Topic of this homework: Acoustics; Fourier Transform; Signal processing;

Deliverable: Show your work.

If you hand it in late, you will get zero credit (I will be handing out my solution at that time). You will only get credit for what you hand in. I want a paper copy, with your name on it. Please **no** files.doc.

No matter how limited your results, on the due date submit what ever you have. Some credit is better than NO credit.

Note: This homework will be discussed by the entire class on Disc: 2/16. You need to be there. Each person is to do there own final writeup, but obviously you can discuss it as much as you like between yourselves.

1 Basic Acoustics

1. What is the formula for the speed of sound?
2. Identify the variables: Names, units, and values of consonants.
3. What is the meaning of ηP_0?
4. Does P_0 depend on temperature? Explain?
5. Does ρ_0 depend on temperature or P_0? Explain.
6. What is the form of the dependence of the speed of sound on temperature? Namely give the formula for $c(T)$, and explain the dependence.
7. decibels
   (a) Express decibels in terms of the pressure ratio
   (b) State the reference pressure for dB-SPL
   (c) What is the attenuator gain, expressed in dB, if the voltage is reduced by a factor of 2?
   (d) How many millibels [mB] in 1 bel [B]?
   (e) Give the formula for the intensity in mB units.
   (f) Give the formula for the sound pressure level in cB (centibel) units.

1.1 Basic equations of sound propagation:

1. Write out the 2x2 matrix equation that describes, in the frequency domain, the propagation of 1 dimensional sound waves in a tube having area $A(x)$, in terms of the pressure $P(x, s)$ and volume velocity $U(x, s)$:
2. Assuming $A(x)$ is constant, rewrite these equations as a second order equation solely in terms of the pressure P (remove U), and thereby find the formula for the speed of sound in terms of Z and Y:

1.2 Helmholtz Resonator

A bottle has a neck diameter of 1 [cm] and is l =1 cm long. It is connected to the body of the bottle "barrel" which is 5 cm in diameter and L =10 cm long. Treat the barrel as a short piece of transmission line, closed at one end, which looks like a compliance $C = V_{barrel}/\eta P_0$, and the neck which look like a mass $M = \rho_0 l/A_{neck}$. These two impedances are in series, since they both see the same volume velocity (flow).

1. Find the resonant frequency of the bottle. That is, write out the formula for the resonant frequency in terms of the dimensions of the bottle. Hint: set the impedance to zero and solve for the resonant frequency in terms of M and C, given above.
2. Calculate the resonant frequency in Hz for the dimensions given.
3. Blow into a bottle and measure the resonant frequency, by recording the tone, and taking the FFT of the resulting waveform (i.e., with Matlab), to find the frequency. Compare this to the formula. Can you explain any error?
4. There are two different definitions of acoustic dB, one based on pressure

   $$dB_p = 20 \log_{10}(P/P_{ref})$$

   and a second based on acoustic intensity

   $$dB_I = 10 \log_{10}(I/I_{ref})$$

   where P is the pressure in Pascals and $I \equiv |P|^2/\rho c$ is the acoustic intensity. Here ρ is the density of air and c is the speed of sound. The product $\rho c = 412$ [Rayls] is called the *Specific acoustic impedance* of air.

   Demonstrate that $P_{ref} \equiv 20\ \mu$Pa is the same as $I_{ref} \equiv 10^{-12}$ [W/m^2].
5. What is the acoustic impedance observed by a plane wave? What are its units?

2 Signal processing and Acoustics

1. A microphone with a sensitivity of 50 [mV/Pa] is in a sound field of very wideband pulses (clicks). The pulses are much wider in bandwidth than the microphone, which has a frequency response that is flat between $f = 0$ to $f = 20$ [kHz] and is zero for $|f| > 20$ [kHz]. The phase response of the microphone is $\phi(f) = -2\pi f 10^{-3}$ radians. The observed peak voltage across the microphone terminals on an oscilloscope is 1 [volt].
   (a) Find the click intensity (wide band) over the bandwidth of the microphone
   (b) Find the *spectral level*, defined as the pressure in a 1 Hz bandwidth. Give this both in terms of [watts] and [dB-SPL].
   (c) You are given independent information that the true bandwidth of the clicks may be well approximated as constant to 50 kHz, and then zero beyond. What is the true intensity of the clicks?
   (d) If the microphone is driven as a loudspeaker, give its impulse response.
   (e) As specified, does the loudspeaker obey all of the network postulates? If not, which are not obeyed, and why?

3 Name that transform

1. You are given a specification of the time and frequency properties of some signals and you are asked to name the Fourier type transform that would be used to analyze these signals

   (a) The time response is zero for $t < 0$ and the frequency response is a function of the radian frequency $\omega = 2\pi f$

   (b) The time response is zero for $t > 0$ and the frequency response is a function of the radian frequency $\omega = 2\pi f$

   (c) The time response is given at points $t_n = nT$, where $T = 1/F_s$ with $F_s = 44100$ kHz, and the frequency response is specified outside the unit circle.

   (d) The time response is given at times $t[n] = nT$ for integer n and constant T, and the frequencies are given at $f[k] = k/T$

2. Find the Fourier series expansion of the periodic function

$$f((t))_T \equiv \sum_{n=-\infty}^{\infty} \delta(t - nT)$$

   Give the formula for $F[k]$. Show your work. Explain how you get the solution.

3. Find the Laplace transform of $h(t) = 3e^{-t/\tau}U(t)$. Give an example of an electrical circuit that has this impulse response.

4. If the impulse response of some system is $h(t) = 3e^{t/\tau}U(t+1)$, describe the interesting things about the system.

5. What is the basic idea behind an *analytic function?* Give an example of a function that is analytic, and one that is not.

6. Laplace vs. Fourier

   (a) When do you use a Laplace transform and when do you use the Fourier transform?

   (b) Give an example where you can use both.

   (c) Give an example where you cannot use the Laplace transform.

7. Given the transform pair $f(t) \leftrightarrow F(\omega)$ one may prove that $F^*(t) \leftrightarrow 2\pi f^*(\omega)$.

   Apply this relationship to the following transform pairs, to derive new transform pairs (I worked out the first question, as an example):

   (a) $\delta(t) \leftrightarrow 1$
   Solution: $f(t) = \delta(t) \leftrightarrow F(\omega) = 1$. Thus applying the above relationship we find that if the time function is 1 then the transform is $2\pi\delta(\omega)$.

   (b) $e^{j\omega_0 t} \leftrightarrow 2\pi\delta(\omega - \omega_0)$

   (c) $U(t) \leftrightarrow \pi\delta(\omega) + 1/j\omega$
   All of the above solutions need a careful checking!

4 Loudspeaker impedance

A small loudspeaker is made from a ball of piezoelectric material 3 cm in diameter.

1. Assume that the loudspeaker is in a baffle. Argue why you can replace the baffled speaker with a half-sphere, and then show that the radiation impedance for the loudspeaker is an inductor L in parallel with a resistance R.

2. Derive the formulas for R and L, and give the numerical values of these constants (diameter = 3 [cm]). Hint: I derived these in class.

3. Write out the formula for $Z(\omega) = R(\omega) + jX(\omega)$ and plot the real $R(\omega)$ and imaginary part $X(\omega)$ part of the impedance, as a function of frequency f, between 100 [Hz] to 10 [kHz]. Explain your results.

5 Power series of Z(s) and s(Z)

If a function $Z(s)$ is analytic in s, then by definition, all its derivatives exist. Namely

$$Z(s) = \sum_n a_n s^n,$$

converges, with

$$a_n = \frac{d^n}{ds^n} Z(s)\bigg|_{s=0},$$

for any $s = s_0$ in the *region-of-convergence* (ROC), e.g.,

$$Z(s - s_0) = \sum_n \frac{d^n Z(s)}{ds^n}\bigg|_{s=s_0} (s - s_0)^n,$$

as long as $Z(s)$ is analytic at s_0. Likewise the *inverse* must also exists. Namely

$$s(Z - Z_0) = \sum b_n (Z - Z_0)^n.$$

That the function *and it inverse* must exist follows from the fact that $Z(s - s_0)$ is *analytic* around $s = s_0$. That is, if it is analytic, all the derivatives wrt s must exist for both the function, and its inverse [e.g., $Z(s)$ and $s(Z)$]. Using a more transparent notation, $w(s) = \sum_k W_k s^k$ and $s(w) = \sum_k S_k w^k$ for complex constants W_k and S_k.

To do: Give an example of this for the *entire* function $w = e^s - 1$ and its inverse $s = \log(1+w)$, find W_k and S_k. That is, write down the the power series for this pair of functions. The discuss the implications of such relationships.

6 History

1. Describe some interesting things about Pythagoras. Be sure to include when, where, and why. What might this have to do with Audio Engineering?

2. Give a few reasons that Newton might be relevant to Audio Engineering.

3. What year did Fourier work out his analysis of heat transfer? How did he do it?