review of the different kinds of levitating systems can be found in the paper of Ing-Yann [3].

## 2. Axial force between magnetized disks

In one of our systems we use magnetized disks to produce the main levitating force. The finite-element (FEM) method [4] was employed to determine the magnitude of the interaction force between two coaxial disks magnetized in attractive or repulsive configurations.

The evaluation of the magnetic field, from a single uniformly magnetized disk, was performed by using the equivalence represented in Fig. 1. The resulting field was obtained in terms of the vector magnetic potential which,

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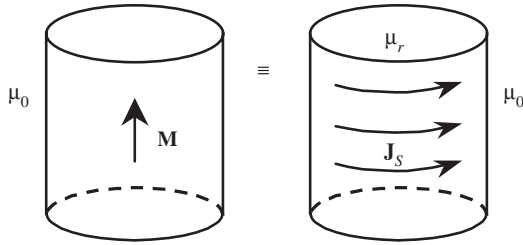


Fig. 1. Equivalence between uniform magnetization and virtual surface currents to produce the magnetic field.

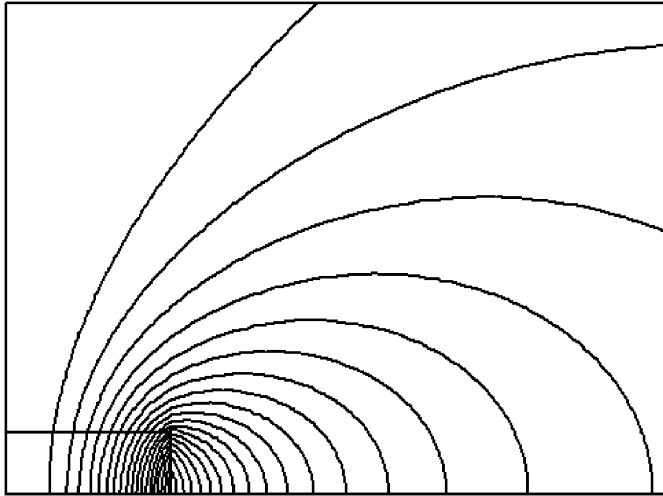


Fig. 2. Representation of the isoflux lines of one quarter of the plane containing the symmetry axis.

in this case, presents a single component  $A_\phi$  in the azimuthal direction. The equation to be solved is,

$$-\frac{\partial}{\partial \rho} \left[ \frac{1}{\rho} \frac{\partial(\rho A_\phi)}{\partial \rho} \right] - \frac{\partial^2 A_\phi}{\partial z^2} = \mu_0 J_\phi, \tag{1}$$

where  $\rho$  is the radial distance to the symmetry axis. In the left-hand side of Fig. 2 (symmetry axis) a homogeneous Dirichlet condition is imposed, whereas in the lower side of the same figure a Neumann homogeneous boundary condition is considered. For points at a distance greater than the disk dimensions it is possible to calculate analytically the magnetic potential [5]. Therefore Eq. (2), was used to introduce appropriate boundary conditions in our FEM processor.

$$A_\phi = \frac{\mu_0 J_S R}{2\pi} \int_{-T/2}^{T/2} \int_{\alpha=0}^{\pi} \frac{\cos \alpha \, d\alpha}{(r^2 + R^2 - 2Rr \sin \theta \cos \alpha)^{1/2}} \, dz. \tag{2}$$

In Eq. (2),  $R$  and  $T$  are the disk radius and thickness,  $J_S$  is the virtual surface current density which must equal the material magnetization  $M = 979 \text{ kA/m}$ ,  $r$  and  $\theta$  are the distance and zenithal angle between each disk layer and the point where  $A_\phi$  is being calculated. With these conditions the configuration of the magnetic field, represented in Fig. 2 as a set of isoflux lines, was obtained.

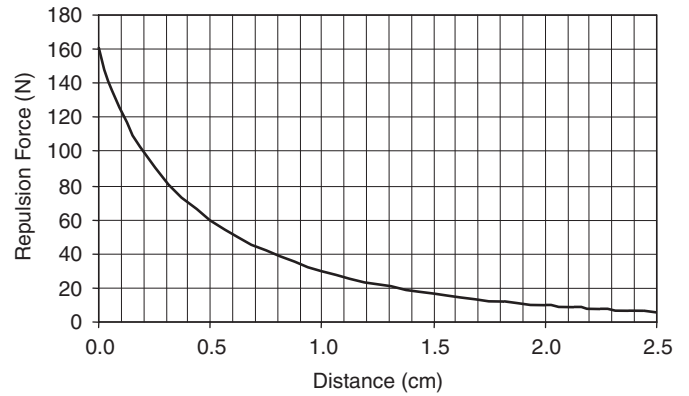


Fig. 3. Repulsive force between two disks as a function of the separation distance between them.

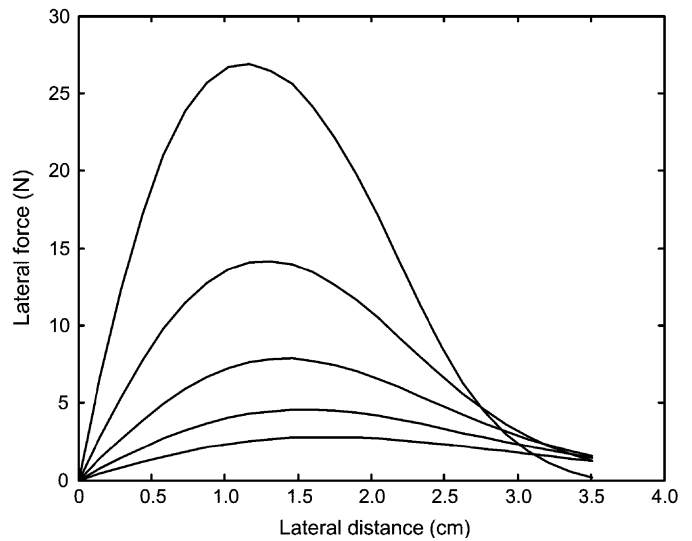


Fig. 4. Lateral forces for two misaligned disks, for longitudinal separations increasing of 0.5 cm between curves.

The axial force between two coaxial magnets is given by integration of the external product between the magnetic induction from one magnet and the virtual currents of the other one. It is possible to show [6] that the axial force is proportional to the flux  $\psi_L$ , produced by one of the magnets, that passes through the lateral surface of the other magnet.

$$F = 2\pi J_S R \Delta A_\phi = J_S \psi_L. \tag{3}$$

For Nd–Fe–B magnets with a radius  $R = 1.25 \text{ cm}$  and a thickness  $T = 1.0 \text{ cm}$  the axial force versus separation distance is shown in Fig. 3.

### 3. Lateral force between magnetized disks

When the two disks are perfectly aligned there is no lateral force. However this is an unstable situation. In fact, the lateral forces are very strong when the alignment between the two disks fails. In Fig. 4 we represent the lateral force as a function of the lateral distance between

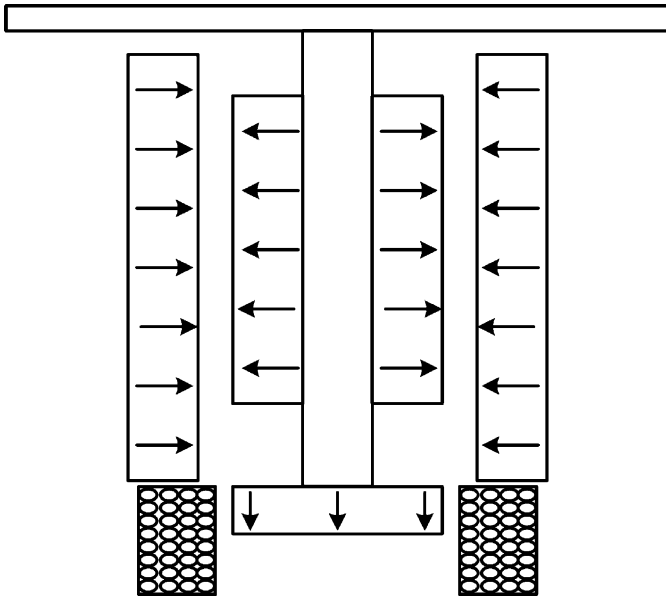


Fig. 5. Cross-section of the magnetic spring with magnets and a bottom solenoid.

the axes of the two magnets considered before. The represented curves correspond to different distances between the disks in the axial direction. Due to the difficulty in the stabilization of such lateral forces, this kind of magnets can be used in an attractive mode, because in this way the instability is only kept in the axial direction. However, the repulsive mode is still possible if a centering device is included.

#### 4. Inclusion of centering magnets

We know that two coaxial cylinders which are radially magnetized in opposite directions, as shown in Fig. 5, tend to align their axes. Being unstable along the common axis, it is possible to use this configuration alone, or in conjunction with opposite repelling disks, to obtain a structure whose unstable movement is controllable. We decided to use the radial magnets to center and levitate because, as shown in the Fig. 6, we obtain a vertical force which is large enough for our purposes. To stabilize the structure, a solenoid is placed in the bottom level which interacts with a magnetized disk.

#### 5. Conclusion

We designed and constructed a levitating device which works as a magnetic spring. A number of these springs will

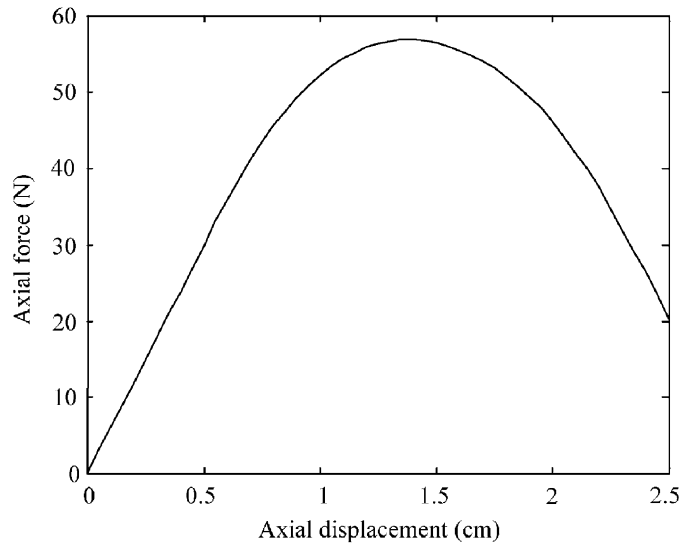


Fig. 6. Levitating force for the structure represented in Fig. 5, with no current in the bottom solenoid.

be used later to levitate and stabilize a platform. In the design, the FEM was used to evaluate the fields and the resulting forces. The geometry of our problem is axisymmetric with open boundaries. Being the processor developed by ourselves, we could consider closed boundaries. The appropriate Dirichlet boundary conditions were calculated analytically as far-field values. The forces were also measured, when possible, and the experimental data agreed well with the numerical results.

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