– *12.2: Calculate x_1 and x_2. What number is the algorithm approaching?*
Sol: First we must find $P_3'(x) = -3x^2$. Thus the equation we must iterate is:

$$x_{n+1} = x_n + \frac{1 - x_n^3}{3x_n^2}.$$

Given a first guess for the root x_0, the next are $x_1 = x_0 + \frac{1-x_0^3}{3x_0^2}$ and $x_2 = x_1 + \frac{1-x_1^3}{3x_1^2}$. Note that if $x + 0$ is the root, then $x_1 = x_0$ and we are done. However, if $x_0 = 0$, then $x_1 = \infty$, since $x_0 = 0$ is a root of $P_3'(x)$. Thus we must not start at the roots of $P_n'(x_0) = 0$. ■

– *12.3: Here is an Octave/Matlab script for the $P_2(x)$ case. Modify it to find $P_3(x)$:*

```
x(1)=1/2; %x(1)=0.9; %x(1)=-10
y(1)=x(1);
 for n=2:10
x(n) = x(n-1) + (1-x(n-1)^2)/(2*x(n-1));
y(n) = (1+y(n-1)^2)/(2*x(n-1));
 end
semilogy(abs(x)-1); hold on
semilogy(abs(7)-1,'or'); hold off
```

Sol:

```
x=1/2;
for n = 1:3
x = x+(1-x*x*x)/(3*x*x);
end
```

■

– *12.4: (1 pts) For $n = 4$, what is the absolute difference between the root and the estimate, $|x_r - x_4|$?*
Sol: 4.6E-8 (very small!) ■

– *12.5: Does Newton's method work for $P_2(x) = 1 + x^2$? If so, why?* Hint: The poles and zeros are exactly known!
Sol: Here $P_2'(x) = 2x$. Thus the iteration gives

$$x_{n+1} = x_n - \frac{1 + x_n^2}{2x_n}.$$

In this case the roots are $x_\pm = \pm 1\jmath$—namely, purely imaginary. The solution will converge for complex roots as long as the starting point is complex. If we start with a real number for x_0, and use real arithmetic, Newton's method fails because there is no way for the answer to become complex. Real in = Real out. ■

– *12.6: What if we let $x_0 = (1 + \jmath)/2$ for the case of $P_2(x) = 1 + x^2$?*
Sol: By starting with a complex initial value, we fix the Real in = Real out problem. ■

Problem # *13: In this problem we consider the case of fractional roots, and take advantage of this fact during the itteration. Given that the roots are integers, composed of primes, we may uniquely identify the primes by factoring the numerator and denominator of the rational approximation of the root.*

The method is:

1. Start the Newton itteration
$$s_{n+1} = s_n - \frac{M(s_n)}{M'(s_n)}$$

2. Apply the CFA to the next output `rats`(s_{n+1})

3. Factor the Num and Dem of the CFA

4. Terminate when the factors converge

Using this method, show that we can find either the best possible fractional approximation to the roots (or even the exact roots, when the answer is within machine accuracy).

– *13.1: Find the roots of a Monic having coefficents $m_k \in \mathbb{F}$.*

Let
$$M_3(x) = (x - 254/17)(x - 2047/13)(x - 17/13)$$

In this case the root vector R becomes
$$R = [14.9412, 157.4615, 1.3077].$$

Verify that `rats(M)` returns the rational set of roots. **Sol:** In double precision this returns M_3. (Not sure what happens in single precision.) ■