ECE 298JA Evaluation of Exams Fall 2017

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Abstract

During the Fall 2017 semester, ECE 298JA, Concepts in mathematics was taught for a third time. Twenty-three students finished the course, with 10: A+, 7: A, 4: A-, 1: B+ and 1: B. The class consisted of 9 Seniors, 5 Juniors, 5 Sophomores and 5 Freshmen. The course was designed to give students who were self-identified as having high-performance in STEM course work, a positive rigorous work out in mathematical physics concepts. While many of the student felt the course was difficult, they out performed their highest expectations. The primary goal is to dramatically increase their knowledge of the math they need to succeed in ECE. An important secondary goal is to raise their self confidence in Math and Physics. It will be interesting to track these students to see how they perform in their future course work. For an insiders view, visit: http://www.istem.illinois.edu/news/jont.allen.html.

1 Summary of Student performance

The following is a summary of the student performance on the three midterms and final exam. The final letter grades are provided and explained in the caption of Table A.1. This report extends and updates the Dec 2016 summary of a questionnaire that was administered to the class with a new questionnaire of Nov 13, 2017, administered immediately following midterm 3, 2017.

Performance depended critically on 1) Attending class, where the basic concepts were first developed 2) Doing the homework assignments, where they were discussed, in finer detail 3) Doing the assignments on time, since each exam depended critically on the homework 4) Attending office hours when a concept of assignment was not clear, or was difficult. 5) Reading the class notes before each lecture.

Class attendance was generally good, with only a few students missing the lectures. For example, one student never came to class (A, Jr), one always came to class except for the last two weeks (A+, Soph). Two came to class about 50% of the time (A-, Jr) and (A+, Sr). The remaining 19 students attended regularly, with a good excuse (e.g., a class conflict) when they were a no-show.

The assignments were generally handed in on time, and the solution was given out on the day the HW was due. Thus students had access to the solutions on the day they were due. Occasionally a student would hand in the HW after the due date. No points were taken off for this. In the end, to get a good grade, the assignments needed to be mastered, as they were tested on the three midterms and the final. There were too many questions, and the assignments were difficult, that memorization was not practical (nor possible).

Reading assignments were the written matching component of the lectures. For example, Lecture 20 was assigned for reading on the day that Lecture 20 was delivered. Reviews of previous ideas were frequent, and typically restricted to the first 20 mins of the 50 min lecture period. There were three lectures per week. When a student handed in their completed exam, a written solution was provided, which they were to keep for their records. There were three midterms, one after each of the main sections, and a final. A lecture was canceled on the day of an exam. The three main sections were I number systems, II Algebra, III Scalar calculus and IV Vector calculus. The three midterms covered I, II and III while the final exam covered all four sections. Thus section IV was only tested for the Final whereas the other sections were tested twice.

1.1 Analysis of the final exam

The scores [%] for each student for the three midterms and final are provided in Table A.1 of the Appendix. The final scores for the 23 students are also presented as histograms in Fig. 1. The rows are the students and the columns are the

1http://auditorymodels.org
2https://jontalle.web.engr.illinois.edu/uploads/298.17/Grades.16
Figure 1: This figure shows the histograms of the final grades (Appendix Table A.1), along with a breakdown of the 5 question areas: I. Number Systems (NumSys), II. Algebra (AlgEq), III. Scalar Calculus (DiffEq), IV. Vector calculus (VecCalc), and V. History (Hist). The first three sections were previously tested on midterm exams (Exam 1, 2, 3), and covered again in the final, except for last section, VecCalc, which was tested for the first time on the final. The highest performance was on the NumSys and Hist questions. For example, for the NumSys questions, 14 students had a perfect score. Only 4 students failed to answer all 10 history questions. The most difficult set of questions were in problem 4 (VecCalc) (which had not been previously tested). This section asked detailed questions about Gauss’ and Stokes’ laws, Maxwell’s equations, and fundamental theorems of vector calculus (Appendix A.3). But even given the difficulty of the VecCalc questions, 18 students had a score above 78%, while 5 (< 1/3) had a score below 70%. Note that the abscissa and ordinate scales are different for each panel.

scores of the three midterms and final, the weighted average, and the final grade.

Overall the final grades were high, with 19 out of 23 students receiving an A (Fig. 1). It is instructive to review the questions for Part IV, along with the 10 assignments, to fully appreciate the accomplishment.

The three primary areas of NumSys, AlgEq and DiffEq are above 90% correct. These were the topics of the earlier three midterms. The problems for parts I, II and III of the final were taken from the three midterm exams. Vector Calculus (VC, Part IV) was tested for the first time on the final. All the VC questions were based on two HW assignments VC1 and VC2, which may be downloaded from the class website. Copies of the final, the three midterms and all assignments, with solutions, are available from the author.

There were 4 parts to VecCalc (IV) portion of the exam: 1) Maxwell’s Equations (8 pts), 2) Scalar fields (6 pts), 3) Webster Horn equation (3 pts) and 4) Vector Algebra (3 pts), summing to 20 out of 98 total points (20%). A compressed version of Parts IV (VecCalc) and V (History) are provided in the Appendix. The final question on the exam was a history question (Hist), which shows that 19 students have mastered history basics by the time of the final.

1.2 Analysis of the three midterm exams

The questions on the midterms were taken from the homework, thus the scores directly reflect the extent to which the students mastered the ten assignments.

Figure 2: Histograms of midterm 1 grades. The 7 problems were 1: History I, 2: Prime numbers and GCD, 3: Pythagorean triplets, 4: Continued Fraction algorithm, 5: Pell’s equation solution, 6: Matrix diagonalization, and 7: Fibonacci equation solution.

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3 https://auditorymodels.org/index.php?n=Courses.ECE298JA-F17#Week
4 https://jontalle.web.engr.illinois.edu/uploads/298.17/VC1-F17.pdf
Midterm 1: Number systems  

Fig. 2 shows the analysis of the first midterm, based on the first 9 Lectures on Number System. There were 7 questions, as described in the figure caption. Other than the History question, the scores were high. History questions caught the students off guard, even though I repeated emphasize its importance throughout the course. Eventually they caught on, as may be seen in Question 9 of midterm 2 and question 5 of the final. With a few exceptions [questions 3 (Pythagorean triplets) and 5 (Pell’s equation)] they had almost perfect scores.

![Histograms of midterm 2 grades. There were 9 problems on midterm 2: 1: Analytic functions, 2: Residue Expansion, 3: Convolution and polynomials, 4: Gaussian Elimination, 5: Ohm’s Law, 6: 2-port analysis (Transmission lines), 7: Complex function colorized mapping, 8: Fourier and Laplace transforms, and 9: History II.](image)

Midterm 2: Algebra, complex analysis  

Fig. 3 shows the scores for midterm 2. The upper left panel gives a summary of midterm 2 while the remaining 9 panels show the scores on each of the 9 questions. Question 7 inadvertently gave away the solution, so this question must be ignored. Unfortunately it was an important question, so we lack data on this important concept.

The midterm questions covered the topics as outlined in the figure caption. The distributions were high for all the questions except the last two. Question 8 was on Fourier and Laplace transforms, and the student had just been introduced to that topic the same week of the midterm, splitting the scores into high and very low scores. In retrospect this question should have been pushed to midterm 3, but at that time this was not practical. As a result half the class did poorly on question 8. Problem 9, History II, had many high scores, but there were also still a few students that had not come up to speed on the History questions. Based on the final exam question 5, by the end of the course most had mastered the History questions.

![Histograms of midterm 3 grades. There were 5 problems on midterm 3: 1: Cauchy-Riemann equations, 2: Branch cuts and Riemann sheets, 3: Fund. Th. of complex Integration (FTCC), 4: Cauchy’s theorems, and 5: Brune Impedance.](image)

Midterm 3: Scalar calculus  

In Fig. 4 the results of midterm 3 on scalar calculus, (i.e., differential equations with constant coefficients). A breakdown of the topics for midterm 3 are outlined in the figure caption. At this point, 3/4 of the way through the course, the students had learned the basic of Laplace transforms, integration in the complex plane, the basic residue theorems of Cauchy, branch cuts, and basic properties of impedance. In many ways these topics, based
on complex analysis (complex functions of the complex frequency $s$) are the corner-stone of the course. This material was used for every topic once it was introduced. For example, two-port network theory, which describes transmission lines, requires $2 \times 2$ matrices, having Transmission elements $[A(s), B(s); C(s), D(s)]$ that are functions of $s$. To solve these problems one must find the $2 \times 2$ eigen values and eigen vector of the matrix, a technique first introduced to solve both Pell’s and Fibonacci’s equations. Eigen analysis is then used repeatedly throughout the remainder of the course.

As mentioned in the above section, on the analysis of the Final, the material on vector calculus had not been tested until the final. This was complicated by the fact that Section IV (lectures 33-41 of the book) had not yet been finished, complicating the last quarter of the course. A major upgrade to the vector calculus section (lectures 33-41) was made during the Thanksgiving break, in a “just-in-time” effort to provide written material toward a final set of lectures. This update was largely newly written material, not yet completed.

Comparisons across exams: It is interesting to compare across exams to see how the understanding of the topics evolved. This is attempted in Fig. 5. More needs to be done using the final as the standard of comparison, but this analysis has yet to be done.

Figure 5: This figure shows how the students improved from midterm 1 ($\circ$) to midterm 2 ($+$) by using midterm 3 as a control. When the scores for the first and second exam are plotted as a function of the scores for the third exam, for each student (the points on the graph represent each of the 23 students), there is a systematic increase in the scores over time. When exam 1 is plotted against midterm 3, the distribution shows about half of the class with low scores. By exam 2 many of these scores increase. It will be really interesting to see how the distribution of scores increase when plotted against the Final exam. This curve demonstrates the leaning effect of the students. By midterm 3, many have started to improve their scores, between exam 1 and 2.

1.3 Final comments:

Two students (one a freshman) volunteered that the course was very difficult, requiring lots study to do the 10 assignments. I believe the high grades have not come easy, for any of the students. The point of the course is not to make the material easy, it is to teach the deep concepts of mathematics from the ages, using history as the guide. It is likely that the first year students had more difficulty than the 4th year students. One older student (Jr) stated that he first understood the material he already studied in his ECE courses, following the treatment by 298JA.

Yes, there is a lot of material in this course, that is not treated either superficially or dumbed down. The students must work hard, but in the end they understand it, as demonstrated by the analysis of the Final. The Nov, 2017 questionnaire summary of section 2 provides insight on the student’s views of the course. Since they did not have their final grad, they were in many cases worried about how they were doing. This is obvious in some of their comments. As stated in the abstract “It will be interesting to track these students to see how they perform in their future course work.”

The student with the grade of B wrote the following note on the last page of his final exam:

Thank you for this semester. There were a few times where I lost my way around the course, but your attitude towards learning helped me a lot, and I appreciated it.
2 Questionnaire

Immediately following the third midterm (Nov 13, 2017), an anonymous questionnaire was provided, to evaluate the student’s views of the class. As shown in Fig. 6, there were six basic question aimed at quantifying the class’s success with respect to the the

1. Lectures
2. Homework assignments
3. Course notes (book)
4. The TA’s performance
5. Office hours
6. Perception of the course’s value to the student.

The lectures were...
- -- - -/ + ++ very valuable to me
- - not valuable to me at all
- -- - -/ + ++ very useful for the homework
- not useful for the homework
- -- - -/ + ++ very useful for the course
- - -/ + ++ not useful for the course

The homework assignments were...
- -- - -/ + ++ very useful for the homework
- not useful for the homework
- -- - -/ + ++ too long
- not too long
- -- - -/ + ++ graded unfairly
- graded fairly

The course notes were...
- -- - -/ + ++ helpful
- not helpful
- -- - -/ + ++ contained very few errors
- riddled with errors
- -- - -/ + ++ graded fairly
- graded unfairly

The course TA was...
- -- - -/ + ++ very helpful
- not helpful at all
- -- - -/ + ++ sustained very few errors
- graded unfairly

Office hours were...
- -- - -/ + ++ enough office hours
- too few office hours
- -- - -/ + ++ very helpful
- not helpful at all
- -- - -/ + ++ sustained very few errors

Overall, this course’s material has...
- -- - -/ + ++ will help me in future courses
- will not help me in future classes
- -- - -/ + ++ sustained very few errors
- helped me in other classes
- -- - -/ + ++ not helpful in other classes

Figure 6: A questionnaire was filled out by 15 of the 23 students of ECE 298IA during the first week of Dec., 2017. There were six basic question (bold letters), with multiple sub-questions, for a total of 14 questions. The results of the study were analyzed using Matlab/Octave, first using histograms across the students for each of the 14 questions. The responses were anonymous, except for two students who identified themselves.

2.1 Analysis

The raw data from the questionnaire are shown in Table 2. The rows are the students and the columns are the responses to the questions. Each question was given a number from 1 to 5. Occasionally (6 times) no response was given. In general a 1 represents a negative response, while a 5 is the most positive while ‘0’ means no response was provided. There were 6 ‘0’ responses, three of which were given by student 1.

Student 12 gave the lowest response. Six out of 12 students gave the highest overall evaluation the course (5,5 for question 6). These results are presented as 14 histograms, condensed down to 3 figures.

2.2 Summary of Fig. 7

The upper row shows the histograms for questions 1.1, 1.2, 1.3: “The lectures were . . .” a) valuable, b) useful for HW, c) useful for midterms. These had modes of 4, 4, 3. The ideal answer to 1.1 is a 5, The mode was 4. The ideal answer to 1.2 is 5. However the lectures were somewhat independent of the HW in that they concepts needed to be explained before the
HW could even be understood. The lecture was not a good place to be working out HW solutions. Some small time was spent on this. Office hours are the ideal time for working on HW that can not be understood from examples in the Notes.

The obvious explanation for 1.3 having mode of 3 (re lectures and midterms) is that the midterms were directly based on the homeworks. the students acknowledge understood the roles of Lecture and HW re the midterms. The most relevant input to the midterms is the homework (i.e., Lectures take second place).

The lower row shows the response histograms for questions 2.1, 2.2, 2.3: “The homework was . . .” a) difficult, b) long, c) fair. The ideal response to this question is 2.1, which is close to what was given. In retro-respect 2.2 should have asked the somewhat obvious question “Are the HWs relevant to the exams.” However this was obvious, thus there was little point in even asking this question. Question 2.3 had the ideal response of 5 (very fair).

### Figure 7
The upper row shows the distributions from question 1 (a,b,c): “The lectures were:” a) valuable to me (1.1), b) useful for the HW (1.2), c) useful for the midterms (1.3). The lower panels are the distribution for question 2 (a,b,c): “The homeworks were:” a) difficult (2.1) b) long (2.2) c) fairly graded (2.3). For each of these questions a 1 is negative (- -) and a 5 is positive (++).

### Figure 8
The next six questions were about the class notes (questions 3abc and the TA’s performance (questions 4abc)) and office hours and utility of course (Helpful now or in the future)

#### 2.3 Summary of Fig. 8
The upper left row of Fig. 8 is about the class notes: a) are the helpful and b) the number of errors. The class “notes” refer to the book (class notes) (Allen, 2017a) that has been written over the last 1.5 years. In the first year Stillwell (2010) was the class text, and the second year, a set of slides was developed. In 2017 the class notes Allen (2017b) was written.

The lower right row of Fig. 8 is about the TA: a) was he fair and b) was he helpful. The mode of “helpful” was 4 and the mean is slightly below 4.5.

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### A Tables

#### A.1 Table of Final scores

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Table 1: Scores in % for the three midterms (E1, E2, E3) and final. The final letter grades were assigned based on this distribution, with A+ within the 5 point spread starting from the top grade (96), where A is the 5 point spread starting with 90, and A− is the 5 point spread starting with 85. Since the homework would lower the grades, it was removed from the analysis. The distribution of student year is 5 Fresh, 5 Soph, 5 Jr and 8 Sr. Names and IDs have been removed for the public version.

#### A.2 Table of questionnaire responses

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Table 2: Raw data transcribed from the questionnaires. The first column gives the subject index while the remaining columns are the response to the six questions, along with the sub-questions.
A.3 Sections IV and V of Final Exam

4 Part IV: Vector Calculus & Partial Differential Equations (20 pts)

4.1 Maxwell's Equations (8 pts)

The variables have the following names and defining equations:

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<th>Name</th>
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<tr>
<td>( \mathbf{E} )</td>
<td>( \mathbf{E} = -\nabla \phi + \frac{1}{c^2} \frac{\partial \mathbf{B}}{\partial t} )</td>
<td>Electric field strength</td>
<td>N/C</td>
</tr>
<tr>
<td>( \mathbf{H} )</td>
<td>( \mathbf{H} = \nabla \times \mathbf{E} + \frac{1}{c^2} \frac{\partial \mathbf{D}}{\partial t} )</td>
<td>Magnetic field strength</td>
<td>A/m</td>
</tr>
<tr>
<td>( D )</td>
<td>( D = \varepsilon_0 \varepsilon \mathbf{E} + \mathbf{P} )</td>
<td>Electric Displacement</td>
<td>C/m²</td>
</tr>
<tr>
<td>( B )</td>
<td>( B = \mu_0 \mu \mathbf{H} )</td>
<td>Magnetic Induction</td>
<td>T</td>
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</tbody>
</table>

Note that \( \mathbf{J} = \nabla \times \mathbf{E} \) is the current density, which has units of (Ampere/m²), where \( \sigma \) is the conductivity. Also, \( \mathbf{n} \) is a unit normal vector | \( |\mathbf{n}| = 1 \), typically normal to a surface \( S \).

To do:
1. When was Maxwell born (and died)? How long did he live (within \( \pm 10 \) years)?
2. The speed of light in a vacuum is \( c_0 = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \times 3 \times 10^8 \text{ m/s} \). The characteristic resistance in insulators is \( \rho = \frac{1}{\sqrt{\varepsilon_\infty \mu_\infty}} \times 3 \times 10^8 \text{ m/s} \).
3. In a few words, identify the law, define what it means, and explain the following formula:

\[ \nabla \phi + \frac{1}{c^2} \frac{\partial \mathbf{B}}{\partial t} = \mathbf{0}. \]

4 Electric field strength equation:

Consider Maxwell's equation \( \mathbf{E} = -\nabla \phi + \frac{1}{c^2} \frac{\partial \mathbf{B}}{\partial t} \) integrated over a two-dimensional surface \( S \), where \( \mathbf{n} \) is a unit normal vector to the surface

\[ \int_S \mathbf{E} \cdot d\mathbf{S} = \int_S \left( -\nabla \phi + \frac{1}{c^2} \frac{\partial \mathbf{B}}{\partial t} \right) \cdot \mathbf{n} \, dS. \]

(a) Apply Stokes' theorem to the left-hand side of the equation.

(b) Consider the right-hand side of the equation. How is it related to the magnetic flux \( \Phi \) through the surface \( S \)?

5 Magnetic field strength equation:

Consider Maxwell's equation \( \mathbf{H} = \nabla \times \mathbf{E} + \frac{1}{c^2} \frac{\partial \mathbf{D}}{\partial t} \), where \( \mathbf{D} \) is the magnetic field strength, \( \mathbf{C} = J \cdot d \) is the total current density, \( J \times \mathbf{E} \) is the conductivity (resistivity) current density and \( \mathbf{D} \) is the time rate of change of the electric flux density \( \mathbf{E} \), also known as the displacement current.

(a) First consider the equation over a two-dimensional surface \( S \),

\[ \int_S \mathbf{H} \cdot d\mathbf{S} = \int_S \left( \nabla \times \mathbf{E} + \frac{1}{c^2} \frac{\partial \mathbf{D}}{\partial t} \right) \cdot \mathbf{n} \, dS. \]

(b) Apply Stokes' theorem to the left-hand side of this equation. In a sentence or two, explain the meaning of the resulting equation.

4.3 Webster Horn Equation (3 pts)

Horns provide an important generalization of the solution of the 1D wave equation, in regions where the properties (i.e., \( n \), the speed of propagation) are not constant. The 3D wave equation for pressure \( p(x, y, z) \) is

\[ \nabla^2 p(x, y, z) - \frac{1}{\rho_c} \frac{\partial^2 p(x, y, z)}{\partial t^2} = 0. \]

Here \( \rho \) is the density (mass per unit volume).

To do:
1. Write out the formula for the Webster Horn equation in terms of the area function \( A(x) \).
2. Substitute \( A(x) \) for the case of the spherical (mono-conal) horn.
3. Discuss the nature of the solution.

4.4 Vector algebra in \( \mathbb{R}^3 \) (2 pts)

The definitions of the dot, cross, and triple product of vectors \( \mathbf{A}, \mathbf{B}, \mathbf{A} \times \mathbf{B} \) and \( \mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) \) are defined in Fig. 1.

4.5 Part VI: History (8 pts)

Write the individual's name on the line next to the contribution/date. *(If you do not know the year a person died, you will get full credit for 10 correct answers, and no points off for incorrect answers.)*

<table>
<thead>
<tr>
<th>Person</th>
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<tbody>
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Note: In the following problem, we will work in the frequency domain.

5 Part VI: History (8 pts)

Write the individual's name on the line next to the contribution/date. *(If you do not know the year a person died, you will get full credit for 10 correct answers, and no points off for incorrect answers.)*

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References

Allen, J. B. (2017a), An invitation to mathematical physics, and its history (Allen, 1404 Sunny Acres Rd, Mahomet IL, 61853), jontallen@ieee.org.
