Topic of this homework: Model of the human middle ear

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Some credit is better than NO credit.

Note: Due: Due: Lect 24. Each person is to do there own final writeup, but obviously you can discuss it as much as you like between yourselves. However, your crossing the line if you share computer files. The general rule is you need to process all the words you write through your eyes and your fingers. When ever you use material from someone else, you must give them credit, as I do here.

This homework is important because the middle ear is the acoustic load on an in the ear earphone (earbud). The middle ear model has many thing in common with a loudspeaker, but it runs in reverse.

1 Transducer Thevenin Parameters

The Hunt model of a electromechanical motor was first introduced by **?**, and fully developed by **?**. It contains a gyrator, proposed by **?**. The history is carefully discussed in Chapter 1 of Hunt's book, as well in our text (to a lesser extent).

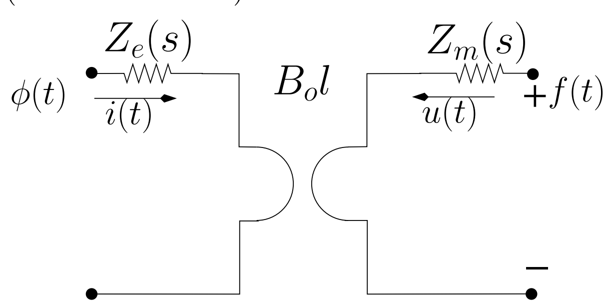

Figure 1: *The Hunt model is composed of an electrical impedance $Z_e(s)$, a Gyrator having parameter $T = lB_o$, and a mechanical impedance $Z_m(s)$. The electrical side is driven by a voltage $\phi(t)$ and a current $i(t)$, while the mechanical side produces a force $f(t)$ and a velocity $v(t)$.*

1.1 To do:

1. Define and then find the ABCD (Transmission) $\mathbf{T}(s)$ matrix. Solution: The definition of the Transmission matrix is

$$\begin{bmatrix}\Phi(\omega)\\ I(\omega)\end{bmatrix} = \begin{bmatrix}1 & Z_e\\ 0 & 1\end{bmatrix}\begin{bmatrix}0 & lB_o\\ lB_o & 0\end{bmatrix}\begin{bmatrix}1 & Z_m\\ 0 & 1\end{bmatrix}\begin{bmatrix}F(\omega)\\ U(\omega)\end{bmatrix} = \frac{1}{T}\begin{bmatrix}Z_e & \Delta_Z\\ 1 & Z_m\end{bmatrix}\begin{bmatrix}F(\omega)\\ U(\omega)\end{bmatrix}$$

where Δ_Z is the determinent of the impedance $\mathbf{Z}(s)$ matrix.

2. Define and then find the impedance matrix $\mathbf{Z}(s)$. Solution: This may be found via the table of relations between **T** and **Z** (VanValkenberg62). The answer is

$$\begin{bmatrix}\Phi(\omega)\\ F(\omega)\end{bmatrix} = \begin{bmatrix}Z_e & -T\\ T & Z_m\end{bmatrix}\begin{bmatrix}I(\omega)\\ U(\omega)\end{bmatrix}$$

where $T = lB_o$, with l [m] the length of the wire and B_0 [Web] the strength of the magnet [Wb/m^2].

3. Compute Δ_Z Solution: $\Delta_Z = Z_e(s)Z_m(s) + T^2$ and Δ_T. Solution: $\Delta_T = -1$. What is the significance of these determinents?

4. Define, and then find the Thevenin equivalent force $F_{\text{thev}}(s)$ [Nt/Amp]. The thevenin force

5. Define, and then find the Thevenin equivalent mechanical impedance $Z_{\text{thev}}(s)$ [Mech-Ω].

2 Simulation of the middle ear

Acoustic constants: $\rho_o c = 407$ [Rayls], $c = 345$ [m/s].

The middle ear may be treated as an $L = 2.5$ cm long stub of transmission line, terminated in a series combination of a stiffness, shown as a capacitor C_{al} and damping, shown as a resistor R_c. The experimental details of the cat ear have been extensively explored by Guinan and Peake (1967), and first modeled by Zwislocki (1962, 1957) and much later by Lynch *et al.* (1982). The model shown here is a significant simplification, but adequate for our purpose. This homework was the basis for the masters thesis of Pierre Parent (Parent and Allen, 2007).

The diameter of the ear canal is $d_c = 0.75 = 2 * r_c$ cm (area $A_c = \pi r_c^2$). As shown in the figure, the free field sound pressure, defined as $P_0(\omega)$ acts as a source in series with the radiation resistance R_{rad}. The total radiation impedance $Z_{rad}(s, A_{rad})$ is a combination of the resistance and a reactive component L_{rad}, which represents the local stored field. The two impedances are in parallel

$$Z_{rad}(s) = sL_{rad}R/(sL_{rad} + R_{rad}) = 1/Y_{rad}(s).$$

where $s = \sigma + j\omega$ is the Laplace complex frequency variable. The *radiation admittance for a sphere* is

$$Y_{rad} = 1/Z_{rad} = \frac{A_{rad}}{s r_c \rho_o} + \frac{A_{rad}}{\rho_o c},$$

where r_c is the radius of the sphere and A_{rad} is the *effective area* of the radiation.

If we assume that the pinna (the flap of skin we call our ear) changes the radiation area, then we can account for this pinna-horn effect (a transformer), changing the effective area. The effective area is equivalent to *resonate scattering* characterized by $kr_{rad} = 1$, where $k = \omega/c = 2\pi/\lambda$. Thus the larger the effective area the smaller the impact of the mass reactance L_{rad} on the radiation efficiency. Namely, the size of L_{rad} decreases as the effective area is made larger via the action of the horn transformer. We can chose the area to match the frequency where the radiation load switches from a mass to a resistance. This frequency was found to be 3 kHz in the cat ear (Rosowski *et al.*, 1988, Fig. 10).

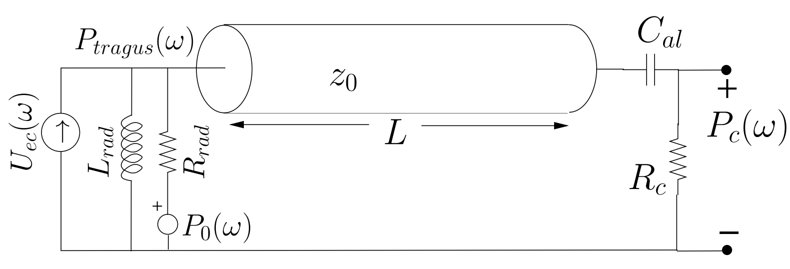

Figure 2: *Model of the ear canal, terminated by the radiation impedance $Z_{rad}(s)$ at the tragus ($x = 0$), and by the eardrum and cochlea at $x = L$. The horn transformation is the transformer that represents the concha, that converts the area of the canal to the area of the pinna/concha opening. This transformer ratio seems necessary to reduce the effective radiation mass seen by the ear canal, to improve the matching of energy between the canal and free-space. It does this by making the area defining L_{rad} larger by the transformer turns ratio. The effective turns ratio needs to be estimated from measured data (Rosowski* et al.*, 1988, Fig. 10).*

The model: To model the middle ear we first need to simulate the transmission line by using d'Alembert solution of the wave equation. This simulation will operate in the time domain.

Create two arrays that represent the forward and backward velocity traveling waves $u_+(x - ct)$ and $u_-(x + ct)$. Let $c = 345$ m/s.

The boundary conditions at each end of the line will be realized via reflection coefficients $R(L, s)$ at the cochlear end and $R(0, s)$ at the input. Initially assume that P_0 is zero, and drive the middle ear with a velocity source U_{ec} at the tragus.

On the left we terminate the ear in a radiation impedance

$$Z_{rad}(s) = \frac{sL_{rad}R_{rad}}{sL_{rad} + R_{rad}}.$$

At the cochlear end we terminate the line with an impedance

$$Z_c = R_c + 1/sC_{al},$$

where R_c is the cochlear impedance and C_{al} is the stiffness of the annular ligament, which is the ligament that holds the stapes in the oval window. The cochlear resistance (R_c) is assumed to be twice the characteristic impedance of the ear canal.

Each impedance may be converted into a reflection coefficient, defined as

$$R(s) \equiv \frac{U_-}{U_+} = \frac{Z(s) - z_0}{Z(s) + z_0},$$

where $z_0 = \rho_o c/A_c$ is the characteristic impedance of the ear canal, having area A_c. For example, the reflection coefficient at $x = L$ is the transfer function between the wave reflected $U_-(L, s)$ over the incident wave $U_+(L, s)$, as defined above. The two reflection coefficients are a function of complex frequency s.

Use the bilinear z transform to convert $R(s)$ into a discrete time IIR filter having numerator polynomial $N(z)$ (zeros) and denominator polynomial $D(z)$ (poles), so that the reflectance filters (e.g., $R(0, s)$ and $R(L, s)$) are implemented in the time domain. I recommend that you implement the filter manually rather than use the `filter()` command. [If you chose to use the `filter()` command, *you must save state* between calls. See `help filter` for the details on how to do this, and *carefully verify that it is working.*]

Finally is the question of the boundary conditions. This is where things are a little tricky. These are dealt with by defining the end reflection coefficients

$$R(L, s) \equiv \frac{U_-(L, s)}{U_+(L, s)}$$

and

$$R(0, s) \equiv \frac{U_+(0, s)}{U_-(0, s)}.$$

In the frequency domain we can find

$$U_-(L, s) = R(L, s)U_+(L, s)$$

and

$$U_+(0, s) = R(0, s)U_-(0, s).$$

These relations give the junction reflected wave in terms of the incident wave. For example, at $x = L$ the incident wave is U_+ and the reflected wave is U_-. At $x = 0$ the signs must change. Since this relation is in the frequency domain, it represents a convolution in the time domain. Since we wish to run the middle ear model in the time domain, we must design the filter represented by these reflection coefficients. To do this we may use the bilinear z transform.

Each reflection coefficient R is given by

$$R(s) = \frac{Z(s) - z_0}{Z(s) + z_0}.$$

I derived this formula in class and showed that $Z(s)$ is the load impedance and $z_0 = \rho_o c/A$ is the *characteristic impedance* of the transmission line. Taking the case of $x = L$ we find

$$R(L, s) = \frac{R_c + 1/sC_{al} - z_0}{R_c + 1/sC_{al} + z_0}.$$

When using the bilinear z transform one substitutes $2F_s \frac{z-1}{z+1}$ for s. Try `doc bilinear` from Matlab©.

Show your work. Check all the filters and all the components carefully.

Show your work. For example, show the impulse response for each of these filters $r(0,t) \leftrightarrow R(0,s)$ and $r(L,t) \leftrightarrow R(L,s)$. Also test your time domain transmission line impulse response by setting the reflection coefficients to 1, and then compute the impulse response of $p_c(L,t)$ with an impulse at the input (e.g., find $p_{ec}(0,t) = \delta(t)$). Show your work. Write this up with a full description of what you did, so that I can easily understand how you got your results. I will grade this homework partially on the readability of the presentation.

Solution: There are two areas that need to be distinguished here, the canal area and the effective area of the tragus looking out into the pinna. Please be clear about these two areas. The canal area is $A_c = \pi r_c^2$ while *the effective radiation area is half the area of a sphere*, having a diameter of the canal) is $A_{rad} = \frac{\pi}{2} d_c^2$.

1. To-do:

   (a) Find z_0, the characteristic impedance of the ear canal. (1 min) Solution: The radius of the canal is $r_c = d_c/2 = 0.0075/2$ [m]. The canal area is $\pi r_c^2 = 4.42 \times 10^{-5}$, so $z_0 = \rho_o c/A = 407/\pi r_c^2$ or 9.2 [MΩ] (read this as "MKS Acoustic meg-ohms")

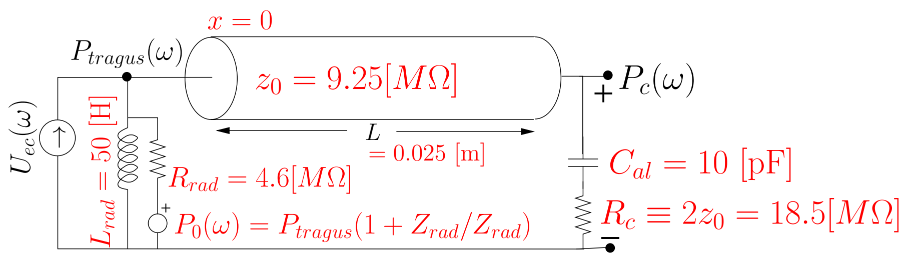

Figure 3: *Left: Model of the ear canal, terminated by the radiation impedance $Z_{rad}(s)$ at the tragus ($x = 0$), and by the eardrum and cochlea at $x = L$.*

   (b) Assume that the cochlear load resistor R_c is equal to twice the characteristic impedance of the ear canal. Find C_{al}, by assuming that the impedance of this element ($Z_{al} \equiv 1/sC_{al}$) is equal to the cochlear impedance R_c at a frequency of 0.800 kHz. Show all your work. (1 min) Solution: To find C_{al} set the impedance of C_{al} equal to $2z_0$ at 800 [Hz]: $|1/2\pi 800 C_{al}| = 2z_0 = 2\rho_o c/A_c$ giving $C_{al} = 10.8$ [pF].

   (c) Determine the radiation impedance looking out the ear canal, and compute the resonant frequency, defined as the frequency $f_c \equiv s/2\pi j$, for which the real and imaginary parts are equal. For an example of the frequency response of this impedance look at Rosowski *et al.* (1988, Fig. 10) provided on the class web site.

   Solution: In this case we use the effective area of the pinna given by $A_{rad} = \frac{\pi}{2} d_c^2 = 88 \times 10^{-6}$ [m^2] and

$$Y_{rad} = 1/sL_{rad} + 1/R_{rad}, \tag{1}$$

   with $R_{rad} \equiv \rho_o c/A_{rad}$=4.6 MΩ, $L_{rad} = \rho_o r_{rad}/A_{rad} = 1.18(d_c/2)/0.5\pi(d_c)^2 = 50$ [H].

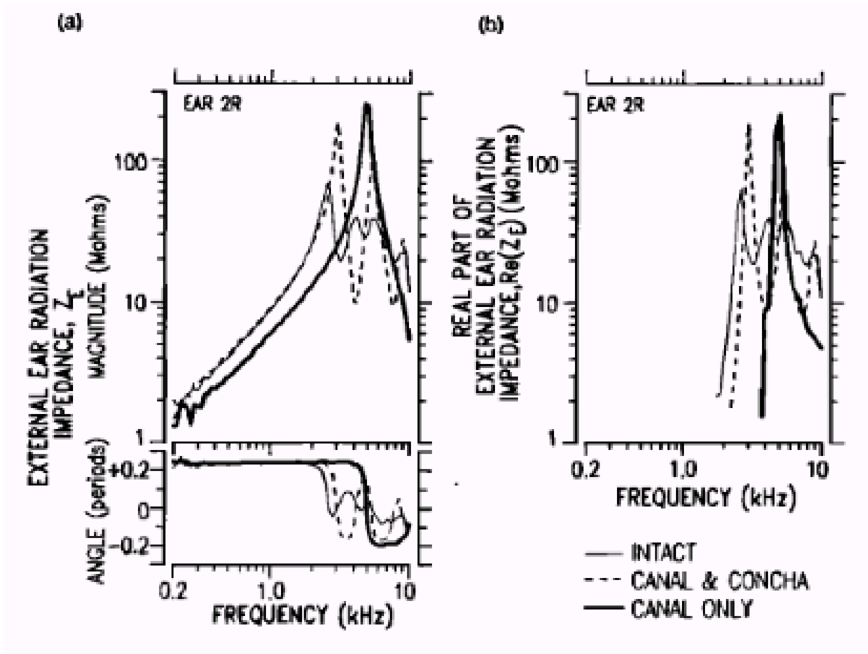

FIG. 10. Radiation impedance measured in one ear (2R), before surgical modification (intact), after removal of the pinna flange (canal and concha), and after removal of the concha (canal only). (a) Magnitude and angle of the impedance. (b) The real parts Re$\{Z_E\}$'s of the impedances

Figure 4: *$Z_{rad}(L,s)$ for an actual cat ear, looking out from the eardrum, with and without the pinna. (Rosowski* et al.*, 1988, Fig. 10).*

Details: In the problem specification we assume that the effective radiation area, looking out of the canal into the pinna from the tragus, is half the area of a sphere having the same diameter as the canal.

The cutoff frequency, defined such that the impedance of the mass of the air equals the radiation resistance, is given by $\omega_c/2\pi = \rho_o c/2\pi\rho_o r_0 = c/d_c\pi \approx 14.6$ [kHz].

Thus in this model, the radiation load is well approximated by a mass. Thus

$$R_{tragus} \approx \frac{sL_{rad} - z_0}{sL_{rad} + z_0} = -1 \tag{2}$$

since the resonant frequency determined by $f_0 \equiv z_0/L2\pi \approx 50$ [kHz]. The cat data indicate that the ear radiates well above about 3 kHz, and the canal of the cat is half that of the human ($d_{cat} \approx 3.6$ [mm]). My basis for this is Fig. 10 which shows a matched radiation impedance above about 3 kHz ("intact" line in Fig. 10 (a) (left panel).

Something seems to be off by an order of magnitude, and I can't find it. R_{tragus} should approach 0 above 3 kHz according the the data of Fig.10(a). This would happen if $f_0 \equiv z_0/L_{rad}2\pi$ were 3 kHz rather than 50. I have double checked this, and have not found an error. If the mass were 3 H rather than 50, the model would agree with the cat data. I do not know of corresponding human data.

It is the job of a horn to match two impedances. If the pinna is playing the role of a horn, and our formula for the radiation mass is off, due to the influence of this pinna-horn, then that would account for the differences we observe here. I think this is an interesting research project.

You might think that the radiation resistance can be ignored, *until you want to compute the power.* Thus the real part, though small, is critical and cannot be ignored!

The pinna modifies this characteristic load, as it must act as a horn, transforming the canal impedance, to better match the air. We have taken this effect into account by assuming that the area of the radiation impedance is determined by a half-sphere having a diameter the canal. This gives a cutoff frequency of 14 kHz. In the cat data shown in the include figure from Rosowski et al., the pinna resonance is more like 3-5 kHz. Some how the effective area is much larger, even for the cat pinna. I conclude that this pinna is very important to hearing. Try putting up your hands about your ears, to make larger pinna, and you will see what I mean. For more details download the paper (Rosowski *et al.*, 1988) posted on the class website.

(d) The simulation is to be done in the time domain. Define two arrays in Matlab© called $u_+(x - ct)$ and $u_-(x + ct)$, which represent the velocity of the waves in the ear canal, traveling in the two directions. At each time step, the data in the first array is shifted to the right, while the second is shifted to the left. The distance between two points corresponds in the array is $dx = cT$ where sampling rate $T = 1/F_s$.Choose the sampling rate F_s such that there are at least 5 points along the length of the ear canal (you can modify the length of the canal slightly to make the samples come out at the ends).

Solution: To have 5 points, there must be 4 sections of delay, so $F_s = 1/dT$ with $dT = 0.025/4$. Thus F_s =55.2 kHz. An alternative is to change the length of the canal to reduce this frequency to something that your sound card can work at, like $F_s = 44.100$ [kHz]. You might do this if you wish to listen to sounds processed by the middle ear model.

(e) Find the formula for the input impedance $Z(0, s)$ of the middle ear at the entrance of the ear canal, when the cochlea is "blocked" ($Z_c = \infty$ or $R(L, s) = 1$)? (Hint, this has a simple answer that you can easily derive).

Solution: When the end of the acoustic line is blocked there is a "short" across the end, namely the velocity (current) is zero. The reflectance at $x = 0$ is a delayed version of the reflectance at the cochlea ($x = L$), thus $R(0, s) = -e^{-j\omega 2L/c}$, which has an inverse Fourier transform of $r(0, t) = -\delta(t - 2L/c)$. Make sure you understand why this is! Do you understand where the delay is coming from?

It follows that the impedance is

$$Z(0, s) = z_0 \frac{1 + R(0, s)}{1 - R(0, s)} = z_0 \frac{1 + e^{-s2L/c}}{1 - e^{-s2L/c}}.$$

A little algebra and we find

$$Z(0, s) = jz_0 \frac{\cos(sL/c)}{\sin(sL/c)} = j\frac{\rho_o c}{A} \cot(\omega L/c) = z_0 \coth(sL/c).$$

This may be written in the time domain by a Taylor series, and it is a train of impulses spaced $2L/c$ apart. In other words, $R(0, s) \leftrightarrow r(x = 0, t) = \delta(t - 2L/c)$ is the same as impedance $Z(\omega) = -j\frac{\rho_o c}{A} \cot(\omega L/c)$.

Personally I find this trivially obvious, once you think hard about it, yet "kinda" cool.

(f) Find the two reflection coefficients and plot their magnitude frequency response.

Solution: In each case

$$R(s) = \frac{Z(s) - z_0}{Z(s) + z_0},$$

with $z_0 = \rho_o c/A$ and $Z(s)$ as the impedance load, at each end.

The reflectance at each end is expressed as a numerator over denominator polynomial, namely we need four polynomials, $N(0, s)$ and $D(0, s)$ at the stapes end, defined as

$$R(0, s) = \frac{N(0, s)}{D(0, s)} = \frac{sL_{rad}||R_{rad} - z_0}{sL_{rad}||R_{rad} + z_0} \approx \frac{L_{rad}s - z_0}{L_{rad}s + z_0}$$

which has a first order pole and a zero as roots of $N(L,s)$ and $D(L,s)$. $R(0,s) \approx 1$ due to the high mass. Because of the horn of the pinna, this may be unrealistic Rosowski *et al.* (1988).

At the cochlear end

$$R(L,s) = \frac{N(L,s)}{D(L,s)} = \frac{R_c + 1/sC_{al} - z_0}{R_c + 1/sCal + z_0}$$

which also has a first order pole and zero. This last expression is 1 at low frequencies ($f < 0.8$ [kHz]), where the impedance of the annular ligament dominates, and becomes 1/3 at high frequencies ($f > 0.8$ [kHz]).

The two reflectance functions may be inverse Laplace transformed to give impulse responses $r(0,t)$ and $r(L,t)$, which then may be convolved with $u^+(t)$ to give $u^-(t)$ at each end of the line. These filter are designed next.

(g) What are the coefficients of the time domain filters that implement the reflection coefficients?

(Hint 1: Use Matlab's `bilinear()` command. Hint 2: here is the answer to 1 decimal place, to help you debug up to this point. I've used a sampling rate of ≈70 kHz to compute this: $R(0,z^{-1}) = (.5 - .08z^{-1})/(1 - .4z^{-1})$ and $R(L,z^{-1}) = (0.3 - 0.3z^{-1})/(1 - 0.95z^{-1})$.)

Solution: These filters are determined by Matlab's `bilinear` command, with the coefficients for the two reflectance functions given above

[Nd,Dd] = bilinear(N,D,Fs)

To learn how to do this, type:

doc bilinear

(h) What are the poles and zeros in s and in z^{-1} planes?

(i) Compute and plot the input impedance (numerically derived at $x = 0$), using your Matlab model at the point labeled $P_{ec}(\omega)$ (i.e., at $x = 0$). Use log-frequency [Hz] and dB for the magnitude. Use z_0 as the dB reference (e.g., plot $20\log_{10}(|Z_c(0,f)|/z_0)$). Don't forget that $Z \equiv P/U$ where $p = (u^+ - u^-)/z_0$ and $p \equiv u^+ - u^-$. Compute the time sequence for p and u and then FFT these and take the ratio. Be sure that each function has settled to zero or else you will alias the answer. I am not asking for the transmission line model, I'm looking for the numerical solution.

Solution: This is done by finding both $p_{ec}(0,t)$ and $u_{0,t}$ from the time domain model, and then FFT'ing them, and taking ratios. The answer should look like a transmission line terminated in an impedance that is close to its characteristic impedance. In other words, there should be a very shallow standing wave. At low frequencies, the impedance should become stiffness dominated due to the stiffness of the annular ligament, represented by the capacitor C_{al}. The radiation load reflects most of the energy coming from the canal below the resonant frequency of the radiation load $\omega_c = c/r_0$, which is determined by the diameter of the canal (see the solution to question 3, above). This means that below 29.3 kHz, the radiation impedance will be dominated by the inductance, and therefore the reflection coefficient $R(0,s) \approx -1$. It would be good to know the precise value of this resonant frequency, to give a more precise estimate of the reflectance at the input. I should have asked for these numbers in an early part of the problem, but I didn't think of it. Too bad, because that would have helped.

(j) Using the time domain model, compute the *tragus* transfer function of the middle ear, defined as the ratio of the cochlear pressure to the ear canal pressure $P_c(\omega)/P_{ec}(\omega)$. Pressure P_{ec} is more precisely P_{tragus}.

Solution: The intent here was to do this numerically, not analytically. This is done by finding the impulse response to a current pulse in the ear canal, and then computing the cochlear velocity $u_c(L,t) = u^+(L,t) - u^-(L,t)$. From this velocity find the cochlear pressure ($P_c = R_c u_c(L,t)$), and then divide that by the ear canal pressure, which is computed from the canal velocity $u_{ec}(0,t) = u^+(0,t) - u^-(0,t)$ with the relation $p_{ec}(0,t) = z_{rad}(t) \star u_{ec}(0,t)$. To find the transfer function in question, you need to FFT() the two impulse responses, and take their ratio. Then plot this on log-log coordinates. It should look like a high pass filter, with a 6-9 dB high pass slope, and an in-band ripple of not more than a few dB, between 0.8 kHz and $F_s/2$.

(k) Compute the free field transfer function of the middle ear, defined as the ratio of the cochlear pressure $P_c(\omega)/P_0(\omega)$. I suggest you work backward, and find the pressure P_0, the assumed source, required to give the given source U_{ec}.

Note that this question may be just too hard, and take to long, so do this problem last (or just skip it if you don't have the time to deal with it. (30-60 min)

Solution:

Method 1: Solve the problem with a velocity source at the entrance to the ear canal. Find all the velocities and pressures. From the impedance divider equation

$$\frac{P_{ec}}{P_0} = \frac{Z_{ec}}{Z_{ec} + Z_{rad}}. \tag{3}$$

One may then solve this for $P_0(\omega)$, given P_{ec} and the two impedances.

Method 2: One may also write the system in chain matrix form:

$$\begin{bmatrix} P_0 \\ U_0 \end{bmatrix} = \begin{bmatrix} 1 & Z_{rad} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \sinh(\gamma L) & z0\cosh(\gamma L) \\ y_0\cosh(\gamma L) & \sinh(\gamma L) \end{bmatrix} \begin{bmatrix} 1 & sC_{al} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ Z_c & 1 \end{bmatrix} \begin{bmatrix} P_c \\ U_c \end{bmatrix} \tag{4}$$

where $\gamma = \omega/c$, $Z_{rad} = sL_r R_{rad}/(sL_r + R_{rad})$ and $y_0 = 1/z_0 = A/\rho_o c$. It is assumed that $U_c = 0$ since the cochlear impedance is the load, and there is no additional load. At the input end, the radiation impedance looks like a short circuit compared to the characteristic impedance of the canal (i.e., $R(0,x) = -1$). Thus $U_0 = P_0/R_{rad}$. If we were to make these substitutions, we would have two equations (one matrix equation) in two variables, P_0 and P_c. There would still be lots of algebra to do.

(l) A 16 electrical ohm earphone delivers 120 dB SPL into the ear canal with 1 volt RMS input at 1 kHz. Using your middle ear model, calculate the efficiency (Acoustic/Electrical power ratio, in %) of the earphone, at 1 kHz. Solution: The idea behind this problem is that since you know the power in the ear canal and the electrical power, you can compute the efficiency of the earphone by taking the ratio. The Acoustic power is just $0.5|P|^2/R$ where P is the pressure at 120 dB SPL, and R is the real part of the ear canal impedance, which you got from the model. The electrical power is $|V|^2/2R_e$, where V=1 volt RMS, and $R_e = 16$ ohms. Thus the electrical power is 1/32 watts (31 mW).

Since the frequency is 1 kHz, we may safely ignore the annular ligament stiffness sC_{al}, and just assume that the power delivered is into the cochlear resistance $R_c = 2\rho_o c/A$ ohms. Also 120 dB SPL is 120-94=26 dB (a factor of 20) larger than 1 Pa (94 dB SPL). Thus 120 dB SPL is just 20 Pa, and the acoustical power is $20^2/R_c$ Watts, or 22 μ watts. The power ratio is 0.66×10^{-3} (0.06% efficiency). This earphone is really inefficient, by a factor of 1000. I believe that these numbers are realistic.

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