ECE 403 **ExamII – Version 1.00 April 18, 2017** **Spring 2017**

Univ. of Illinois **Tue, April 18, 2017; 7-10PM** **Prof. Allen**

Topics: Acoustics, Transforms, Two-port networks

Instructions:

- If you need more space for calculation, use the back of any page.
- NO Cell phones.
- NO Calculators
- Note that each problem has points assigned. More points means "harder."

You may open the exam at 7:00 PM; You must close it by 9:00 PM.

NAME: ________________________ NETID: ________________________

I will not share information during this exam,
and I will discretely report others if I observe
cheating: ________________________ (sign here)

#	value	score
1	15	
2	7	
3	16	
4	17	
5	15	
6	15	
$\sum$	85	

Terminology, constants & transforms

Definitions of common acoustic variables,[1] the mathematical symbols and the units, as used in the text (see p. 180-181).

Variable	Symbol	[Units]
Total pressure	$P = P_o + p(t)$	[N/m^2 = Pa]
temperature	$T = T_o + \tau(t)$	[K°] Kelvin
density	$\rho = \rho_o + \delta(t)$	[kg/m^3]
adiabatic law	p/δ^γ = Const.	
Boyle's Law	$P_o/(\rho_o T_o)$ = Const.	
Constants (air)		
Atm Pressure	$P_o = 10^5$	[Pa]
Abs temperature	$T_o = 273$	[K°]
density	$\rho_o = 1.18$	[kg/m^3]
sound speed	$c = 345 = \sqrt{\gamma P_o/\rho_o}$	[m/s]
specific impedance	$\rho_o c = 407$	[Rayls]
viscosity	$\mu = 1.86 \times 10^{-5}$	[Ns/m^2] (Poiseuille)
thermal conductivity	$\kappa = 25.4 \times 10^3$	[N/sK]
specific heat cap @V$_o$	c_v	[J/kg]
specific heat cap @P$_o$	c_p	[J/kg]
Boltzman's const.	$k = 1.38 \times 10^{-23}$	J/K
ratio of specific heats	$\gamma = c_p/c_v = 1.4$	(unitless)

Transform	Time → Frequency	Frequency → Time
Laplace Transform (LT)	$F(s) = \int_0^\infty f(t)e^{-st}dt$	$f(t) = \frac{1}{2\pi j}\oint_{C_L} F(s)e^{st}ds$
		C_L is the Laplace contour
Fourier Transform (FT)	$F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt$	$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{j\omega t}d\omega$
Fourier Series (FS)	$F[k] = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(t)e^{-jnt}dt$	$f(t) = \sum_{k=-\infty}^{\infty} F[k]e^{jnt}$
Z-Transform (ZT)	$F(z) = \sum_{n=0}^{\infty} f[n]z^{-n}$	$f[n] = \frac{1}{2\pi j}\oint_{C_z} F(z)z^{n-1}dz$
		C_z is the Z-transform contour
Discrete-Time FT (DTFT)	$F(\omega) = \sum_{-\infty}^{\infty} f[n]e^{-j\omega n}$	$f[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} F(\omega)e^{j\omega n}d\omega$
Discrete FT (DFT)	$F[k]\sum_{n=0}^{N-1} x[n]e^{-\frac{2\pi j}{N}kn}$	$f[n] = \frac{1}{N}\sum_{k=0}^{N-1} F[k]e^{\frac{2\pi j}{N}kn}$

Discrete convolution: $f[n] \star g[n] = \sum_{k=-\infty}^{\infty} f[k]g[n-k]$

Continuous convolution: $f(t) \star g(t) = \int_{-\infty}^{\infty} f(\tau)g(t-\tau)d\tau$

[1] c_p, c_v: https://en.wikipedia.org/wiki/Heat_capacity#Specific_heat_capacity

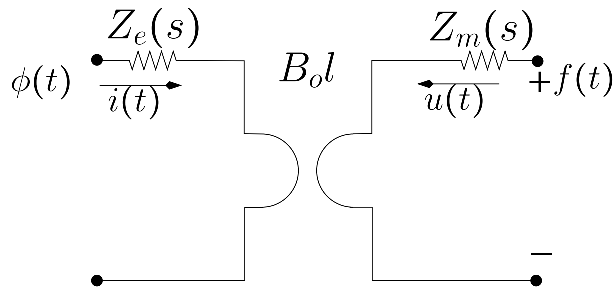

Figure 1: *The Hunt model is composed of an electrical impedance $Z_e(s)$, a Gyrator having parameter $T = lB_o$, and a mechanical impedance $Z_m(s)$. The electrical side is driven by a voltage $\phi(t)$ and a current $i(t)$, while the mechanical side produces a force $f(t)$ and a particle velocity $u(t)$.*

1 (15pts) Transducer Thevenin Parameters

1. **(3pts)** Find the ABCD (Transmission) $\mathbf{T}(s)$ matrix for the Hunt transducer model in Figure 1

$$\begin{bmatrix} \Phi(\omega) \\ I(\omega) \end{bmatrix} = \mathbf{T(s)} \begin{bmatrix} F(\omega) \\ -U(\omega) \end{bmatrix}.$$

2. **(2pts)** Find the impedance matrix $\mathbf{Z}(s)$.

$$\begin{bmatrix} \Phi(\omega) \\ F(\omega) \end{bmatrix} = \mathbf{Z}(s) \begin{bmatrix} I(\omega) \\ U(\omega) \end{bmatrix}.$$

3. **(2pts)** Compute Δ_Z and Δ_T. What is the significance of each of these determinants?

4. **(4pts)** Define, and then find the Thevenin equivalent force $F_{\text{thev}}(s)$ [N], given an input voltage Φ_o. *Note: If you are not sure of your answers to the previous parts, you can solve this problem in terms of an arbitrary ABCD matrix.*

5. **(4pts)** Define, and then find the Thevenin equivalent mechanical impedance in mechanical Ohms, $Z_{\text{thev}}(s)$ [Ω].

2 (7pts) Nyquist Theorem on Thermal noise

A stub of transmission line having characteristic impedance z_0 is terminated in each end with this Thevenin model, with $R = z_0$. Then at $t = 0$, the resistance is short or open circuited. This setup is shown in the figure.

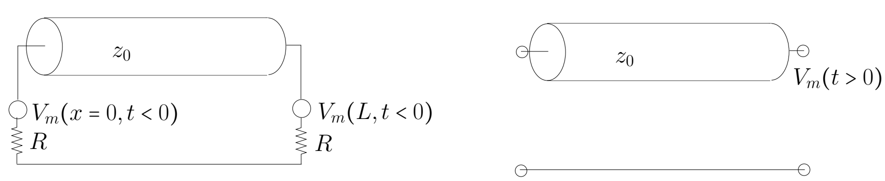

Figure 2: *On the left an ideal lossless transmission line, driven by two resistors. Each resistor has a noise source in series with it, that generates a voltage equal to the thermal noise in the resistor. At time $t = 0$ the resistors are switched out, leaving the voltage in the transmission line frozen in place.*

The transmission line stores the voltage as a function of x at $t = 0$ once the switch is opened removing the resistors (and also the Thevenin source). At that point voltage $V_m(L,t)$ becomes frozen.

1. **(1pt)** Discuss what happens after $t = 0$.

2. **(2pt)** What is the fundamental period of the noise voltage, $v_m(L, t > 0)$?

3. **(2pts)** Every periodic signal has a *Fourier series*. If the period is T [s], what can you say about the Fourier series frequencies?

4. **(2pts)** The noise spectrum will differ depending on the time at which the switch is opened. Why?

3 (16pts) Hilbert transform

Analyze the real, causal impulse response

$$h(t) = e^{-t/\tau_0} u(t),$$

with $\tau_0 = 10$ [ms], in terms of its Hilbert transform (integral) relations. *Note: Use the notation* $h(t) \leftrightarrow H(s)$ *and* $H(\omega) = H(s)|_{s=j\omega}$.

1. **(2pts)** Find $H(s)$, the Laplace transform of $h(t)$.

2. **(1pt)** Where are the poles of $H(s) \leftrightarrow e^{-t/\tau_0} u(t)$?

3. **(4pts)** Find and *sketch* the real and imaginary parts of $H(\omega) \equiv H(s)|_{s=j\omega}$.

4. **(1pt)** Write out the even $h_e(t) = h(t) + h(-t)$ and odd $h_o(t) = h(t) - h(-t)$ functions. Describe why these equations are symmetric and antisymmetric functions.

5. **(4pts)** Find the Fourier transforms of $h_e(t) \leftrightarrow H_e(\omega)$ and $h_o(t) \leftrightarrow H_o(\omega)$. How do they relate to the real and imaginary parts of $H(\omega)$?

6. **(4pts)** Find the Hilbert (integral) relations between $H_r \equiv \Re H(\omega)$ (real part) and $H_i \equiv \Im H(\omega)$ (imag part) of $H(\omega)$. *Hint: These integrals come from a frequency-domain convolution.*

4 (17pts) Wave equation

4.1 History of the wave equation

1. **(1pt)** What year did d'Alembert derive his solution to the wave equation?

2. **(1pt)** What is the form of D'Alembert's solution?

3. **(1pt)** Who was the first person to calculate the speed of sound, and what was the result?

4.2 The Webster horn equation:

In the time domain, in 2x2 matrix form, the Webster horn equation is given by

$$\frac{\partial}{\partial x}\begin{bmatrix} p(x,t) \\ \nu(x,t) \end{bmatrix} = -\begin{bmatrix} 0 & \frac{\rho_o}{A(x)} \\ \frac{A(x)}{\gamma P_o} & 0 \end{bmatrix} \frac{\partial}{\partial t}\begin{bmatrix} p(x,t) \\ \nu(x,t) \end{bmatrix}. \tag{1}$$

where $p(x,t) \leftrightarrow \mathcal{P}(x,\omega)$ is the pressure and $\nu(x,t) \leftrightarrow \mathcal{V}(x,\omega)$ is the *volume velocity*.

1. **(2pts)** Transform the Horn equation to the frequency domain.

2. **(4pts)** Assuming a *conical horn*, having area $A(x) = A_o(x/x_o)^2$ with $A_o \le 4\pi$, rewrite Equation 1 as a second order equation solely in terms of the pressure $\mathcal{P}(x,\omega)$ (remove $\mathcal{V}$), and thereby find the frequency domain solutions $P_\pm(x,s)$ for the conical horn equation.

3. **(4pts)** Assuming an exponential area function

$$A(x) = A_0 e^{2mx}$$

(m is a positive constant, called the *horn flair parameter*), derive the exponential horn equation for the pressure.

$$\frac{\partial^2 p(x,t)}{\partial x^2} + 2m \frac{\partial p(x,t)}{\partial x} = \frac{1}{c^2} \frac{\partial^2 p(x,t)}{\partial^2 t} \tag{2}$$

4. **(4pts)** In general the solution to a wave equation is of the form

$$p(x,t) = P^+(\kappa) e^{\kappa(s)x} e^{st} + P^-(\kappa) e^{-\kappa(s)x} e^{st}$$

where $s = \sigma + \jmath\omega$ is the Laplace frequency and $\kappa(s)$ is the complex "wave number."

(a) What is $\kappa(s)$ for the conical horn?

(b) What is the significance of $\kappa(s)$?

(c) Why is it a function of s?

(d) What is the role of $P^\pm(\kappa, s)$?

5 (15pts) Reflectance

1. **(6pts)** A tube transmission line with characteristic impedance z_0 and length L is terminated in a load impedance $Z_L(s)$.

   (a) The reflectance at any location x along the tube transmission line is

   $$\Gamma(x,s) = \frac{Z(x,s) - z_0}{Z(x,s) + z_0}.$$

   Starting with this formula, show that the impedance

   $$Z(x,s) \equiv \frac{\mathcal{P}(x,s)}{\mathcal{V}(x,s)}.$$

   Hint: What is z_0 in terms of the forward and reverse traveling waves $\mathcal{P}^{\pm}$ and $\mathcal{V}^{\pm}$?

   (b) Find the formula for the reflectance at the load (simplify your answer, if applicable), $\Gamma(x,s)|_{x=L} = \Gamma(L,s) = \Gamma_L(s)$ for

      i. $Z_L(s) = r$ [Nt-s/m^5]

      ii. $Z_L(s) = 1/sC$ [Nt-s/m^5]

      iii. $Z_L(s) = r \| sM$ [Nt-s/m^5]

2. **(3pts)** For the transmission line described in the previous problem, let $L = 1$, $Z_L(s) = 1$ and $z_0 = 2$.

   (a) Find the frequency domain reflectance $\Gamma(0, s)$ at $x = 0$.

   (b) Find the time-domain reflectance $\gamma(0, t) \leftrightarrow \Gamma(0, s)$ at $x = 0$.

3. **(3pts)** Two transmission lines are in cascade, the first one having an area of 1 [cm^2] and a second having an area of 2 [cm^2], with lengths L_1 and L_2 respectively, terminated with a resistor $r = \rho c/A$, where $A = 2$ [cm^2]. Find $R(x = 0, s)$.

4. **(3pts)** What is the inverse Laplace transform of

   (a) $H(s) = 1/(s + 1)$? Find $h(t)$.

   (b) $H(s) = s/(s + 1)$?

6 (15pts) Model of the middle ear

As shown in the figure, the free field sound pressure, defined as $P_0(\omega)$ acts as a source in series with the radiation resistance R_{rad}. The total radiation impedance $Z_{rad}(s)$ is a combination of the resistance and a reactive component L_{rad}, which represents the local stored field. The two impedances are in parallel

$$Z_{rad}(s) = sL_{rad}R_{rad}/(sL_{rad} + R_{rad}) = 1/Y_{rad}(s).$$

where $s = \sigma + j\omega$ is the Laplace complex frequency variable. The *radiation admittance for a sphere* is

$$Y_{rad} = 1/Z_{rad} = \frac{A_{rad}}{sr_c\rho_o} + \frac{A_{rad}}{\rho_o c} = \frac{1}{sL_{rad}} + \frac{1}{R_{rad}},$$

where r_c is the radius of the sphere and A_{rad} is the *effective area* of the radiation.

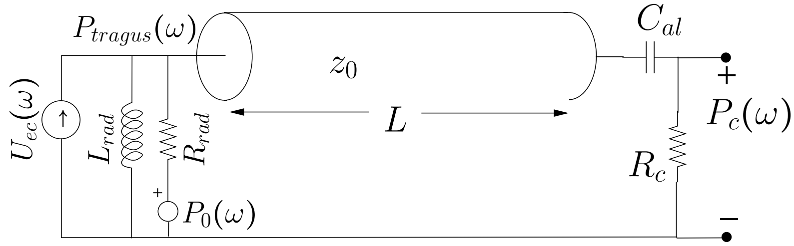

Figure 3: *Model of the ear canal, terminated by the radiation impedance $Z_{rad}(s)$ at the tragus ($x = 0$), and by the eardrum and cochlea at $x = L$.*

On the left we terminate the ear in a radiation impedance

$$Z_{rad}(s) = \frac{sL_{rad}R_{rad}}{sL_{rad} + R_{rad}}.$$

At the cochlear end we terminate the line with an impedance

$$Z_c = R_c + 1/(sC_{al}),$$

where R_c is the cochlear impedance and C_{al} is the stiffness of the annular ligament, which is the ligament that holds the stapes in the oval window. The cochlear resistance (R_c) is assumed to be twice the characteristic impedance of the ear canal.

To Do:

1. **(4pts)** Find the formula for the reflection coefficient at $x = L$. Sketch the magnitude $|R(L,s)|$ as a function of frequency. *Hint: You should label any constants, or set them equal to 1 for plotting.*

2. **(4pts)** Find the formula for the reflection coefficient at $x = 0$ (looking out towards the radiation impedance). Sketch its magnitude as a function of frequency.

3. **(3pts)** Consider the radiation impedance looking out the ear canal, Y_{rad}.

   (a) What is the frequency for which its real and imaginary parts are equal (in terms of L_{rad} and R_{rad})?

   (b) Describe the dependence of Y_{rad} on L_{rad} and R_{rad} below and above this frequency.

4. **(4pts)** Find the formula for the input impedance $Z(0, s)$ of the middle ear at the entrance of the ear canal, when the cochlea is "blocked" ($Z_c = \infty$ or $\Gamma(L, s) = 1$)?