Chapter 1

Number systems

1.0.1 Exercises NS-1

Topic of this homework: Introduction to MATLAB/OCTAVE (see the Matlab or Octave tutorial for help).
Deliverable: Report with charts and answers to questions. Hint: Use LATEX.¹

Plotting complex quantities in Octave/Matlab

Problem #1: Consider the functions \( f(s) = s^2 + 6s + 25 \) and \( g(s) = s^2 + 6s + 5 \).

– Q 1.1: Find the zeros of functions \( f(s) \) and \( g(s) \) using the command \texttt{roots()}.
\textbf{Sol:} The roots of \( f(s) \) are \(-3 \pm 4i\) (in Matlab: \texttt{roots([1 6 25])}). The roots of \( g(s) \) are \(-1\) and \(-5\) (in Matlab: \texttt{roots([1 6 5])}). You will find the program that generates all these figures at \url{http://jontalle.web.engr.illinois.edu/uploads/298.17/NS1.m}

– Q 1.2: Show the roots of \( f(s) \) as red circles and of \( g(s) \) as blue plus signs.
The x-axis should display the real part of each root, and the y-axis should display the imaginary part. Use \texttt{hold on} and \texttt{grid on} when plotting the roots. \textbf{Sol:}

![Complex Roots of f(s) and g(s)](image)

– Q 1.3 Give your figure the title ‘Complex Roots of \( f(s) \) and \( g(s) \)’ Label the x- and y-axis ‘Real Part’ and ‘Imaginary Part.’
\textbf{Hint:} use \texttt{xlabel} and \texttt{ylabel}. Type \texttt{ylim([-10 10])} and \texttt{xlim([-10 10])}, to expand the axes.

Problem #2: Consider the function \( h(t) = e^{j2\pi ft} \) for \( f = 5 \) and \( t=[0:0.01:2] \).

¹http://www.overleaf.com
\[ e^{j2\pi t} = \cos(10\pi t) + j\sin(10\pi t). \]

\[ = 0 \quad 0.5 \quad 1 \quad 1.5 \quad 2 \]
\[ -1 \quad -0.5 \quad 0 \quad 0.5 \quad 1 \]

- Time (s)
- Real Part

- Time (s)
- Imaginary Part

- Q 2.1: Use subplot to show the real and imaginary parts of \( h(t) \)
Make two graphs in one figure. Label the x-axes ‘Time (s)’ and the y-axes ‘Real Part’ and ‘Imaginary Part’.

\[ e^{j2\pi t} = \cos(10\pi t) + j\sin(10\pi t). \]

- Q 2.2: Use subplot to plot the magnitude and phase parts of \( h(t) \).
Use the command \texttt{angle} or \texttt{unwrap(angle())} to plot the phase. Label the x-axes ‘Time (s)’ and the y-axes ‘Magnitude’ and ‘Phase (radians)’. \textbf{Sol:}

Prime numbers, infinity, etc. in Octave/Matlab

\textbf{Problem # 1: Prime numbers, infinity, etc.}

- Q 1.1: Use the Matlab function \texttt{factor} to find the prime factors of 123, 248, 1767, and 999,999.
\textbf{Sol:} Factors: 123 (3, 41), 248 (2,2,2,31), 1767 (3,19,31), 999999 (3,3,3,7,11,13,37)

- Q 1.2: Use the Matlab function \texttt{isprime} to check if 2, 3 and 4 are prime numbers.
What does the function \texttt{isprime} return when a number is prime, or not prime? Why?
\textbf{Sol:} \texttt{isprime(2)} returns 1, \texttt{isprime(3)} returns 1, and \texttt{isprime(4)} returns 0. 1 means ‘yes’ and 0 means ‘no’

- Q 1.3: Use the Matlab/Octave function \texttt{primes.m} to generate prime numbers between 1 and \( 10^6 \)
Save them in a vector \( x \). Plot this result using the command \texttt{hist(x)}. \textbf{Sol:}
Now try \([n, \text{bincenters}] = \text{hist}(x)\). Use \text{length}(n)\) to find the number of bins. \textbf{Sol:} \text{length}(n)\) is 10

Set the number of bins to 100 by using an extra input argument to the function \text{hist}. Show the resulting figure and give it a title and axes labels. \textbf{Sol:}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{primes.png}
\caption{Primes between 1 and \(10^5\)}
\end{figure}

\textbf{Problem # 2:} \textit{Inf, NaN and logarithms in Matlab}

Try \(1/0\) and \(0/0\) in the command window. What are the results? What do these ‘numbers’ mean in Matlab? \textbf{Sol:} \(1/0\) returns \text{Inf} (infinity) and \(0/0\) returns \text{NaN} (‘not a number’).

Try \(\log(0)\) in the command window. In Matlab, the natural logarithm \(\ln(\cdot)\) is computed using the function \(\log(\cdot)\), and \(\log_{10}\) and \(\log_2\) are computed using \(\text{log10}()\) and \(\text{log2}()\). \textbf{Sol:} \(\log(0)\) is \(-\text{Inf}\).

Try \(\log(-1)\) in the command window. Do you get what you expect for \(\ln(-1)\)? Show how Matlab arrives at the answer by considering \(-1 = e^{i\pi}\). \textbf{Sol:} \(\log(-1)\) is \(0 + i\pi\), because \(\ln(-1) = \ln(e^{i\pi}) = i\pi \ln(e) = i\pi\).

(Not graded) \textit{What is a decibel? Look up decibels on the internet.} \textbf{Sol:} The decibel is very important in engineering (unused in mathematics). It is defined as the log of a power ratio. If a power ratio is 2, the dB value is 6 [dB]. A ratio of 10 is 20 [dB]. Thus the formula for the dB-ref is \(10 \log_{10} \frac{P}{P_{\text{ref}}}\). Thus the decibel is defined on the log (i.e., ratio) scale. Engineers quickly learn to “think” in dB units, because its so easy (once they learn to think in terms of ratios).

\textbf{Problem # 3:} \textit{Very large primes on Intel computers}
–Q 3.1: Find the largest prime number that can be stored on an Intel 64 bit computer, which we call $$\pi_{\text{max}}$$.

Hint: As explained in the Matlab/Octave command `help flintmax`, the largest positive integer is $$2^{53}$$, however the largest integer that can be factored is $$2^{32} = \sqrt{2^{64}}$$. Explain the logic of your answer. Hint: `help isprime()`.

Sol: Using Matlab/Octave, start with the largest integer $$2^{32}$$ and check if its prime. Then work down by subtracting 1, and again check. Stop when you get to the first prime below the largest integer. The answer I get is: $$2^{32} - 5 = 4,294,967,291$$ is the first prime below $$2^{32}$$ prime.

Problem #4: Suppose you are interested in primes that are greater than $$\pi_{\text{max}}$$. How can you find them on an Intel computer (i.e., one using IEEE-floating point)?

–Q 4.1: Thus consider a sieve containing only odd numbers, starting from 3 (not 2).

Hint 1: Since every prime number greater than 2 is odd, there is no reason to check the even numbers. $$n_{\text{odd}} \in \mathbb{N}/2$$ contain all the primes other than 2. Sol: At this time, I don’t see any way to do this, due to the matlab limitation that it cannot factor numbers larger than $$2^{32}$$.

Problem #5: The following identity is interesting:

\[
1 = 1^2 \\
1 + 3 = 2^2 \\
1 + 3 + 5 = 3^2 \\
1 + 3 + 5 + 7 = 4^2 \\
1 + 3 + 5 + 7 + 9 = 5^2 \\
\vdots \\
\sum_{n=0}^{N-1} 2n + 1 = N^2.
\]

–Q 5.1: Can you find a proof?\(^2\)

Sol: Subtracting any line from the line following it, gives:

\[
(1 - 1) + 3 = 2^2 - 1^2 \\
5 = 3^2 - 2^2 \\
7 = 4^2 - 3^2 \\
9 = 5^2 - 4^2 \\
\vdots \\
\sum_{n=0}^{N-1} 2n + 1 - \sum_{n=0}^{N-2} 2n + 1 = N^2 - (N - 1)^2 \\
2N - 1 = N^2 - (N^2 - 2N + 1) \\
2N - 1 = 2N - 1.
\]

Thus the two sides are equal, as suggested by the above formula.

Can you find a simpler more constructive “proof?” Hint: assuming you know what integration by parts is, can you devise a concept called Summation by parts?

\(^2\)This problem came from an exam problem for Math 213, Fall 2016.