ECE 403 **ExamII – Version 1.00 April 17, 2017** **Spring 2017**

Univ. of Illinois **Tue, Feb 7, 2017; 7-10PM** **Prof. Allen**

Topics: Acoustics, Transforms, Two-port networks

Instructions:

- If you need more space for calculation, use the back of any page.
- NO Cell phones.
- NO Calculators
- Note that each problem has points assigned. More points means "harder."

You may open the exam at 7:00 PM; You must close it by 9:00 PM.

NAME: _____ NETID: _____

I will not share information during this exam,
and I will discretely report others if I observe
cheating: _____ (sign here)

#	value	score
1	9	
2	7	
3	25	
4	5	
5	10	
6	8	
7	18	
8	15	
$\sum$	98	

Terminology and acoustic constants

Definitions of common acoustic variables,[1] the mathematical symbols and the units, as used in the text (see p. 180-181).

Variable name	Symbol	[Units]
pressure	$P = P_o + p(t)$	[N/m^2 = Pa]
density	$\rho = \rho_o + \delta(t)$	[kg/m^3]
temperature	$T_o + \tau(t)$	[K°] Kelvin
Constants		
Atm Pressure	$P_o = 10^5$	[Pa]
Abs temperature	$T_o = 273$	[K°]
sound speed	$c = 345$	[m/s]
density	$\rho_o = 1.18$	[kg/m^3]
specific impedance	$\rho c = 407$	[Rayls]
viscosity	$\mu = 1.86 \times 10^{-5}$	[Ns/m^2] Poise
Boyle's Law	$P_o = \rho_o T_o$ Const.	[Pa]
adiabatic law	$p = \delta^\gamma$ Const.	[Pa]
thermal conductivity	$\kappa = 25.4 \times 10^3$	[N/sK]
specific heat cap @V$_o$	c_v	[J/kg]
specific heat cap @P$_o$	c_p	[J/kg]
Boltzman's const.	$k = 1.38 \times 10^{-23}$	J/molecule
ratio of specific heats	$\gamma = c_p/c_v = 1.4$	
Lossy $\gamma\sqrt{\mu}$	$\gamma' = 6.6180 \times 10^{-3}$	
Propagation function	$\kappa(s) = \left(s + \beta_0\sqrt{s}\,\right)/c$	$\mathcal{P}^+(s) = e^{-\kappa(s)x}$

The lossy wave-number $\kappa(s)$, including thermal and viscous loss, is (p. 154, *anInvitation*) $\kappa(s) = (s + \beta_0\sqrt{s})/c$. If the tube diameter D [m] is the tube diameter, then $\beta_0 D = 2\gamma'/\sqrt{\rho} = 12.0828$, with

$$\gamma' = \sqrt{\mu}\left(1 + \sqrt{5/2}\left(\sqrt{\gamma} - \frac{1}{\sqrt{\gamma}}\right)\right) = 6.618045 \times 10^{-3}.$$

For example, with $D = 1$ [cm], $\beta_0 = 1.2083 \times 10^3$, and for $D = 1$ [mm], $\beta_0 = 12.083 \times 10^3$.

[1] C_p, C_v: `https://en.wikipedia.org/wiki/Heat_capacity#Specific_heat_capacity`

1 Wave equation

1.1 History of the wave equation

1. What year did d'Alembert derive his solution to the wave equation?
2. What is the form of D'Alembert's solution?
3. Who was the first person to calculate the speed of sound, and what was the result?

1.2 The Webster wave equation:

1. Write out the time-domain *Webster Horn equation*. Hint:
   (a) The first equation is Newton's second law (conservation of momentum density), i.e., the negative gradient in the pressure (force density) equals the mass-density times the rate of change of particle velocity.
   (b) For static displacement (compression) the stiffness is the static pressure $P_o = 10^4$ [Pa], i.e., 1 [atm]). But when the heat cannot diffuse in one cycle of the sound wave, the heat is trapped, and then the adiabatic condition changes the stiffness of air to be γP_o.
   (c) The second equation is Hook's law, the equation for a spring, determined by the stiffness of air, namely the divergence of particle velocity equals the pressure over the stiffness density γP_o.
2. Transform the Horn equation to the frequency domain.
3. Assuming a *conical horn*, having area $A(x) = A_o(x/x_o)^2$ with $A_o \leq 4\pi$, rewrite these equations as a second order equation solely in terms of the pressure P (remove U), and thereby find the frequency domain solutions $P_\pm(x, s)$ for the conical horn equation.
4. Assuming an exponential area function

$$A(x) = A_0 e^{2mx}$$

   (m is a positive constant, called the *horn flair parameter*), derive the exponential horn equation for the pressure.

$$\frac{\partial^2 p(x,t)}{\partial x^2} + 2m\frac{\partial p(x,t)}{\partial x} = \frac{1}{c^2}\frac{\partial^2 p(x,t)}{\partial^2 t} \tag{1}$$

   Hint: The horn equation in pressure is

$$\frac{1}{A(x)}\frac{\partial}{\partial x}A(x)\frac{\partial}{\partial x}p(x,t) = \frac{1}{c^2}\frac{\partial^2}{\partial^2 t}p(x,t).$$

5. In general the soluton to a wave equation is of the form

$$p(x,t) = P^+(\kappa)e^{\kappa(s)x}e^{st} + P^-(\kappa)e^{-\kappa(s)x}e^{st}$$

   where $s = \sigma + j\omega$ is the Laplace frequency and $\kappa(s)$ is the complex "wave number." What is $\kappa(s)$ for the

(a) Conical horn?

(b) What is the significance of $\kappa(s)$?

(c) Why is it a function of s?

(d) What is the role of $P^{\pm}(\kappa, s)$?

1.3 Reflectance:

1. Find (derive) the formula for the "input" impedance of a transmission line, having characteristic impedance $z_0(x, s)$, in terms of the reflectance. Define all the terms. Hint, I did this in class several times.

2. Find the formula for the reflectance $R(s)$ in terms of the load impedance $Z_L(s)$ and the characteristic impedance z_0 if:[2]

   (a) $Z_L(x, s) = r$ [Nt-s/m^5]

   (b) $Z_L(x, s) = 1/sC$ [Nt-s/m^5]

   (c) $Z_L(x, s) = r \| sM$ [Nt-s/m^5]

   (d) Two transmission lines are in cascade, the first one having an area of 1 [cm^2] and a second having an area of 2 [cm^2], with lengths L_1 and L_2 respectively, terminated with a resistor $r = \rho c/A$, where $A = 2 \times 10^{-4}$ [m^2]. Find $R(x = 0, s)$.

   (e) What is the inverse Laplace transform of

      i. $H(s) = 1/(s + 1)$? Find $h(t)$.

      ii. $R(s) = \frac{Z - z_0}{Z + z_0}$ where $L = 1$, $Z = 1$ and $z_0 = 2$? Find $r(t)$ at the input.

      iii. $H(s) = s/(s + 1)$?

2 Nyquist Thm on Thermal noise

A stub of transmission line having characteristic impedance z_0 is terminated in each end with this Thevenin model, with $R = z_0$. Then at $t = 0$, the resistance is short or open circuited. If open circuited you may watch the voltage at one end, and if short circuited, you may watch the current. This setup is shown in the figure.

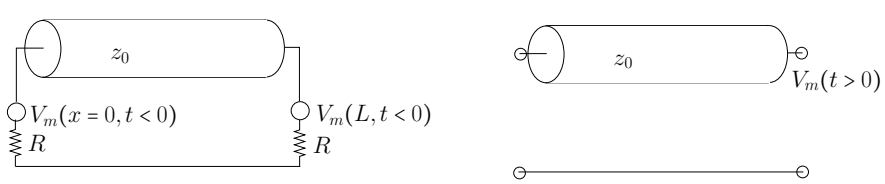

Figure 1: On the left an ideal lossless transmission line, driven by two resistors. Each resistor has a noise source in series with it, that generates a voltage equal to the thermal noise in the resistor. At time $t = 0$ the resistors are switched out, leaving the voltage in the transmission line frozen in place.

[2]Note that r, C and M represent an acoustic resistance, compliance and mass. Namely they are positive constants.

The transmission line stores the voltage at $t = 0$ once the switch is opened removing the resistors (and also the Thevenin source). At that point voltage $V_m(L, t)$ becomes periodic. Discuss what happens after the resistors are removed.

1. What is the fundamental period of the noise?
2. Every periodic signal has a *Fourier series*. If the period is T [s], what are the Fourier series frequencies?
3. Why aren't the values of each spectral peak the same?

3 Hilbert transform

In all parts of this problem $h(t) \leftrightarrow H(s)$ and $H(\omega) = H(s)|_{s=j\omega}$

Analyze the real impulse response

$$h(t) = e^{-t/\tau_0} u(t),$$

with $\tau_0 = 10$ [ms], in terms of its Hilbert transform (integral) relations.

1. Find $H(s)$, the Laplace transform of $h(t)$.
2. Where are the poles of $H(s) \leftrightarrow e^{-t/\tau_0} u(t)$?
3. Find the real and imaginary parts of $H(\omega) \equiv H(s)|_{s=j\omega}$.
4. Write out the symmetric $h_e(t)$ and antisymmetric $h_o(t)$ functions.
5. Find the Fourier transforms of $h_e(t) \leftrightarrow H_e(\omega)$ and $h_o(t) \leftrightarrow H_o(\omega)$.
6. Find the Hilbert (integral) relations between $H_r \equiv \Re H(\omega)$ (real part) and $H_i \equiv \Im H(\omega)$ (imag part) of $H(\omega)$.

4 Transducer Thevenin Parameters

The Hunt model of a electromechanical motor was first introduced by **?**, and fully developed by **?**. It contains a gyrator, proposed by **?**. The history is carefully discussed in Chapter 1 of Hunt's book. The equations are also presented in our lab manual, and HW-C.

4.1 To do:

1. Define and then find the ABCD (Transmission) $\mathbf{T}(s)$ matrix.
2. Define and then find the impedance matrix $\mathbf{Z}(s)$.
3. Compute Δ_Z and Δ_T. What is the significance of each of these determinants?
4. Define, and then find the Thevenin equivalent force $F_{\text{thev}}(s)$ [N], given an input voltage Φ_o.
5. Define, and then find the Thevenin equivalent mechanical impedance in mechanical Ohms, $Z_{\text{thev}}(s)$ [Ω].

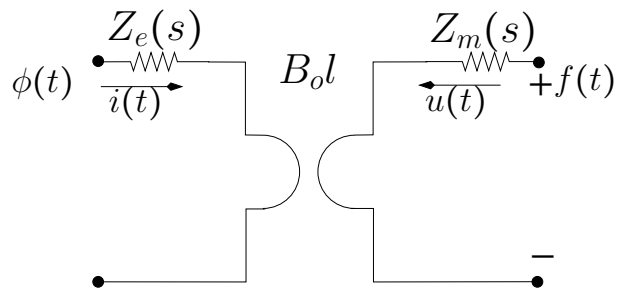

Figure 2: *The Hunt model is composed of an electrical impedance $Z_e(s)$, a Gyrator having parameter $T = lB_o$, and a mechanical impedance $Z_m(s)$. The electrical side is driven by a voltage $\phi(t)$ and a current $i(t)$, while the mechanical side produces a force $f(t)$ and a velocity $v(t)$.*

5 Simulation of the middle ear

As shown in the figure, the free field sound pressure, defined as $P_0(\omega)$ acts as a source in series with the radiation resistance R_{rad}. The total radiation impedance $Z_{rad}(s, A_{rad})$ is a combination of the resistance and a reactive component L_{rad}, which represents the local stored field. The two impedances are in parallel

$$Z_{rad}(s) = sL_{rad}R_{rad}/(sL_{rad} + R_{rad}) = 1/Y_{rad}(s).$$

where $s = \sigma + j\omega$ is the Laplace complex frequency variable. The *radiation admittance for a sphere* is

$$Y_{rad} = 1/Z_{rad} = \frac{A_{rad}}{sr_c\rho_o} + \frac{A_{rad}}{\rho_o c} = \frac{1}{sL_{rad}} + \frac{1}{R_{rad}},$$

where r_c is the radius of the sphere and A_{rad} is the *effective area* of the radiation.

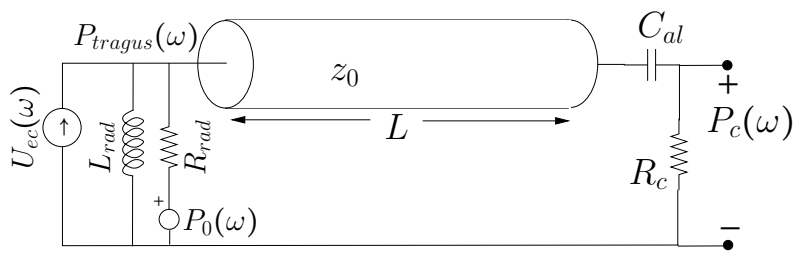

Figure 3: *Model of the ear canal, terminated by the radiation impedance $Z_{rad}(s)$ at the tragus ($x = 0$), and by the eardrum and cochlea at $x = L$.*

On the left we terminate the ear in a radiation impedance

$$Z_{rad}(s) = \frac{sL_{rad}R_{rad}}{sL_{rad} + R_{rad}}.$$

At the cochlear end we terminate the line with an impedance

$$Z_c = R_c + 1/(sC_{al}),$$

where R_c is the cochlear impedance and C_{al} is the stiffness of the annular ligament, which is the ligament that holds the stapes in the oval window. The cochlear resistance (R_c) is assumed to be twice the characteristic impedance of the ear canal.

Each impedance may be converted into a reflection coefficient, defined as

$$\Gamma(s) \equiv \frac{U_-}{U_+} = \frac{Z(s) - z_0}{Z(s) + z_0},$$

where $z_0 = \rho_o c/A_c$ is the characteristic impedance of the ear canal, having area A_c.

To Do:

1. Find z_0, the characteristic impedance of the ear canal.
2. Derive $\Gamma(s)$ from first principles.
3. Find the formula for the reflection coefficient at $x = L$

$$\Gamma(L, s) \equiv \frac{U_-(L, s)}{U_+(L, s)}.$$

4. Find the formula for the reflection coefficient at $x = 0$

$$\Gamma(0, s) \equiv \frac{U_+(0, s)}{U_-(0, s)}.$$

5. Find the formula for the input impedance $Z(0, s)$ of the middle ear at the entrance of the ear canal, when the cochlea is "blocked" ($Z_c = \infty$ or $R(L, s) = 1$)? (Hint, this has a simple solution, that you can easily derive.Start at $x = L$ and work backwards, in the time domain.)
6. Find the two reflection coefficients and <u>sketch</u> out their magnitude frequency responses.