ECE 403 **HWa – Version 1.07 January 17, 2019** **Spring 2019**

Univ. of Illinois **Due: Lec 7** **Prof. Allen**

Topic of this homework: Convolution, Fourier and Laplace Transforms, Impedance functions of Laplace frequency $s = \sigma + \jmath\omega$, Filter classes.

Deliverable: Show your work. If you hand it in late, you will get zero credit. I need a **stapled paper copy**, with your name on it. Please **NO** files.doc

Note: Each person is to do their own final writeup, but you may (and should) discuss it as much as you wish between yourselves. You're crossing the line if you share computer files. The general rule is, "look, understand, but do not copy." In otherwords, you need to process all the words you write through your eyes and fingers. If you use material from elsewhere, you must cite the source.

Terminology and acoustic constants

Definitions of common acoustic variables,[1] the mathematical symbols and the units, as used in the text (see p. 180-181).

Variable name	Symbol	[Units]
pressure	$P(t) = P_o + p(t)$	[N/m^2 = Pa]
density	$\rho = \rho_o + \delta(t)$	[kg/m^3]
temperature	$T(t) = T_o + \tau(t)$	[K°] Kelvin
Constants		
Atm Pressure	$P_o = 10^5$	[Pa]
Abs temperature	$T_o = 273$	[K°]
sound speed	$c_o = 345$	[m/s]
density	$\rho_o = 1.18$	[kg/m^3]
specific impedance	$\rho_o c_o = 407$	[Rayls]
viscosity	$\mu_o = 1.86 \times 10^{-5}$	[Ns/m^2] Poise
Boyle's Law	$P_o = \rho_o T_o$ Const.	[Pa]
adiabatic law	$p(t) = \delta^\gamma$ Const.	[Pa]
thermal conductivity	$\kappa_T = 25.4 \times 10^3$	[N/sK]
specific heat cap @V$_o$	c_v	[J/kg]
specific heat cap @P$_o$	c_p	[J/kg]
Boltzman's const.	$k = 1.38 \times 10^{-23}$	[J/molecule]
ratio of specific heats	$\gamma = c_p/c_v = 1.4$	[ ]
Loss constant	$\gamma' = 6.618 \times 10^{-3}$	
Propagation function	$\kappa(s) = \left(s + \beta_0\sqrt{s}\,\right)/c_o$	$\mathcal{P}^+(s) = e^{-\kappa(s)x}$

where

$$\gamma' = \sqrt{\mu_o}\left(1 + \sqrt{5/2}\left(\sqrt{\gamma} - \frac{1}{\sqrt{\gamma}}\right)\right) = 6.618045 \times 10^{-3}.$$

For the case of a cylindrical waveguide the radius is $R = 2S/P$ (S is the area and P the perimeter). Then

$$\beta_o = \frac{1}{R}\frac{\gamma'}{\sqrt{\rho_o}} = \frac{1}{R}6.0924 \times 10^{-3}.$$

The lossy wave-number $\kappa(s) = (s + \beta_0\sqrt{s})/c_o$ includes thermal and viscous loss.[2]

If the tube diameter D [m] is the tube diameter, then $\beta_0 D = 2\gamma'/\sqrt{\rho}$.

[1] c_p, c_v: `https://en.wikipedia.org/wiki/Heat_capacity#Specific_heat_capacity`

[2] Appendix F, *An Invitation to Mathematical Physics*, Allen, 2019.

1 Basic Acoustics

1. What is the formula for the speed of sound?
   (a) Identify the variables in the formula for the speed of sound: Names, units, and values of consonants.
   (b) What is the meaning of γP_0?
   (c) Does P_0 depend on temperature? Explain?
   (d) Does ρ_0 depend on temperature? Explain.
2. State the Gas law, in terms of the universal gas constant R_0.
   (a) Specifically discuss the value and nature of the constant PV/T.
   (b) What is 1 [mol] of air?
   (c) What is the relationship between the density of air ρ and the volume V.
   (d) Describe the relationship between the gas constant R, Boltzman's constant k and Avagadro's number N_A.
3. What is the form of the dependence of the speed of sound on temperature? Namely give the formula for $c(T)$, and explain the dependence.

1.1 Basic equations of sound propagation:

1. Write out the 2x2 matrix equation that describes, in the frequency domain, the propagation of 1 dimensional sound waves in a tube having area $A(x)$, in terms of the pressure $P(x,s)$ and volume velocity $U(x,s)$:
2. Assuming $A(x)$ is constant, rewrite these equations as a second order equation solely in terms of the pressure P (remove U), and thereby find the formula for the speed of sound in terms of Z and Y:

2 deciBels [dB]

1. Express decibels in terms of the pressure ratio
2. State the reference pressure for dB-SPL
3. What is the attenuator gain, expressed in dB, if the voltage is reduced by a factor of 2?
4. How many millibels [mB] in 1 bel [B]?
5. Give the formula for the intensity in mB units.
6. Give the formula for the sound pressure level in cB (centibel) units.
7. There are two different definitions of acoustic dB, one based on pressure

   $$dB_p = 20\log_{10}(P/P_{ref})$$

   and a second based on acoustic intensity

   $$dB_I = 10\log_{10}(I/I_{ref})$$

   where P is the pressure in Pascals and $I \equiv |P|^2/\rho c$ is the acoustic intensity. Here ρ is the density of air and c is the speed of sound. The product $\rho c = 407$ [Rayls] is called the *Specific acoustic impedance* of air.

   Demonstrate that $P_{ref} \equiv 20\ \mu$Pa is the same as $I_{ref} \equiv 10^{-12}$ [W/m^2].
8. What is the acoustic impedance observed by a plane wave? What are its units?

3 The Helmholtz Resonator

A bottle has a neck diameter of 1 [cm] and is l =1 cm long. It is connected to the body of the bottle "barrel" which is 5 cm in diameter and L =10 cm long. Treat the barrel as a short piece of transmission line, closed at one end, which looks like a compliance $C = V_{barrel}/\gamma P_0$, and the neck which look like a mass $M = \rho_0 l/A_{neck}$. These two impedances are in series, since they both see the same volume velocity (flow).

1. Set the impedance to zero and solve for the bottle's resonant frequency, in terms of M and C.
2. Write out the formula for the resonant frequency in terms of the physical dimensions of the bottle.
3. Calculate the resonant frequency in Hz for the dimensions given.
4. **Extra credit:** Blow into a blottle and measure the resonant frequency by recording the tone, and taking the FFT of the resulting waveform, and finding the frequency.

4 Fourier and Laplace transforms

1. Derive the Fourier transform for the step function $u(t-1)$.
2. Derive the Laplace transform for the step function $u(t-1)$.
3. Find the Fourier transform $X(\omega)$ of
$$x(t) = u(t) - u(t-.001) \leftrightarrow X(\omega)$$
4. Find the Laplace transform $\mathcal{X}(s)$ of $x(t) = u(t) - u(t-.001) \leftrightarrow \mathcal{X}(s)$
5. If $\tilde{u}(t) \leftrightarrow \widetilde{U}(\omega)$ is the $\mathcal{F}$ step function, what is $\tilde{u}(t) \star \tilde{u}(t)$?
6. Hand-plot (or describe the plot of) $|X(\omega)|$ and $|\mathcal{X}(s)|$.
7. Where are the poles and zeros in each case above.

5 Laplace Transforms

Laplace transforms: given that $f(t) \leftrightarrow F(s)$

1. Find the Laplace transform of $\delta(t)$, $df(t)/dt$, $\int_{-\infty}^{t} \delta(t)dt$, and $\int_{-\infty}^{t} u(t)dt$.
2. If $f(t) = 1/\sqrt{\pi t}$ has a Laplace transform $F(s) = 1/\sqrt{s}$:
   (a) What is the inverse Laplace transform of $\sqrt{s}$?
   (b) What is $f(-1)$?
   (c) Integrate $I = \int_C \frac{1}{s} ds$ around the unit circle centered on $s = 0$.
   (d) Integrate $\int_C \frac{1}{s} ds$ around the unit circle centered on $s = .5$ (i.e., $\sigma = .5, \omega = 0$), and $s = -2$.

6 Convolution

Given two "causal" sequences $a_n = [\cdots, \therefore, 0, 1, 0, -1, 0, \cdots]$ and $b_n = [\cdots, \therefore, 1, -1, 0, 0, \cdots]$. Here the rising dots $\therefore$ define $t = 0$, before and at which time the signal is zero.

1. Find causal sequence $c \equiv a \star b$ by direction convolution
2. Form the polynomials $A(z) = \sum a_n z^n$ and $B(z) = \sum b_n z^n$, and find $C(z) = A(z) \cdot B(z)$
3. What can you say about the sequence c_n and the coefficients of $C(z)$?