Problems NS-1

Plotting complex quantities in Octave/Matlab

Problem *# 1: Consider the functions* $f(s) = s^2 + 6s + 25$ *and* $g(s) = s^2 + 6s + 5$.

–Q *1.1*
Find the zeros of functions $f(s)$ and $g(s)$ using the command `roots()`.

–Q *1.2: Show the roots of* $f(s)$ *as red circles and of* $g(s)$ *as blue plus signs.*
The x-axis should display the real part of each root, and the y-axis should display the imaginary part. Use `hold on` and `grid on` when plotting the roots.

–Q *1.3 Give your figure the title 'Complex Roots of f(s) and g(s)' Label the x- and y-axis 'Real Part' and 'Imaginary Part.'*
Hint: use `xlabel` and `ylabel`. Type `ylim([-10 10])` and `xlim([-10 10])`, to expand the axes.

Problem *# 2: Consider the function* $h(t) = e^{j2\pi ft}$ *for* $f = 5$ *and* `t=[0:0.01:2]`.

–Q *2.1: Use* `subplot` *to show the real and imaginary parts of* $h(t)$
Make two graphs in one figure. Label the x-axes 'Time (s)' and the y-axes 'Real Part' and 'Imaginary Part'.

–Q *2.2: Use* `subplot` *to plot the magnitude and phase parts of* $h(t)$.
Use the command `angle` or `unwrap(angle())` to plot the phase. Label the x-axes 'Time (s)' and the y-axes 'Magnitude' and 'Phase (radians)'.

Prime numbers, infinity, etc. in Octave/Matlab

Problem *# 3: Prime numbers, infinity, etc.*

–Q *3.1: Use the Matlab/Octave function* `factor` *to find the prime factors of 123, 248, 1767, and 999,999.*

–Q *3.2: Use the Matlab/Octave function* `isprime` *to check if 2, 3 and 4 are prime numbers.*
What does the function `isprime` return when a number is prime, or not prime? Why?

–Q *3.3: Use the Matlab/Octave function* `primes.m` *to generate prime numbers between 1 and* 10^6
Save them in a vector `x`. Plot this result using the command `hist(x)`.

–Q *3.4: Now try* `[n,bincenters] = hist(x)`.
Use `length(n)` to find the number of bins.

–Q *3.5: Set the number of bins to 100 by using an extra input argument to the function* `hist`.
Show the resulting figure and give it a title and axes labels.

Problem *# 4:* `Inf`, `NaN` *and logarithms in Octave/Matlab*

–Q *4.1: Try* `1/0` *and* `0/0` *in the Octave/Matlab command window.*
What are the results? What do these 'numbers' mean in Octave/Matlab?

–Q *4.2: Try* `log(0), log10(0)` *and* `log2(0)` *in the command window.*
In Matlab/Octave, the natural logarithm $\ln(\cdot)$ is computed using the function `log`. Functions $\log_{10}$, and $\log_2$ are computed using `log10` and `log2`.

–Q *4.3: Try* `log(1)` *in the command window. What you expect for* `log10(1)` *and* `log2(1)`*?*

–Q *4.4: Try* `log(-1)` *in the command window. What do you expect for* `log10(-1)` *and* `log2(-1)`*?*

–Q *4.5: Show how Matlab/Octave arrives at the above answer because* $-1 = e^{i\pi}$.

–Q *4.6: Try* `log(exp(j*sqrt(pi)))` *(i.e.,* $\log e^{\jmath\sqrt{\pi}}$*) in the command window. What do you expect?*

–Q *4.7: What does* inverse *mean in this context? What is the inverse of* $\ln f(x)$*?*

–Q *4.8: What is a decibel? (Look up decibels on the internet.)*

Problem *# 5: Very large primes on Intel computers*

–Q *5.1: Find the largest prime number that can be stored on an Intel 64 bit computer, which we call* $\pi_{\max}$.
Hint: As explained in the Matlab/Octave command `help flintmax`, the largest positive integer is 2^{53}, however the largest integer that can be factored is $2^{32} = \sqrt{2^{64}}$. Explain the logic of your answer. Hint: help isprime().

Problem *# 6: Suppose you are interested in primes that are greater than* $\pi_{\max}$*. How can you find them on an Intel computer (i.e., one using IEEE-floating point)?*

–Q *6.1: Thus consider a sieve containing only odd numbers, starting from 3 (not 2).*
Hint 1: Since every prime number greater than 2 is odd, there is no reason to check the even numbers. $n_{\text{odd}} \in \mathbb{N}/2$ contain all the primes other than 2.

Problem *# 7: The following idenity is interesting:*

$$\begin{aligned} 1 &= 1^2 \\ 1+3 &= 2^2 \\ 1+3+5 &= 3^2 \\ 1+3+5+7 &= 4^2 \\ 1+3+5+7+9 &= 5^2 \\ &\cdots \\ \sum_{n=0}^{N-1} 2n+1 &= N^2. \end{aligned}$$

–Q *7.1: Can you find a proof?*[1]

[1] This problem came from an exam problem for Math 213, Fall 2016.