Chapter 1

Number systems

1.1 Problems NS-1

Topic of this homework:

Introduction to Matlab/Octave (see the Matlab or Octave tutorial for help)

Deliverables: Report with charts and answers to questions.

Plotting complex quantities in Octave/Matlab

Problem # *1: Consider the functions* $f(s) = s^2 + 6s + 25$ *and* $g(s) = s^2 + 6s + 5$.

– *1.1: Find the zeros of functions* $f(s)$ *and* $g(s)$ *using the command* `roots()`.

Ans:

– *1.2: Show the roots of* $f(s)$ *as red circles and of* $g(s)$ *as blue plus signs.*

The x-axis should display the real part of each root, and the y-axis should display the imaginary part. Use `hold on` and `grid on` when plotting the roots.

Ans:

– 1.3 Give your figure the title "Complex Roots of f(s) and g(s)." Label the x- and y-axes "Real Part" and "Imaginary Part." Hint: Use `xlabel`, `ylabel`, `ylim([-10 10])`, and `xlim([-10 10])` to expand the axes.

Problem # *2: Consider the function* $h(t) = e^{\jmath 2\pi f t}$ *for* $f = 5$ *and* `t=[0:0.01:2]`.

– 2.1: Use `subplot` *to show the real and imaginary parts of* $h(t)$.
Make two graphs in one figure. Label the x-axes "Time (s)" and the y-axes "Real Part" and "Imaginary Part."

Ans:

– 2.2: Use `subplot` *to plot the magnitude and phase parts of* $h(t)$.
Use the command `angle` or `unwrap(angle())` to plot the phase. Label the x-axes "Time (s)" and the y-axes '"Magnitude" and "Phase (radians)."

Ans:

Prime numbers, infinity, and special functions in Octave/Matlab

Problem # *3: Prime numbers, infinity, and special functions.*

– 3.1: Use the Matlab/Octave function `factor` *to find the prime factors of 123, 248, 1767, and 999,999.*
Ans:

– 3.2: Use the Matlab/Octave function `isprime` *to determine whether 2, 3, and 4 are prime numbers. What does the function* `isprime` *return when a number is prime or not prime? Why?*
Ans:

– 3.3: Use the Matlab/Octave function `primes` *to generate prime numbers between 1 and* 10^6*. Save them in a vector* `x`*. Plot this result using the command* `hist(x)`.
Ans:

– 3.4: Now try `[n,bincenters] = hist(x)`*. Use* `length(n)` *to find the number of bins.*
Ans:

– 3.5: Set the number of bins to 100 by using an extra input argument to the function `hist`*. Show the resulting figure, give it a title, and label the axes.* Hint: `help hist` and `doc hist`.
Ans:

Problem # 4: *`Inf`, `NaN`, and logarithms in Octave/Matlab.*

– 4.1: Try `1/0` *and* `0/0` *in the Octave/Matlab command window.*
What are the results? What do these "numbers" mean in Octave/Matlab? **Ans:**

– 4.2: Try `log(0)`, `log10(0)`, *and* `log2(0)` *in the command window.*
In Matlab/Octave, the natural logarithm $\ln(\cdot)$ is computed using the function `log`. Functions $\log_{10}$ and $\log_2$ are computed using `log10` and `log2`. **Ans:**

– *4.3: Try* `log(1)` *in the command window. What do you expect for* `log10(1)` *and* `log2(1)`*?*
Ans:

– *4.4: Try* `log(-1)` *in the command window. What do you expect for* `log10(-1)` *and* `log2(-1)`*?*
Ans:

– *4.5: Explain how Matlab/Octave arrives at the answer in problem 4.4. Hint:* $-1 = e^{i\pi}$.
Ans:

– *4.6: Try* `log(exp(j*sqrt(pi)))` *(i.e.,* $\log e^{\jmath\sqrt{\pi}}$*) in the command window. What do you expect?*
Ans:

– *4.7: What does* inverse *mean in this context? What is the inverse of* $\ln f(x)$*?*
Ans:

– *4.8: What is a decibel? (Look up* decibels *on the internet.)*
Ans:

Problem # *5: Very large primes on Intel computers. Find the largest prime number that can be stored on an Intel 64-bit computer, which we call $\pi_{\max}$.* Hint: As explained in the Matlab/Octave command `help flintmax`, the largest positive integer is 2^{53}; however, the largest integer that can be factored is $2^{32} = 2^{54} - 6$. Explain the logic of your answer. Hint: `help isprime()`.
Ans:

Problem # *6: We are interested in primes that are greater than $\pi_{\max}$. How can you find them on an Intel computer (i.e., one using IEEE floating point)?* Hint: Consider a sieve that contains only odd numbers, starting from 3 (not 2). Since every prime number greater than 2 is odd, there is no reason to check the even numbers. $n_{\text{odd}} \in \mathbb{N}/2$ contain all the primes other than 2.
Ans:

Problem # *7: The following identity is interesting. Can you find a proof?*

$$\begin{aligned}
1 &= 1^2 \\
1+3 &= 2^2 \\
1+3+5 &= 3^2 \\
1+3+5+7 &= 4^2 \\
1+3+5+7+9 &= 5^2 \\
&\vdots \\
\sum_{n=0}^{N-1} 2n+1 &= N^2.
\end{aligned}$$

Ans: