ECE-298CLA: *Complex Linear Algebra*; Spring 2019

Instructor: Jont Allen
Course Coordinator: Jont Allen
Prerequisites: ECE-210 or ECE-310 concurrent.
Target Audience: Sophomores & Juniors from Engineering
Text: Boas, R., (2009), *Invitation to complex analysis*, Mathematical Association of America, 2d edition.
Allen, Jont, (2017) *An invitation to Mathematical Physics and its History*

Outline: This course teaches complex variables in the context of 2x2 matrix analysis, providing both a basic understanding of complex linear algebra and complex linear algebra (2x2 matrices). Topics:

1. Linear algebra of 2x2 complex matrices
2. Complex analysis [e.g., frequency and time domain methods]; complex impedance $Z(s)$; transfer function $H(s)$; differential equations

This course emphasizes engineering insight and intuition building to help the students expand their natural creative skills. For context, the specific mathematical contributions of Newton, Euler, Cauchy, Gauss, Riemann, and Helmholtz are discussed. There are 7 homework sets, roughly 1 per week, covering key engineering problems. There is one midterm exam, and a final. This course is not a substitute for Math 241, 286, 292.

I. **Number systems:** Integers, rationals, real vs. complex numbers, vectors, matrices.

II. **Complex linear algebraic and calculus:** Time and frequency domains (e.g., Laplace transforms), complex impedance (e.g., complex impedance as a function of the complex variable $s = \sigma + j\omega$).

Final Grade: The final grade will be based on the midterm, final and home-works.

Course outline by topic:[1]

	Part I. Introduction to complex 2x2 matrices
L	Description
1	Integers, rationals, real vs. complex numbers, vectors, matrices.
2	Polynomials and Newton's complex root finding method
3	Complex analytic functions, geometry and scalar products
4	Inverse matrix via Gaussian Elimination
5	Analysis of simple LRC circuits by matrix composition; ABCD (transmission matrix);
6	Pell's equation: $m^2 - Nn^2 = 1$ $(m, n, N \in \mathbb{N})$ & Fibonacci Series: $f_{n+1} = f_n + f_{n-1}$ $(n, f_n \in \mathbb{N})$.
	Companion matrix and eigen-analysis (eigenvalues, eigenvectors)
	Part II. Complex Algebraic analysis
7	*Fourier Transforms* for signals vs. *Laplace transforms* for systems.
8	Laplace transforms and Causality; Residue expansions
9	The ten system postulates: e.g., (P1) causality postulate, (P2) linearity
10	Exam I
11	Integration in the complex plane; Fundamental theorems of real vs. complex calculus
12	Differentiation in the complex plane; Complex Taylor series; Cauchy-Riemann conditions
13	Complex analytic functions; Brune Impedance.
14	Multi-valued complex functions; Riemann sheets; Branch cuts
15	Complex analytic mapping (Domain coloring)
16	Riemann's extended plane
17	Cauchy's Integral theorem & Formula
18	Cauchy Residue theorem; Green's theorem in the plane
19	Inverse Laplace transform $t \leq 0$; Case for causality
20	Inverse Laplace transform via the Residue theorem $t > 0$
21	Properties of the Laplace Transform: Modulation, convolution, etc.
22	Final Exam

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[1] L: Lecture