Instructors: Jont Allen, Steve Levinson, John DAngelo (Math) and others from ECE, Math, and Physics
Course Coordinator: Jont Allen
Prerequisites: Calculus I, II. (Concurrent registration in Calc. III or Differential Equations is encouraged)
Target Audience: Sophomores & precocious freshmen from Engineering, Physics and Mathematics
Text: Allen, Jont, (2017) *An invitation to Mathematical Physics and its History*

Outline: This course provides a "mathematical road map" to help students strengthen their understanding of engineering mathematics, at the conceptual level. A broad review of the development of classical mathematical theories used in contemporary engineering is presented, by emphasizing the historical discovery and development of the mathematics of number theory, nonlinear and linear algebra, complex analysis (e.g., frequency domain methods, impedance) and differential equations (both scalar and vector). This course is not a substitute for Math 241, 286, 292, 406, 415, 448.

This course will emphasize engineering insight and intuition building, rather than proofs. Intuitive insights into seven fundamental theorems of mathematics will be presented, to help the students expand their natural creative skills. The specific mathematical contributions of Newton, Euler, Cauchy, Gauss, Riemann, Helmholtz and Maxwell, will be discussed, in depth. Problem sets will be based on engineering problems, and how they relate to classical mathematics. An extra hour of credit is given for a student project (with approval of the instructor).

ECE-298JA is presented in four parts:

I. **Number systems:** Integers, rationals, real vs. complex numbers, vectors, matrices.

II. **Algebraic equations:** Topics will include time and frequency domains (e.g., Laplace transforms), complex impedance (e.g., the impedance of a capacitor $Z(s) = 1/sC$ is a function of the complex variable $s = \sigma + j\omega$), how electrical, mechanical, acoustical and thermal networks, are described by matrices, eigenvalues and impedance-based integral equations.

III. **Scalar Differential equations:** Ordinary differential equations for LRC circuits, Newton's & Kirchhoffs laws, etc.

IV. **Vector Differential equations:** Gradient, divergence, curl, Laplacian and vector Laplacian. Partial differential equations, i.e., Laplace, diffusion, wave and Maxwell's Equations, including dispersive wave propagation (e.g., Webster horn equation, Brillouin zones).

Final Grade: The final grade will be based on a weighted average of the three midterm exams, the final exam (95%), and a 5% weight for the homeworks and class participation.

Course outline by topic:[1]

		Part I. Number systems
L	c.	Description
1	(50)	The discovery of Number systems
	(3)	Introduction: Integers, rationals, real vs. complex numbers, vectors, matrices.
		Number systems, Geometry, Calculus (∞)
2	17	Taxonomy of Numbers, from Primes π_k to Complex $\mathbb{C}$; Floating point numbers (IEEE 754)
3	3	Math is a language, designed to do physics; Seven Fundamental theorems of Mathematics.
4	(5)	Two Prime Number Theorems: 1) Fundamental Thm of Arith 2) Prime Number Theorem
5	(3)	Euclidean Algorithm for the GCD; Coprimes.
6	(3)	Continued Fraction algorithm (rational approximation).
7	(3)	Euclid's formula for Pythagorean triplets: $l^2 = m^2 + n^2$ ($[l, m, n] \in \mathbb{N}$).
8	(5)	Pell's equation: $m^2 - Nn^2 = 1$ ($N \in \mathbb{N}$).
9	(5)	Fibonacci Series: $f_{n+1} = f_n + f_{n-1}$ ($n, f_n \in \mathbb{N}$).
10		Exam I: Number Systems

[1]L: Lecture; c.: Century (BCE), CE; Page numbers are for Stillwell 2^d edition.

		Part II. Algebraic Equations
L	c.	Description
11	7	Geometry as physics; The first "algebra" al-Khwarizmi (830CE).
12	18	Equations of physics, quadratic in several variables.
13	17	Polynomial root classification by convolution.
14	17	Analytic geometry; scalar and vector products.
15	(2)	Gaussian Elimination.
16		Matrix composition; ABCD method; Commuting vs. Non-commuting operator.
17		Riemann Sphere and the extended plane (1851.
18		Complex analytic mapping (Domain coloring.
19		*Fourier Transforms* (Hilbert space) for signals vs. *Laplace transforms* for systems.
20		Laplace transforms and Causality.
21	20	The nine postulates of Systems: e.g., (P1) causality postulate
22		Exam II: Algebraic equations

		Part III. Scalar Differential Equations
L	c.	Description
23		Integration in the complex plane; Complex Taylor series.
24	19	The Cauchy-Riemann conditions (*Residue theorem*); Green's theorem in the plane
25	17	Complex analytic functions and Brune Impedance.
26		Multi-valued complex functions; Riemann sheets & Branch cuts.
27	17	Fundamental Thms of complex integration (Part I); Cauchy's Integral theorem & Formula
28	17	Fundamental Thms of complex integration (Part II); Residue Theorem;
29		Inverse Laplace transform $t \le 0$; Case for causality
30		Inverse Laplace transform via the Residue theorem $t > 0$
31		Properties of the Laplace Transform: Modulation, convolution, etc.
32		Properties of Brune impedance.
33	18	Euler's vs. Riemann's Zeta Function (i.e., poles at the log-primes) & Euler's Sieve.
34		Exam III: Scalar differential equations

		Part IV Vector (Partial) Differential Equations
L	c.	Description
35	17	The acoustic wave equation: Newton's and d'Alembert's solution
36		Webster Horn equation.
37		Define *gradient* ∇, *divergence* $\boldsymbol{\nabla}\cdot$ and *curl* $\boldsymbol{\nabla}\times$, and *vector Laplacian* $\boldsymbol{\nabla}\cdot\nabla$
38		Stokes' (curl) and Gauss' (divergence) Theorems, Vector Laplacian
39	20	J.C. Maxwell unifies Electricity and Magnetism (1861); Heaviside's role.
40	20	*The Fundamental theorem of vector calculus* & its many applications.
41		Quasi-static approximation: Newton's laws, KCL/KVL, Telephone equation, ...
42		Guest lecture
		Final Exam: All topics

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