ECE 403 **HWd** – **Version 1.00 March 29, 2016** **Spring 2016**

Univ. of Illinois **3/3, Disc: 3/10, Due: 3/17** **Prof. Allen**

Topic of this homework: Acoustics; Helmholtz resonator; Transmission lines (TL) & Horns & 1D solution; Reflectance (Impedance change on TL)

Deliverable: Show your work.

If you hand it in late, you will get zero credit. I would like a paper copy, with your name on it. **No** doc files.

Some credit is better than NO credit.

1 Basic Acoustics

1. What is the formula for the speed of sound?
2. Identify the variables: Names, units, and values of consonants.
3. What is the meaning of ηP_0?
4. Does P_0 depend on temperature? Explain?
5. Does ρ_0 depend on temperature or P_0? Explain.
6. State the Gas law, in terms of the universal gas constant R_0.
   (a) Specifically discuss the value and nature of the constant PV/T.
   (b) What is 1 [mole] of air?
   (c) What is the relationship between the density of air ρ and the volume V.
   (d) Describe the relationship between the gas constant R, Boltzman's constant k and Avagadro's number N_A.
7. What is the form of the dependence of the speed of sound on temperature? Namely give the formula for $c(T)$, and explain the dependence.

2 deciBels [dB]

1. Express decibels in terms of the pressure ratio
2. State the reference pressure for dB-SPL
3. What is the attenuator gain, expressed in dB, if the voltage is reduced by a factor of 2?
4. How many millibels [mB] in 1 bel [B]?
5. Give the formula for the intensity in mB units.
6. Give the formula for the sound pressure level in cB (centibel) units.

7. There are two different definitions of acoustic dB, one based on pressure

$$dB_p = 20\log_{10}(P/P_{ref})$$

and a second based on acoustic intensity

$$dB_I = 10\log_{10}(I/I_{ref})$$

where P is the pressure in Pascals and $I \equiv |P|^2/\rho c$ is the acoustic intensity. Here ρ is the density of air and c is the speed of sound. The product $\rho c = 412$ [Rayls] is called the *Specific acoustic impedance* of air.

Demonstrate that $P_{ref} \equiv 20$ μPa is the same as $I_{ref} \equiv 10^{-12}$ [W/m^2].

8. What is the acoustic impedance observed by a plane wave? What are its units?

3 The Helmholtz Resonator

A bottle has a neck diameter of 1 [cm] and is l =1 cm long. It is connected to the body of the bottle "barrel" which is 5 cm in diameter and L =10 cm long. Treat the barrel as a short piece of transmission line, closed at one end, which looks like a compliance $C = V_{barrel}/\eta P_0$, and the neck which look like a mass $M = \rho_0 l/A_{neck}$. These two impedances are in series, since they both see the same volume velocity (flow).

1. Set the impedance to zero and solve for the bottle's resonant frequency, in terms of M and C.

2. Write out the formula for the resonant frequency in terms of the physical dimensions of the bottle.

3. Calculate the resonant frequency in Hz for the dimensions given.

4. **Extra credit:** Blow into a blottle and measure the resonant frequency by recording the tone, and taking the FFT of the resulting waveform, and finding the frequency.

4 Wave equation

4.1 History of the wave equation

1. What year did d'Alembert derive his solution to the wave equation?

2. What is the form of D'Alembert's solution?

4.2 The Webster wave equation:

The *Webster Horn equation* may be written in the time domain as 1D transmission line equation:

$$\frac{\partial}{\partial x}\begin{bmatrix} p(x,t) \\ \nu(x,t) \end{bmatrix} = -\begin{bmatrix} 0 & \frac{\rho_0}{A(x)} \\ \frac{A(x)}{\eta_0 P_0} & 0 \end{bmatrix} \frac{\partial}{\partial t}\begin{bmatrix} p(x,t) \\ \nu(x,t) \end{bmatrix}, \tag{1}$$

where $\nu(x,t) = A(x)u(x,t)$ is the *volume velocity*, more generally defined as the integral over the normal component of the particle velocity $u(x,t)$, over the cross-sectional area $A(x)$ of the tube. Transforming to the frequency domain we have

$$\frac{d}{dx}\begin{bmatrix} P(x,\omega) \\ V(x,\omega) \end{bmatrix} = -\begin{bmatrix} 0 & Z_s(s,x) \\ Y_s(s,x) & 0 \end{bmatrix} \begin{bmatrix} P(x,\omega) \\ V(x,\omega). \end{bmatrix} \tag{2}$$

Here we use the complex *Laplace frequency* s when referring to the *per-unit impedance*

$$Z_s(s,x) \equiv s\frac{\rho_0}{A(x)} = sM(x) \tag{3}$$

and *per-unit admittance*

$$Y_s(s,x) \equiv s\frac{A(x)}{\eta_0 P_0} = sC(x), \tag{4}$$

where $M(x) = \rho_0/A(x)$ is the horn's *per-unit-length mass*, and $C(x) = A(x)/\eta_0 P_0$ *per-unit-length compliance*, to remind ourselves that these functions must be *causal*, and except at their poles, *analytic* in s.

The horn of a loudspeaker cone is either conical $A(x) = A_0(x/x_0)^2$, or in the shape of an exponential.

4.3 To Do:

1. Starting from the transmission line equations given above, and assuming an exponential area function,

$$A(x) = A_0 e^{2mx},$$

(m is a positive constant, called the *horn flair parameter*) derive the exponential horn equation for the pressure

$$\frac{\partial^2 p}{\partial x^2} + 2m\frac{\partial p}{\partial x} + \frac{\omega^2}{c^2}P(x,\omega) = 0 \tag{5}$$

2. Show that the solution to Eq. 5 is of the form

$$P^{\pm}(x,s) = e^{-mx}e^{\mp\sqrt{m^2+(s/c)^2}\,x}.$$

4.4 Reflectance:

1. Find (derive) the formula for the "input" impedance of a transmission line, having characteristic impedance $z_0(x,s)$, in terms of the reflectance. Define all the terms. Hint, I did this in class several times.

2. Find the formula for the reflectance $R(s)$ in terms of the load impedance $Z_L(s)$ and the characteristic impedance z_0 if:[1]

   (a) $Z_L(x,s) = r$ [Nt-s/m^5]

   (b) $Z_L(x,s) = 1/sC$ [Nt-s/m^5]

   (c) $Z_L(x,s) = r||sM$ [Nt-s/m^5]

   (d) Two transmission lines are in cascade, the first one having an area of 1 [cm^2] and a second having an area of 2 [cm^2], with lengths L_1 and L_2 respectively, terminated with a resistor $r = \rho c/A$, where $A = 2\times 10^{-4}$ [m^2]. Find $R(x=0,s)$.

   (e) What is the inverse Laplace transform of

      i. $H(s) = 1/(s+1)$? Find $h(t)$.

      ii. $R(s) = \frac{Z-z_0}{Z+z_0}$ where $Z = 1$ and $z_0 = 2$? Find $r(t)$.

      iii. What if the line is 1 meter long and the speed of sound is 1 m/s?

      iv. $H(s) = s/(s+1)$?

[1] Note that r, C and M represent an acoustic resistance, compliance and mass. Namely they are positive constants.

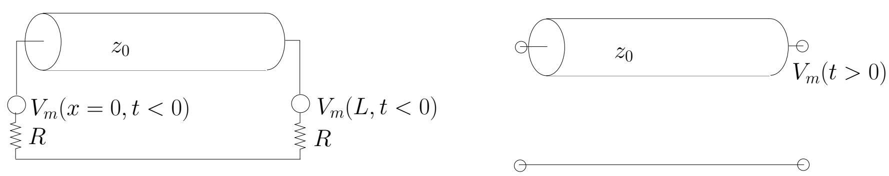
4.5 Basic equations of sound propagation:

1. Write out the 2x2 matrix equation that describes, in the frequency domain, the propagation of 1 dimensional sound waves in a tube having area $A(x)$, in terms of the pressure $P(x, s)$ and volume velocity $U(x, s)$:

2. Assuming $A(x)$ is constant, rewrite these equations as a second order equation solely in terms of the pressure P (remove U), and thereby find the formula for the speed of sound in terms of Z and Y:

5 Nyquist Thm on Thermal noise

The purpose of this problem is to do a simulation of Harry Nyquist's famous result on the noise of a resistor. The experiment is to use a Thevenin equivalent model of a resistor as a resistance R in series with a voltage source. A stub of transmission line having characteristic impedance z_0 is terminated in each end with this Thevenin model, with $R = z_0$. Then at $t = 0$, the resistance is short or open circuited. If open circuited you may watch the voltage at one end, and if short circuited, you may watch the current. Let's monitor the voltage with an open circuit. This setup is shown in the figure.



The transmission line stores the voltage at $t = 0$ once the switch is opened removing the resistors (and also the Thevenin source). At that point voltage $V_m(L, t)$ becomes periodic with a period of $2L/c$, where L is the length of the line and c is the speed of the wave.

Define an array that has a duration of the period, and load it with thermal random samples [`x=randn(1,N)` with N the number of noise samples]. Set the RMS to 1. The RMS may be computed using `std(x,1)`. Use a sampling period $T = 1/f_s$ with $f_s = 10 \times 10^3$ Hz for this experiment, and let $c = 345$ [m/s] be the speed of sound (an acoustic transmission line), with $L = 10$ [m].

1. What is the fundamental period of the noise?

2. Plot two periods of the time domain signal. Using `fft()`, find the spectrum of the periodic noise, for 1, 4 and 10 periods. Be sure to properly label all axes (with units)! For each of the FFT plots, show one figure of the full FFT (from 0 Hz up to the half-sample-rate), and one figure zoomed in on the range 0 to ∼300 Hz (linear axis, not log axis). Comment/explain any observations.

   Note: Let's say your array length is N. When creating 10 periods of random noise, don't use `randn(1,10*N)`. Use `randn(1,N)` to randomly populate 1 period, and then repeat that noise 10 times, so that it is identical. Remember, its in a delay line.

3. Why won't the values of your spectral peaks be the same as your fellow students? Average the values of your spectral peaks over many noise samples (that is, many initializations of the `randn(1,N)` array), and plot the resulting spectral average. Just do this for the case of 10 periods.

4. Given $T = 300$ degrees Kelvin and $k = 1.38 \times 10^{-23}$ [J/degree K], what would the RMS value of the voltage be? Justify your answer. Hint: "Johnson-Nyquist Noise". Note that by sampling we inherently band-limit the signal.