Chapter 6 – Reflection and Transmission

(6.1) When an acoustic wave traveling in one medium encounters another medium, reflected and transmitted waves are generated from the boundary. The ratios of the intensities and pressure amplitudes of the reflected and transmitted waves to the incident wave depend on the characteristic acoustic impedances ($r_0 c$) of the two media and the angle the incident wave makes with the interface.

Let's start with a few general definitions that you need to know (the next few sections will assume a plane wave formulation):

We define $z_n = r_n c_n$ as the characteristic acoustic impedance for medium 'n'.
The pressure transmission and reflection coefficients are defined as:

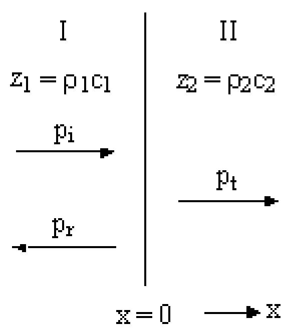
$$\tilde{T} = \frac{\tilde{P}_t}{\tilde{P}_i} \qquad \text{and} \qquad \tilde{R} = \frac{\tilde{P}_r}{\tilde{P}_i}.$$

Recall that for a plane wave the intensity is given by $\frac{P^2}{2 r_0 c}$ so that the intensity transmission and reflection coefficients are given by:

$$T_I = \frac{I_t}{I_i} = \frac{z_1}{z_2}\left|\tilde{T}\right|^2 \qquad \text{and} \qquad R_I = \frac{I_r}{I_i} = \left|\tilde{R}\right|^2.$$

(6.2) Transmission from one fluid to another: normal incidence.

We are going to consider several different cases. The first case is a simple reflection/transmission when an acoustic plane wave is incident normally on a single interface between two different (lossless) fluids. The second case will consider transmission through a fluid layer. A third case will consider a plane wave incident on a fluid-fluid interface at oblique incidence. A final case will look at reflection/transmission from a fluid-solid interface.
Looking at the first case:



$$\tilde{p}_i = \tilde{P}_i e^{j(wt - k_1 x)} \qquad \text{incident wave}$$

$$\tilde{p}_r = \tilde{P}_r e^{j(wt + k_1 x)} \qquad \text{reflected wave}$$

$$\tilde{p}_t = \tilde{P}_t e^{j(wt - k_2 x)} \qquad \text{transmitted wave}$$

We define the three waves, an incident wave traveling in the '+' x-direction, the reflected wave traveling in the '-' x-direction and the transmitted wave traveling in the '+' x-direction. The acoustic wave in each medium has a different wavenumber given by

$$k_1 = \frac{w}{c_1} = \frac{2p\, f}{c_1} = \frac{2p}{l_1} \qquad \text{and} \qquad k_2 = \frac{w}{c_2} = \frac{2p\, f}{c_2} = \frac{2p}{l_2}$$

where the wavenumber and wavelength are determined from the frequency of the acoustic wave and the speed of sound in each medium.

There are two boundary conditions that must be satisfied for all times across the interface, these boundary conditions allow us to determine the magnitudes of the reflected and transmitted waves in terms of the incident amplitude.

The first boundary condition states that the acoustic pressure must be equal on both sides of the interface or

$$\left(\tilde{p}_i + \tilde{p}_r \right)_{x=0} = \tilde{p}_t$$

which essentially says that there is no net force on the massless interface (Newton's 3rd Law). Examining this further we see

$$\tilde{P}_i e^{j(wt - k_1 0)} + \tilde{P}_r e^{j(wt + k_1 0)} = \tilde{P}_t e^{j(wt - k_2 0)}$$

$$\tilde{P}_i + \tilde{P}_r = \tilde{P}_t \qquad \text{(I)}$$

The second boundary condition states that the normal components of the particle velocity must be equal on both sides of the boundary (x-direction for the drawing above) or

$$\left(\tilde{u}_i + \tilde{u}_r \right)_{x=0} = \tilde{u}_t$$

which essentially says that there is no separation between "particles" at the interface. Examining this further we have (for a plane wave)

$$\frac{\tilde{P}_i}{z_1} e^{j(wt - k_1 0)} \hat{x} - \frac{\tilde{P}_r}{z_1} e^{j(wt + k\, 0)} \hat{x} = \frac{\tilde{P}_t}{z_2} e^{j(wt - k_2 0)} \hat{x}$$

$$\frac{\tilde{P}_i}{z_1} - \frac{\tilde{P}_r}{z_1} = \frac{\tilde{P}_t}{z_2} \qquad \text{or} \qquad z_2 \tilde{P}_i - z_2 \tilde{P}_r = z_1 \tilde{P}_t \ . \qquad \text{(II)}$$

Taking (I) and multiplying by $\frac{1}{\tilde{P}_i}$ gives

$$1 + \frac{\tilde{P}_r}{\tilde{P}_i} = \frac{\tilde{P}_t}{\tilde{P}_i}$$

or $\quad 1 + \tilde{R} = \tilde{T}$. (an important relation to remember)

Taking (II) and multiplying by $\frac{1}{\tilde{P}_i}$ gives

$$z_2 - z_2\frac{\tilde{P}_r}{\tilde{P}_i} = z_1\frac{\tilde{P}_t}{\tilde{P}_i}$$

or $\quad z_2 - z_2\tilde{R} = z_1\tilde{T}$.

Substituting in for $\tilde{T}$ from above yields

$$z_2 - z_2\tilde{R} = z_1\left(1+\tilde{R}\right).$$

We seek the reflection coefficient so we solve for $\tilde{R}$ giving

$$\boxed{\tilde{R} = \frac{z_2 - z_1}{z_2 + z_1}}$$

Likewise, solving for $\tilde{T}$ by using $1+\tilde{R} = \tilde{T}$

$$\tilde{T} = 1 + \frac{z_2 - z_1}{z_2 + z_1} = \frac{z_2 + z_1}{z_2 + z_1} + \frac{z_2 - z_1}{z_2 + z_1}.$$

$$\boxed{\tilde{T} = \frac{2z_2}{z_2 + z_1}}$$

Let's consider some 5 different cases and see what they mean:

1) $z_2 = z_1$
2) $z_2 > z_1$
3) $z_2 < z_1$
4) $\frac{z_2}{z_1} \to \infty$, that is, $z_2 \gg z_1$
5) $\frac{z_2}{z_1} \to 0$, that is, $z_2 \ll z_1$

Case (1): $z_2 = z_1$

$$\tilde{R} = \frac{z_2 - z_1}{z_2 + z_1} = 0 \quad \to \quad \tilde{P}_r = 0$$

There is no reflected wave and

$$\tilde{T} = \frac{2z_2}{z_2 + z_1} = 1 \quad \to \quad \tilde{P}_t = 1$$

or the transmitted wave has the same amplitude as the incident wave.
This is as it should be, so all is right with the world and the mathematical description.

Case (2): $z_2 > z_1$

$$\tilde{R} = \frac{z_2 - z_1}{z_2 + z_1} > 0 \quad \rightarrow \quad \frac{\tilde{P}_r}{\tilde{P}_i} > 0$$

In other words, the reflected wave is in phase with the incident wave. This is reminiscent of the string reflection from a free end. For the transmitted wave

$$\tilde{T} = \frac{2z_2}{z_2 + z_1} > 1 \quad \rightarrow \quad \frac{\tilde{P}_t}{\tilde{P}_i} > 0 \quad \text{and} \quad \frac{\tilde{P}_t}{\tilde{P}_i} > 1.$$

This means that the transmitted wave is in phase with the incident wave at the boundary. Furthermore, in this instance, the amplitude of the transmitted pressure wave is greater than the amplitude of the incident wave. Does this seem right? Has the transmitted wave gained energy?

************************* Example 6.1 ***************************

Consider a boundary between two media where $z_2 = 2z_1$ and an acoustic wave is incident on the boundary. This means that the transmitted wave will have larger amplitude than the incident wave. Let's look at the energy in the wave.

The pressure and intensity reflection coefficients are

$$\tilde{R} = \frac{z_2 - z_1}{z_2 + z_1} = \frac{2z_1 - z_1}{2z_1 + z_1} = \frac{1}{3} \qquad \text{and} \qquad R_I = |R|^2 = \frac{1}{9}$$

Likewise the pressure and intensity transmission coefficients are given by

$$\tilde{T} = \frac{2z_2}{z_2 + z_1} = \frac{4z_1}{2z_1 + z_1} = \frac{4}{3} \qquad \text{and} \qquad T_I = \frac{z_1}{z_2}\left|\tilde{T}\right|^2 = \frac{16}{9}\left(\frac{1}{2}\right) = \frac{8}{9}$$

For a plane wave recall that the energy density is:

$$e = \frac{p^2}{2r_0 c} = \frac{p^2}{2z}$$

So, the incident energy is given by:

$$e_i = \frac{\left|\tilde{P}_i\right|^2}{2z_1}$$

The total energy after the incident wave passes the boundary is

$$e_i = \frac{\left|\tilde{P}_r\right|^2}{2z_1} + \frac{\left|\tilde{P}_t\right|^2}{2z_2}.$$

Setting the two energy equal gives

$$\frac{\left|\tilde{P}_i\right|^2}{2z_1} = \frac{\left|\tilde{P}_r\right|^2}{2z_1} + \frac{\left|\tilde{P}_t\right|^2}{2z_2}.$$

Dividing by $\frac{1}{2z_1}\left|\tilde{P}_i\right|^2$ gives

$$1 = \frac{\left|\tilde{P}_r\right|^2}{\left|\tilde{P}_i\right|^2} + \frac{z_1}{z_2}\frac{\left|\tilde{P}_t\right|^2}{\left|\tilde{P}_i\right|^2}$$

or

$$1 = |R|^2 + \frac{z_1}{z_2}|T|^2 = R_I + T_I .$$

From above $R_I = \frac{1}{9}$, $T_I = \frac{8}{9}$ so that

$$1 = R_I + T_I = \frac{1}{9} + \frac{8}{9} = 1 .$$

So, even though the transmitted amplitude is larger than the incident pressure wave amplitude, the total energy in the waves remains conserved.

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Case 3: $z_2 < z_1$

$$\tilde{R} = \frac{z_2 - z_1}{z_2 + z_1} < 0 \quad \rightarrow \quad \frac{\tilde{P}_r}{\tilde{P}_i} < 0$$

In other words, the reflected wave is 180° out of phase with the incident wave. This is reminiscent of the string reflection from a rigid end. For the transmitted wave

$$\tilde{T} = \frac{2z_2}{z_2 + z_1} < 1 \quad \rightarrow \quad \frac{\tilde{P}_t}{\tilde{P}_i} > 0 \quad \text{and} \quad \frac{\tilde{P}_t}{\tilde{P}_i} < 1 .$$

This means that the transmitted wave is in phase with the incident wave at the boundary.

Case 4: $\frac{z_2}{z_1} \rightarrow \infty$, that is, $z_2 \gg z_1$

$$\tilde{R} = \frac{z_2 - z_1}{z_2 + z_1} = \frac{\left(\frac{z_2}{z_1} - 1\right)}{\left(\frac{z_2}{z_1} + 1\right)} \rightarrow +1 \qquad \tilde{P}_r = \tilde{P}_i$$

$$\tilde{T} = \frac{2z_2}{z_1 + z_2} = \frac{2\left(\frac{z_2}{z_1}\right)}{\left(\frac{z_2}{z_1} + 1\right)} \rightarrow 2 \qquad \tilde{P}_t = 2\tilde{P}_i$$

All energy reflected at boundary because the reflection coefficient is +1 and the amplitude of the reflected wave is the amplitude of the incident wave. Medium 2 is virtually incompressible; highly dense. There is a doubling of pressure amplitude at the boundary. However, energy is still conserved because no wave propagates in medium 2. Recall from earlier

$$e_i = \frac{|\tilde{P}_r|^2}{2z_1} + \frac{|\tilde{P}_t|^2}{2z_2} = \frac{|\tilde{P}_i|^2}{2z_1}$$

$$|\tilde{P}_r|^2 + \frac{|\tilde{P}_t|^2}{\frac{z_2}{z_1}} = |\tilde{P}_i|^2 \qquad \text{but } \frac{z_2}{z_1} \rightarrow \infty \text{ so}$$

$$\left|\tilde{P}_r\right|^2 = \left|\tilde{P}_i\right|^2$$ which is true so energy is conserved.

Case 5: $\dfrac{z_2}{z_1} \to 0$ that is, $z_2 << z_1$

$$\tilde{R} = \frac{z_2 - z_1}{z_2 + z_1} = \frac{\left(\frac{z_2}{z_1} - 1\right)}{\left(\frac{z_2}{z_1} + 1\right)} \to -1 \quad \tilde{P}_r = -\tilde{P}_i$$

$$\tilde{T} = \frac{2z_2}{z_1 + z_2} = \frac{2\left(\frac{z_2}{z_1}\right)}{\left(\frac{z_2}{z_1} + 1\right)} \to 0 \quad \tilde{P}_t = 0$$

All energy reflected at boundary and reflected wave is completely out of phase with incident wave (think rigid end for a string). Medium 2 is easily compressed or rarefied. There is zero pressure amplitude at surface.

************************** Example 6.2 ***************************

Consider what happens at an air/water boundary:

$z_1 = 415$ and $z_2 = 1.48 \times 10^6$ (MKS rayl)

$$R = \frac{z_2 - z_1}{z_2 + z_1} = \frac{1.48 \times 10^6 - 415}{1.48 \times 10^6 + 415} = 0.99944$$

$$T = \frac{2z_2}{z_1 + z_2} = \frac{2\left(1.48 \times 10^6\right)}{415 + 1.48 \times 10^6} = 1.99944$$

Why can't you hear people if you are underwater and they are above the water talking to you?

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************************** Example 6.3 ***************************

Consider what happens at a water/air boundary

$z_1 = 1.48 \times 10^6$ and $z_2 = 415$ (MKS rayl)

$$R = \frac{z_2 - z_1}{z_2 + z_1} = \frac{415 - 1.48 \times 10^6}{415 + 1.48 \times 10^6} = -0.99944$$

$$T = \frac{2z_2}{z_1 + z_2} = \frac{2(415)}{1.48 \times 10^6 + 415} = 0.00056$$

Also called a "pressure release" boundary.

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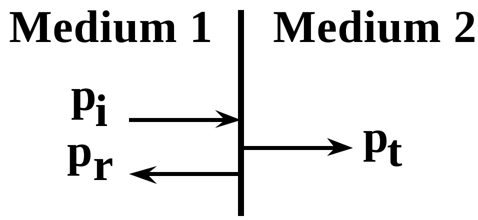
Let's examine the pressure distribution in Medium 1 when $\frac{z_2}{z_1} \to 0$ that is, $z_2 << z_1$ for which ,

then, $\tilde{P}_r = -\tilde{P}_i$ and $\tilde{P}_t = 0$



$$p_i\left(x,t\right) = \tilde{P}_i e^{j\left(wt - k_1 x\right)}$$
$$p_r\left(x,t\right) = \tilde{P}_r e^{j\left(wt + k_1 x\right)}$$

where again $k_1 = \frac{w}{c_1} = \frac{2p\, f}{c_1} = \frac{2p}{l_1}$.

The instantaneous acoustic pressure in Medium 1 is going to be the sum of the incident and reflected waves (assuming CW):

$$p_1\left(x,t\right) = p_i\left(x,t\right) + p_r\left(x,t\right) = \tilde{P}_i e^{j\left(wt - k_1 x\right)} + \tilde{P}_r e^{j\left(wt + k_1 x\right)}$$
$$= \tilde{P}_i e^{j\left(wt - k_1 x\right)} - \tilde{P}_i e^{j\left(wt + k_1 x\right)} = \tilde{P}_i e^{jwt}\left\{-2j\sin\left(k_1 x\right)\right\}$$

Thus, $p_1\left(x,t\right) = \tilde{P} e^{jwt}\left\{-2j\sin\left(k_1 x\right)\right\}$

The magnitude as a function of position of the acoustic pressure in Medium 1 is:

$$\left|p_1\left(x,t\right)\right| = \left|P_{\max}\right|\left|\sin\left(k_1 x\right)\right|$$
$$u_i\left(x,t\right) = \frac{p_i\left(x,t\right)}{r_1 c_1} = \frac{\tilde{P}_i}{r_1 c_1} e^{j\left(wt - k_1 x\right)}$$
$$u_r\left(x,t\right) = -\frac{p_r\left(x,t\right)}{r_1 c_1} = -\frac{\tilde{P}_r}{r_1 c_1} e^{j\left(wt + k_1 x\right)}$$

Giving for the instantaneous particle velocity in Medium 1 is:

$$u_1\left(x,t\right) = u_i\left(x,t\right) + u_r\left(x,t\right) = \frac{\tilde{P}_i}{r_1 c_1} e^{j\left(wt - k_1 x\right)} - \frac{\tilde{P}_r}{r_1 c_1} e^{j\left(wt + k_1 x\right)} = \frac{\tilde{P}_i}{r_1 c_1} e^{jwt}\left(e^{jk_1 x} + e^{-jk_1 x}\right)$$
$$= \frac{\tilde{P}_i}{r_1 c_1} e^{jwt}\left\{2\cos\left(k_1 x\right)\right\} = \frac{2\tilde{P}_i}{r_1 c_1} e^{jwt}\cos\left(k_1 x\right)$$

Thus, $u_1\left(x,t\right) = \frac{2\tilde{P}_i}{r_1 c_1} e^{jwt}\cos\left(k_1 x\right)$

The magnitude as a function of position of the particle velocity in Medium 1 is:

$$|u_1(x,t)| = |U_{\max}||\cos(k_1 x)|$$

Therefore,

$$|p_1(x,t)| = |P_{\max}||\sin(k_1 x)|$$
$$|u_1(x,t)| = |U_{\max}||\cos(k_1 x)|$$

Let's examine the spatial behavior of $|\sin(k_1 x)|$ and $|\cos(k_1 x)|$.

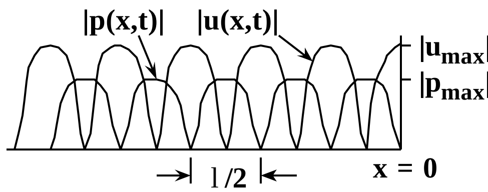
$$|\sin(k_1 x)| = 0 \text{ when } k_1 x = 0, \pi, 2\pi, \ldots, n\pi \text{ , } n = 0,1,2,..$$

so $$x = \frac{n\pi}{k_1} = \frac{n\pi c_1}{\omega} = \frac{n\pi c_1}{2\pi f} = n\frac{\lambda_1}{2}$$

$$|\cos(k_1 x)| = 0 \text{ when } k_1 x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots, \frac{m\pi}{2} \text{ , } m = 1,3,5,..$$

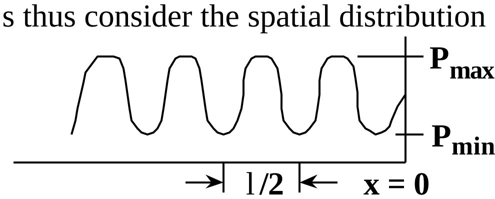
so $$x = \frac{m\pi}{2k_1} = \frac{m\pi c_1}{2\omega} = \frac{m\pi c_1}{4\pi f} = m\frac{\lambda_1}{4}$$

Therefore, for $\frac{z_2}{z_1} \to 0$



What would this plot look like if $\frac{z_2}{z_1} \to \infty$?

Now let's look at what happens when some energy is transmitted into Medium 2. If any energy is transmitted into Medium 2, then the minimums of the acoustic pressure spatial distribution in Medium 1 are non-zero. Let's thus consider the spatial distribution of pressure only in Medium 1.



Note that from $\tilde{P}_i + \tilde{P}_r = \tilde{P}_t$, if $\tilde{P}_t \neq 0$ and $\tilde{P}_i \geq \tilde{P}_r$, then $\tilde{P}_i > \tilde{P}_r$. So,

$$P_{\max} = |\tilde{P}_i| + |\tilde{P}_r|$$
$$P_{\min} = |\tilde{P}_i| - |\tilde{P}_r|$$

where $P_{\max} > P_{\min}$

If $|\tilde{P}_r| = 0$, then $P_{\max} = P_{\min}$

Thus, there are minima but not nulls. The Standing Wave Ratio (SWR) is defined as:

$$\mathrm{SWR} = \frac{P_{\max}}{P_{\min}} = \frac{|\tilde{P}_i| + |\tilde{P}_r|}{|\tilde{P}_i| - |\tilde{P}_r|} = \frac{1 + \dfrac{|\tilde{P}_r|}{|\tilde{P}_i|}}{1 - \dfrac{|\tilde{P}_r|}{|\tilde{P}_i|}}$$

Now recall that

$$\frac{|\tilde{P}_r|}{|\tilde{P}_i|} = |R| \qquad \rightarrow \qquad \mathrm{SWR} = \frac{1+|R|}{1-|R|}$$

Solving for $|R|$ gives

$$|R| = \frac{\mathrm{SWR}-1}{\mathrm{SWR}+1} = \left|\frac{z_2 - z_1}{z_2 + z_1}\right|$$

Consider the 5 cases again: 1) $z_2 = z_1$, 2) $z_2 > z_1$, 3) $z_2 < z_1$, 4) $\frac{z_2}{z_1} \rightarrow \infty$ that is, $z_2 >> z_1$, 5) $\frac{z_2}{z_1} \rightarrow 0$ that is, $z_2 << z_1$.

Case 1: $z_2 = z_1$, $\tilde{P}_r = 0$, $R = \frac{z_2 - z_1}{z_2 + z_1} = 0$

$$\mathrm{SWR} = \frac{1+|\mathrm{R}|}{1-|\mathrm{R}|} = 1$$

$$P_{\max} = P_{\min}$$

Case 2: $z_2 > z_1$, $\frac{\tilde{P}_r}{\tilde{P}_i} > 0$, $R = \frac{z_2 - z_1}{z_2 + z_1} > 0$

$$\mathrm{SWR} = \frac{1+|R|}{1-|R|} > 1$$

$$P_{\max} > P_{\min}$$

Case 3: $\mathrm{z}_2 < \mathrm{z}_1$, $\frac{\tilde{P}_r}{\tilde{P}_i} < 0$, $R = \frac{z_2 - z_1}{z_2 + z_1} < 0$

$$\mathrm{SWR} = \frac{1+|R|}{1-|R|} > 1$$

$$P_{\max} > P_{\min}$$

Case 4: $\frac{z_2}{z_1} \to \infty \quad \tilde{P}_r = \tilde{P}_i \quad R = \frac{z_2 - z_1}{z_2 + z_1} = \frac{\left(\frac{z_2}{z_1} - 1\right)}{\left(\frac{z_2}{z_1} + 1\right)} \to +1$

$$\text{SWR} = \frac{1+|R|}{1-|R|} \to \infty$$

$$P_{\min} \to 0$$

Case 5: $\frac{z_2}{z_1} \to 0 \quad \tilde{P}_r = -\tilde{P}_i \quad R = \frac{z_2 - z_1}{z_2 + z_1} = \frac{\left(\frac{z_2}{z_1} - 1\right)}{\left(\frac{z_2}{z_1} + 1\right)} \to -1$

$$\text{SWR} = \frac{1+|R|}{1-|R|} \to \infty$$

$$P_{\min} \to 0$$

Note ranges: $-1 \le R \le 1$, $1 \le \text{SWR} < \infty$

Let's examine the <u>transfer of energy</u> from Medium 1 to Medium 2
Recall that we defined the Intensity Reflection Coefficient

$$R_I = \frac{\text{Reflected Intensity}}{\text{Incident Intensity}} = \frac{I_r}{I_i}$$

and the intensity Transmission Coefficient

$$T_I = \frac{\text{Transmitted Intensity}}{\text{Incident Intensity}} = \frac{I_t}{I_i}$$

In general, for a plane progressive wave, $I = \frac{p^2}{2rc}$ so

$$R_I = \frac{I_r}{I_i} = \frac{\left(\frac{p_r^2}{2r_1c_1}\right)}{\left(\frac{p_i^2}{2r_1c_1}\right)} = \frac{p_r^2}{p_i^2} = \left|\frac{\tilde{P}_r}{\tilde{P}_i}\right|^2 = |R|^2 = \left(\frac{z_2 - z_1}{z_2 + z_1}\right)^2$$

and

$$T_I = \frac{I_t}{I_i} = \frac{\left(\frac{p_t^2}{2r_2c_2}\right)}{\left(\frac{p_i^2}{2r_1c_1}\right)} = \frac{r_1c_1p_t^2}{r_2c_2p_i^2} = \frac{r_1c_1}{r_2c_2}\left|\frac{\tilde{P}_t}{\tilde{P}_i}\right|^2 = \frac{z_1}{z_2}|T|^2 = \frac{z_1}{z_2}\left|\frac{2z_2}{z_1+z_2}\right|^2 = \frac{4z_1z_2}{(z_1+z_2)^2}$$

Note that: $R_I + T_I = \left(\frac{z_2 - z_1}{z_2 + z_1}\right)^2 + \frac{4z_1z_2}{(z_1+z_2)^2} = \frac{z_2^2 - 2z_1z_2 + z_1^2 + 4z_1z_2}{(z_2+z_1)^2} = \frac{z_2^2 + 2z_1z_2 + z_1^2}{(z_2+z_1)^2} = 1$

Let's next define the Power Reflection Coefficient

$$R_p = \frac{\text{ReflectedPower}}{\text{IncidentPower}} = \frac{A_r I_r}{A_i I_i} = \frac{A_r}{A_i} R_I$$

where A represents the cross sectional area of the beam (reflected, incident or transmitted).

The Power Transmission Coefficient is then

$$T_p = \frac{\text{TransmittedPower}}{\text{Incident Power}} = \frac{A_t I_t}{A_i I_i} = \frac{A_t}{A_i} T_I$$

For normal incidence, $A_i = A_r = A_t$ so

$$R_p = \frac{A_r}{A_i} R_I = \frac{A_r}{A_i} |R|^2 = |R|^2$$

$$T_p = \frac{A_t}{A_i} T_I = \frac{A_t}{A_i} \frac{r_1 c_1}{r_2 c_2} |T|^2 = \frac{r_1 c_1}{r_2 c_2} |T|^2$$

Since $R_I + T_I = 1$, then $R_p + T_p = 1$ for normal incidence.

Summary for normal incidence for the 5 cases considered

1) $z_2 = z_1$

$\mathrm{R} = 0$	$\mathrm{R_I} = 0$	$\mathrm{R_\pi} = 0$
$\mathrm{T} = 1$	$\mathrm{T_I} = 1$	$\mathrm{T_\pi} = 1$
$\mathrm{SWR} = 1$		

2) $z_2 > z_1$

$0 < \mathrm{R} < 1$	$0 < \mathrm{R_I} < 1$	$0 < \mathrm{R_\pi} < 1$
$1 < \mathrm{T} < 2$	$0 < \mathrm{T_I} < 1$	$0 < \mathrm{T_\pi} < 1$
$1 < \mathrm{SWR} < \infty$		

3) $z_2 < z_1$

$-1 < \mathrm{R} < 0$	$0 < \mathrm{R_I} < 1$	$0 < \mathrm{R_\pi} < 1$
$0 < \mathrm{T} < 1$	$0 < \mathrm{T_I} < 1$	$0 < \mathrm{T_\pi} < 1$
$1 < \mathrm{SWR} < \infty$		

4) $\frac{z_2}{z_1} \to \infty$

$\mathrm{R} \to 1$	$\mathrm{R_I} \to 1$	$\mathrm{R_\pi} \to 1$
$\mathrm{T} \to 2$	$\mathrm{T_I} \to 0$	$\mathrm{T_\pi} \to 0$
$\mathrm{SWR} \to \infty$		

5) $\frac{z_2}{z_1} \to 0$

$\mathrm{R} \to -1$	$\mathrm{R_I} \to 1$	$\mathrm{R_\pi} \to 1$
$\mathrm{T} \to 0$	$\mathrm{T_I} \to 0$	$\mathrm{T_\pi} \to 0$
$\mathrm{SWR} \to \infty$		

*************************** Example 6.4 ***************************

Reflection at soft tissue interfaces within the human body

Take z_1 = (1010)(1540) = 1.555x10^6 rayl (liver-like tissue)
z_2 = (999)(1500) = 1.499x10^6 rayl (fat-like tissue)

$$R = \frac{z_2 - z_1}{z_2 + z_1} = \frac{1.499 - 1.555}{1.499 + 1.555} = \frac{-0.056}{3.054} = -0.018$$

$R_I = R^2 = 0.00034$ or only approximately 0.03% of the incident energy is reflected

Check:

$$T_I = \frac{4z_1z_2}{(z_2 + z_1)^2} = \frac{4(1.555)(1.499)}{(1.499 + 1.555)^2} = \frac{9.324}{(3.054)^2} = 0.99966$$

and R_I + T_I = 1.00000 as expected

So in medical imaging we are often just imaging the reflection from different tissue surfaces which are pretty closely matched in impedance and therefore not much reflected energy. Yet, it works really well.

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