1 Problems NS4

Topic of this homework:

Leading edge velocity of neural spike, solutoin of diffusion line

Deliverable: Answers to problems

Problem *# 1: In this problem we wish to show that the solution to the diffusion transmission line (cable equation) is of the form*

$$V(x,t) = V_0(x - c_f t)$$

where c_f is the leading edge velocity of the spike. We will work with the same circuit as presented in home work 3 (NS-3) problem 5, repeated below.

Define the Nyquist *cut-off* wavelength λ_c as

$$\lambda_c f_{\max} = v_e = 23 \text{ [m/s]}$$
$$= \lambda_c/2 \text{ [mm]}$$

From a previous homework we used the Nyquist wavelength that $\Delta = \lambda/2$ and $\tau = RC$. Here Δ_x is taken to be the distance between nodes s, namely $\Delta_x = s$.

We wish to show that Eq. 1 is a solution of the diffusion line.

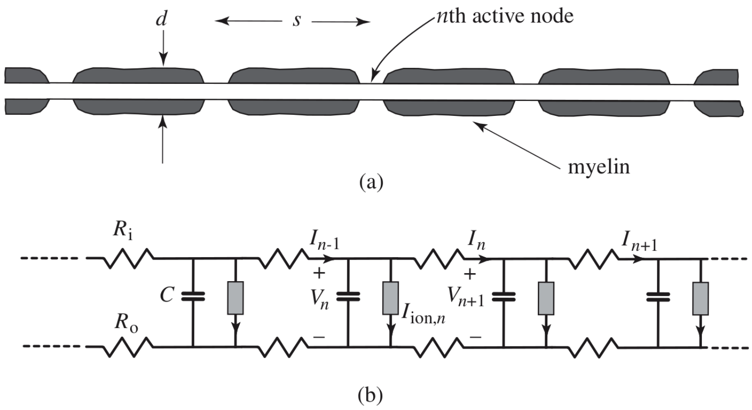

Figure 1: *Diagram of the Frog axon showing the physical and electrical circuit (Scott, p. 142).*

– 1.1: Write out the Transmission line equations for R_1 and C. In this case do not include a second resistor. As a result, the matrix will not by reversable. Assume $R_o = 0$ and suppress the subscript on R_1 (write it as R).

Sol:

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 1 & R \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ sC & 1 \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

Expanding the equation gives the transmission matrix $\mathcal{T}(s)$

$$\mathcal{T}(s) = \begin{bmatrix} 1 + sRC & R_1 \\ sC_1 & 1 \end{bmatrix}$$

■

– *1.2: Define $\tau = RC$. Find the eigen values of $\mathcal{T}(s)$.*

Sol: The general solution for the 2x2 matrix is (Allen, 2020, p. 311)

$$2\lambda_\pm = \mathcal{A} + \mathcal{D} \mp \sqrt{(\mathcal{A} - \mathcal{D})^2 + 4\mathcal{B}\mathcal{C}}$$

Thus

$$\begin{aligned} 2\lambda_\pm &= 2 + s\tau \mp \sqrt{4s\tau + s^2\tau^2} \ \ \text{(verified correct with syms)} \\ &\approx 2 + s\tau \mp 2\sqrt{s\tau} \ \ \text{when} \ \ |s| \ll 4/\tau \\ &= (2 \mp 1) + (\sqrt{s\tau} \pm 1)^2 \end{aligned}$$

Here we assume that if $4s\tau \gg s^2\tau^2$ then $1 \gg |s\tau/4|$, $|s\tau/4| \ll 1$, or $|s| \ll 4/\tau$. This is similar to the Nyquist condition which is a condition on the maximum frequency. ■

– *1.3: Find the transfer function of the difusion line cell $H_{12}(s) = V_2/V_1$.*

The goal is to show that Eq. 1 holds for the diffusion line. But the diffusion line is an infinite cascade of cells. Thus we need to estimate what happens when many cells are cascaded. $H_{12}(s)$ is only one cell.

Sol: Starting from the the two port equation and $\mathcal{T}$, the upper equation is

$$V_1 = (1 + s\tau)V_2 - RI_2.$$

Thus

$$H_{12}(s) = \frac{V_2}{V_1} = \frac{1}{1 + s\tau} + R\frac{I_2}{V_1}.$$

■

Problem *# 2: In this problem we wish to investigate the HH diode model.*

– 2.1: Run the matlab-similink program `hhmodel_7b.slx` *which may be downloaded from the website under Lecture 20. A second slightly different version may be found under Lecture 22.*

In your own words, discuss the evidence as to how it works. I suspect your answer to be at least 100 words (not one sentence). Show the plots of the circuit and the output, and explain what is going on in sufficient detail that a colleague can understand your discussion.