ECE-293: *Complex linear algebra for Engineers*

Instructors: Jont Allen (or Minh Do, Yoram Bresler, Mark HJ, ZP Liang , ...)
Course Coordinator: Jont Allen
Prerequisites: Calculus Concurrent registration in ECE 210/310
Target Audience: Sophomores, Juniors, Seniors in Engineering
Text: Abridgment of: *An introduction to Mathematical Physics and its History, (Springer, 2020)*[1]
Outline: This course provides a "mathematical road map" to help students strengthen their understanding of engineering mathematics, at the conceptual level. A broad review of the development of classical mathematical theories used in contemporary engineering is presented, using historical discovery and development of the mathematics of linear algebra, complex analysis (e.g., frequency domain methods, impedance) and scalar differential equations, using eigen-vector space with complex eigen-values, complex analytic functions, including branch cuts.

What has been sorely missing in the undergraduate curriculum, is *complex linear algebra* (CLA). Complex eigen-values (poles in the left half plane) play a key role when solving differential equations, as used in circuit and control (stability) theory. This course specifically deals with these, using the Cauchy-Riemann conditions, the companion matrix, and eigen-analysis, including the case of over-specified (non-square) systems (i.e, singular value decomposition). This course will briefly discuss the basics needed to understand the mathematical basis for machine-learning (the case of a vector quantized (pruned), Boolean decision space).[2]

The course will also emphasize engineering insight and intuition building. Intuitive insights using five fundamental theorems of mathematics (arithmetic, algebra, calculus, complex calculus, Cauchy's formula) will be presented, engaging the students into expanding their creative skills. Weekly problem sets are based on key engineering problems, and how they relate to classical mathematics. Historical details are used as a pedagogical tool, to layer in needed context. Notable historical figures include Galileo, Newton, Euler, Riemann, Cauchy, Gauss, and others.

ECE has attempted to address this matter via ECE-493/MATH-487. While highly successful, ECE-493 is generally taken too late to provide the basics required to deal with ECE-210, ECE-310, ECE-329, ECE-340, Physics 211–214, etc. ECE-493 works best as a capstone course; ECE-293 can provide the needed basics, much early in the curriculum.

ECE-298JA is presented in 20 lectures, as a 7-week half-semester course, in three parts:

I. **Number systems:** Integers, rationals, real vs. complex numbers, vectors, matrices, the *greatest common factor* and *continued fraction algorithms*.

II. **Algebraic equations:** Topics will include time and frequency domains (e.g., Laplace transforms), complex impedance (e.g., $Z(s)$ is a function of the complex variable $s = \sigma + j\omega$), how electrical, mechanical, and thermal networks are solved using impedance, matrices, eigen-values and vectors.

III. **Differential equations:** Ordinary differential equations (transmission lines) for electrical, mechanical and thermodynamic systems, using Kirchhoff's, Newtons and Gibb's circuit (complex impedance) laws.

Final Grade: The final grade will be based on a weighted average of a midterm exam and the final exam (95%), and a 5% weight for the seven (weekly) homeworks.

[1] https://www.amazon.com/gp/product/3030537587
[2] A topic presented in ECE498-NS (Spring 2020).

Course outline by topic:[3]

W	L	Description
		Part I. Number systems (Ch-2)
1	1	Introduction: Integers, rationals, real vs. complex numbers, vectors, matrices.
	2	Complex analytic functions;
	3	Pythagorean triplets;
2	4	Analysis of LRC circuits using Transmission matrix methods
	5	Pell's equation; Fibonacci series; Matrix formulation of the polynomial;
		Newton's method for finding roots of polynomials.
		Use of Octave's many linear algebra tools: Companion matrix; eigen-values, eigen-vectors.
		Part II. Algebraic Equations (Ch-3)
	6	Fourier Transforms as scalar products; System postulates;
3	7	Laplace Transforms and causality
	8	Fundamental theorems of calculus and complex calculus
	9	A comparison of FT vs. LT; Complex impedance properties;
		Gaussian elimination of 2x2 and 3x3 matrices;
4	10	Complex analytic functions; Brune impedance
	11	Differentiation in the complex plan: complex Taylor series; Cauchy-Riemann conditions
	12	Exam I
		Part III. Scalar Calculus (Ch-4)
5	13	Multi-valued functions; Riemann Sheets; Branch cuts
	14	Complex analytic functions; Cauchy-Riemann conditions
	15	Riemann's extended plane and sphere
6	16	Cauchy's integral theorem & formula
	17	Transmission line problem
	18	Inverse LT: $t < 0$ (causality); Convolution of step functions
7	19	Inverse LT; Properties of the LT for $t > 0$
	20	Properties of the LT, with copious examples

[3]W: Week; L: Lecture;