Loudness and the JND

An introduction to loudness Psychophysics

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ECE-437
The intensity JND is internal uncertainty

- Perception is stochastic: Each time you hear (see) the same short tone (light) pulse, you hear (see) it with a different loudness (brightness)

- The intensity $\text{JND}_I (\Delta I)$ is a measure of this internal perceptual fluctuation (noise) given by $\sigma_L$

\[ \Phi \xrightarrow{\text{"Codec Model"}} \text{OBSERVER} \xrightarrow{} \Psi \]

\( (e.g.: I, \sigma_I) \)

\( (e.g.: L, \sigma_L) \)

- The Loudness JND $\Delta L \propto \sigma_L(L)$
- The loudness JND is proportional to the internal "loudness noise"
Weber’s Law (1846)

- In 1846 Weber showed experimentally that $\Delta I \propto I$
- $I$ is the physical intensity, and $\Delta I$ is called the JND
- Weber used weights of varying relative mass
- Def: $\Delta I/I$ is called the Weber Fraction
- Def: Weber’s Law says the Weber fraction is constant
- Weber’s law sometimes holds:
  - Wide band noise Intensity discrimination (Miller, 1947)
  - The Weber fraction is not constant for pure tones (Riesz, 1928).
- A floating point converter obeys Weber’s Law.
Pure-tone intensity discrimination

- Weber’s “law” says that $\Delta I \propto I$
- Weber’s Law holds for floating point conversion
- For fixed point, $\sigma_I = \Delta I$ is a constant
- Is the ear a fix or floating point converter?

1928 Riesz establishes the near-miss to Weber’s law for tones
Weber’s Law (1846)

**PROBLEM:** Weber formulated his problem in the physical domain, but the noise is internal
Near-miss to Weber’s Law (1846)

Riesz used two beating tones 3 Hz apart for this measurement (i.e., 1000 Hz masker and a low-level 1003 Hz probe)

The near-miss to Weber’s Law results from the fact that the internal noise $\sigma_L \propto \Delta L(L)$ is not independent of $L$.

In fact noise is Poisson-like: [Allen and Neely 1997]

$$\Delta L(L) \approx \sqrt{L}$$
Fechner’s Hypothesis (1860)

- Fechner 1860 is called the father of psychophysics.
- *Fechner’s hypothesis* (or postulate) was that the loudness JND $\Delta L(I)$ is constant:

$$ \Delta L(I, \ell) $$

- Fechner assumed “that the total change in loudness between two intensities $I_1$ and $I_2$ may be found by counting the number of JNDs.”
- From *Fechner’s hypothesis* and the “counting formula:”

$$ N_{JND} \equiv \int_{L_1}^{L_2} \frac{dL}{\Delta L} = (L_2 - L_1) / \Delta L $$
Fechner’s JND theory

Fechner’s idea was that the loudness $\mathcal{L}(I)$ is proportional to the number of JND steps $N_{\text{JND}}$, which is given by:

$$N_{\text{JND}} \equiv \int \frac{d\mathcal{L}}{\Delta \mathcal{L}(\mathcal{L})} = \int \frac{dI}{\Delta I(I)}$$

He assumed that $\Delta I \propto I$, i.e. Weber’s Law.

He assumed that the internal noise $\Delta \mathcal{L} = \sigma_{\mathcal{L}}$ is constant.

These two assumptions give Fechner’s “Law”:

$$\mathcal{L}(I) \propto \log(I)$$
Fechner’s JND theory

- Counting JNDs is a great conceptual start :o)
- Both assumptions
  - Weber’s Law
  - Fechner’s Hypothesis
- are wrong :o(
- Fechner’s “Law” is wrong
L. L. Thurstone 1927 and later David Green 1965:
Formally define the intensity JND as “the relative signal level for detection 75% of the time”

\[ \Delta I \propto \sigma_I \]
## CHRONOLOGICAL DEVELOPMENT

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Loudness Additivity

- Fletcher and Munson 1933 showed that loudness adds
- Adjust $I_2$ so that: $\mathcal{L}(I_1, f_1) = \mathcal{L}(I_2, f_2)$
- Two equally loud tones, played together are twice as loud: $\mathcal{L}(I_1, I_2, f_1, f_2) = 2\mathcal{L}(I_1, f_1)$
- Find gain $\alpha(I)$ such that
  \[ \mathcal{L}(\alpha I_1, f_1) = 2\mathcal{L}(I_1, f_1) \]
- **Results:** $\alpha$ is about 9 dB (actually it depends on intensity)
Loudness additivity

- Fletcher and Munson’s 1933 loudness growth data based on loudness additivity is now called:
  \[ L(I) = I^\nu, \text{ with } \nu \approx 1/3 \]

- Loudness vs. intensity for 1, 2, and 10 equally loud components:
BASIC MODEL OF OBSERVER

Transformation from $\Delta I(I)$ to $\Delta L(L)$

Fechner’s Hypothesis: $\Delta L = \text{const.}$

PIN model: $\Delta L = \sqrt{L}$

WEBER’S LAW: $\Delta I / I = \text{const.}$