Let’s compare the far-field response with the on axis response. From the near-field on axis response

$$\left|p(r,0)\right| = 2r_o cU_o \left|\sin\left\{\frac{kr}{2}\left(\sqrt{1+\left(\frac{a^2}{r^2}\right)}-1\right)\right\}\right|$$

The axial pressure amplitude for the far field (Fraunhofer zone) is given by (for $q = 0$)

$$p(r,q,t) = \frac{jwr_o a^2 U_o}{2r} e^{j(wt-kr)} \left[\frac{2J_1(ka\sin q)}{ka\sin q}\right]$$

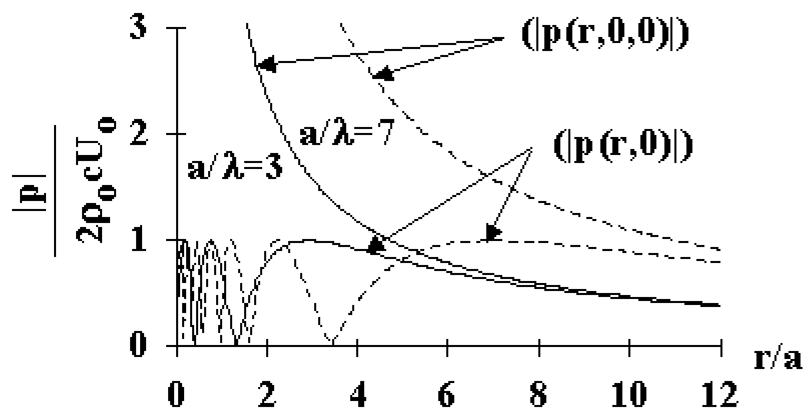
so

$$\left|p(r,0,0)\right| = \frac{wr_o a^2 U_o}{2r} = \frac{(kc)r_o a^2 U_o}{2r} = \frac{1}{2} r_o cU_o \left(\frac{a}{r}\right)ka \quad \text{(Eq 7.4.7)}$$

Normalizing axial-only $\left|p(r,0)\right|$ and far-field axial $\left|p(r,0,0)\right|$ to $2r_o cU_o$ gives:

$$\frac{\left|p(r,0)\right|}{2r_o cU_o} = \left|\sin\left\{\frac{kr}{2}\left(\sqrt{1+\left(\frac{a^2}{r^2}\right)}-1\right)\right\}\right| \qquad \text{and} \qquad \frac{\left|p(r,0,0)\right|}{2r_o cU_o} = \frac{1}{4}\left(\frac{a}{r}\right)ka$$

Plotting these two values



where the solid line represents $a/l = 3$, the dashed line represents the $a/l = 7$. Did one reduce to the other? Why not?

So,

$$p(r,t) = 2jr_o cU_o e^{jwt} e^{-j\frac{k}{2}(R_1+r)} \sin\left\{\frac{k}{2}(R_1 - r)\right\} \text{ where } R_1 = \sqrt{r^2+a^2}$$

and

$$p(r,q,t) = \frac{jwr_o a^2 U_o}{2r} e^{j(wt-kr)} \left[\frac{2J_1(ka\sin q)}{ka\sin q}\right]$$

Homework EXERCISE: Show that these two expressions are the same, on-axis, in the Fraunhofer Zone.

**************************** Example 7.4 ****************************

Compare and contrast the directivity and number of side lobes for the case of a continuous-line source of length L, and a circular transducer of radius L excited at the same frequency and into the same medium.

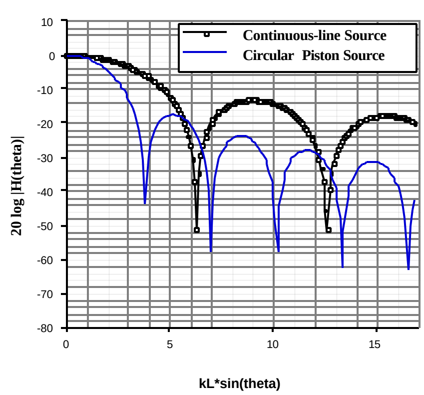
ANSWER: The equations for the line source and circular transducer are listed below:

$$H_L(q) = \left|\operatorname{sinc}\left(\frac{kL\sin q}{2}\right)\right| \qquad \text{(Line source)}$$

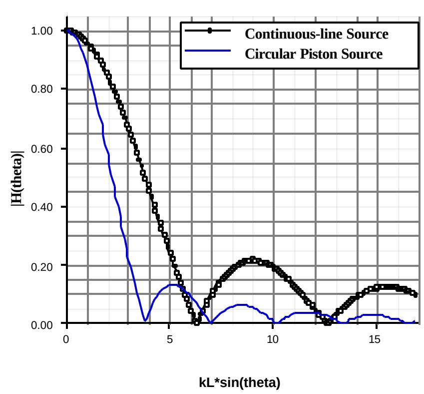
$$H_C(q) = \left[\frac{2J_1(ka\sin q)}{ka\sin q}\right] \qquad \text{(Circular transducer)}$$

The plots for the two cases are shown below. Note that the continuous-line source has the x-axis "expanded out" by a factor of two because of the factor of 0.5 in the expression $\frac{1}{2}kL\sin(q)$. Thus, one can immediately see that the circular transducer (i.e. the circular piston source) has greater directivity, but at the same time has more side lobes than the continuous-line source. Also note that L for the line source is the "full length" whereas L (a) for the piston source is "1/2 the length."

Log Comparison of Directivity



Direct Comparison of No. of Sidelobes



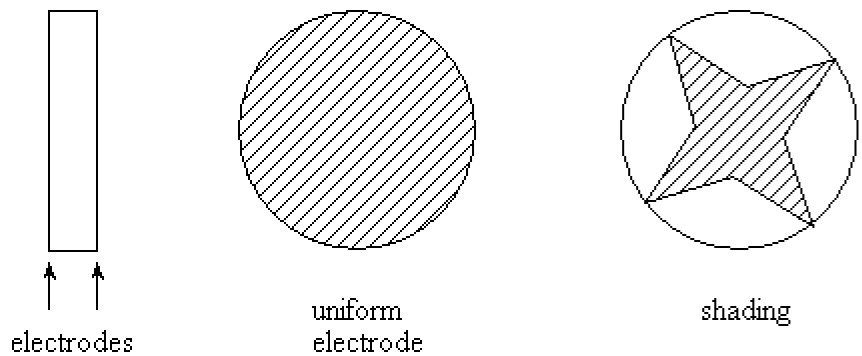
How would the plots be different if $L = D$, the diameter of the circular transducer?

****

The far field pattern can be modified by changing U_0 over the face of the source. For example, if U_0 decreases toward the edges of the circular piston then the side lobe levels will decrease and the main lobe will be broader.

Ex 1 – High frequencies in a loudspeaker move the center more than outer parts.

Ex 2 – "Shading" of a piezoelectric transducer.

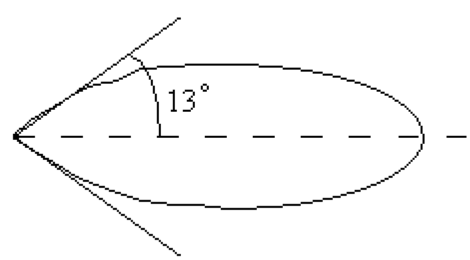


Transducer dimension (thickness) changes in response to electric field. "Shading" lowers the field in the outer portions of the disk.

Beamwidth

Defined as angular width where the intensity is down a specified amount from that on the axis.

Ex	0.5	$\rightarrow$	3 dB down
	0.25	$\rightarrow$	6 dB down
	0.1	$\rightarrow$	10 dB down



Thus if $q = 13°$ for the acoustic amplitude down by 10 dB then the –10 dB beam width is $2q = 26°$.

*************************** Example 7.5 ***************************

Computation of –10 dB beamwidth

For –10 dB the intensity directivity function will have a value of 0.1

Thus, $\left(\frac{2J_1(x)}{x}\right)^2 = 0.1$, where $x = ka\sin(q)$. This occurs at approximately $x = 2.74$.

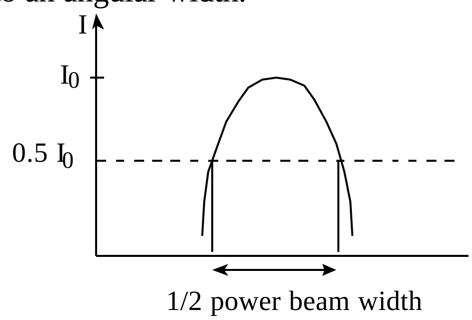
Consider two different values of ka

For $ka = 7$, $q = \sin^{-1}\left(\frac{2.74}{7}\right) = 23^\circ$, so the beamwidth is $2q = 46°$

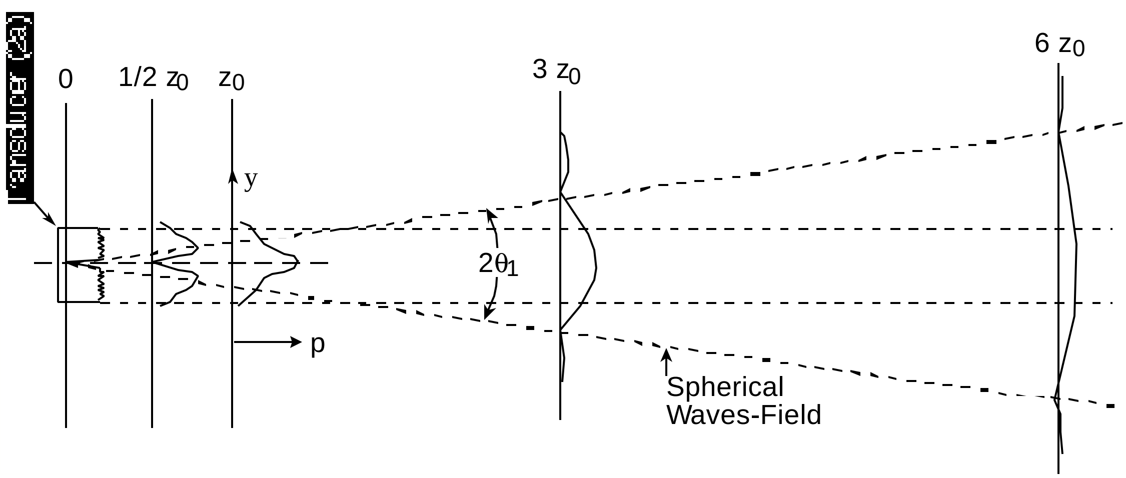
For $ka = 21$, $q = \sin^{-1}\left(\frac{2.74}{21}\right) = 7.5^\circ$, so the beamwidth is $2q = 15°$

****

Alternatively, at a specific distance from the source the beam width might be specified in terms of a linear distance as opposed to an angular width.



Beam patterns for circular sources with uniform velocity amplitude distributions.



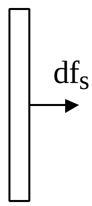
Sound field of a circular piston. $2a/\lambda = 16$.

Transverse pattern in near field is very irregular.

(7.5) Radiation Impedance

An electromechanical transducer has an electrical impedance which is, through some electromechanical coupling parameters (depends on type of transducer), directly dependent upon the mechanical parameters of the transducer and its load.

$\therefore$ Total mechanical impedance = mechanical impedance of transducer (radiating into a vacuum) plus the radiation impedance of the fluid load



comp. $\perp$ surface

$$z_r = \int \frac{\vec{\mathbf{df}}_s}{\vec{\mathbf{u}}}$$

The mechanical impedance of the transducer is analogous to discussion from Chapter 1 for the mass loaded, damped driven harmonic spring system.

Piston $\rightarrow$ mass m
mechanical resistance R_m
stiffness s

Driven by external force $f = Fe^{jwt}$ to give $\vec{u}_0 = U_0 e^{jwt} = jwx_0$ for simple harmonic time dep.

$f_s =$ force of fluid = $\tilde{z}_r u_0$ where $\tilde{z}_r$ is the radiation impedance

We can plug all of this into a second order differential equation (just like the damped, driven harmonic oscillator)

$$\vec{f} - \vec{f}_s - R_m \frac{dx_0}{dt} - sx_0 = m\frac{d^2x_0}{dt^2}$$

or

$$f = f_s + R_m u_0 + s\int u_0 dt + m\frac{du_0}{dt}$$

We can assume a simple harmonic time dependence like the driving force so that

$$f = f_s + \left(R_m + jwm + \frac{s}{jw}\right)u_0$$
$$= \left(\tilde{z}_r + \tilde{z}_m\right)\tilde{u}_0$$

where

$$\tilde{z}_m = R_m + jwm + \frac{s}{jw}$$

is the mechanical impedance. Hence,

$$\tilde{u}_0 = \frac{\tilde{f}}{\tilde{z}_m + \tilde{z}_r}$$

dividing by e^{jwt} gives

$$U_0 = \frac{F}{\tilde{z}_m + \tilde{z}_r}$$

where

$$\tilde{z}_r = (z_r)e^{jq} = \underset{\substack{\uparrow \\ \text{radiation} \\ \text{resistance}}}{R_r} + j\underset{\substack{\uparrow \\ \text{radiation} \\ \text{reactance}}}{X_r} \qquad \frac{\text{N–s}}{\text{m}}$$

Radiation resistance – loss term for the system

two loss terms – $R_m \rightarrow$ loss in transducer

$R_r \rightarrow$ radiated energy

Power radiated $\Pi_r = \frac{1}{2}U_0^2 R_r$

Let's look at the <u>Simple Source</u>

$$R_r = \frac{r_0 c(kS)^2}{4p}$$ unbaffled, where S is the surface area

$$R_r = \frac{r_0 c(kS)^2}{2p}$$ baffled simple source (twice R_r and Π_r)

Looking at $Z_{total} = Z_m + Z_r$

$$= R_m + j\left(wm - \frac{s}{w}\right) + R_r + jX_r$$

a positive X_r means that we are loading the system with mass from the surrounding fluid so that the resonance frequency is decreased for the oscillator.

Thus, we define a radiation mass $m_r = \frac{X_r}{w}$.

Since the resonance frequency of the system is

$$w_0 = \sqrt{\frac{s}{mass}} \Rightarrow w_0 = \sqrt{\frac{s}{m+m_r}}$$

The radiation mass reduces the resonance frequency. m_r is related to mass of material moved in front of source. Less mass at higher frequency.
What does this mean for a transducer operating in air compared to a transducer operating in water?

Let's look briefly at one particular case: The Circular Piston

$$R_r = r_0 c S R_1(2ka)$$
$$X_r = r_0 c S X_1(2ka)$$

where

$$R_1(2ka) = 1 - \frac{2J_1(2ka)}{2ka}$$
$$X_1(2ka) = \frac{2H_1(2ka)}{2ka} \qquad \text{(To be shown in homework)}$$

and $S = p a^2$ (the area of the piston face).

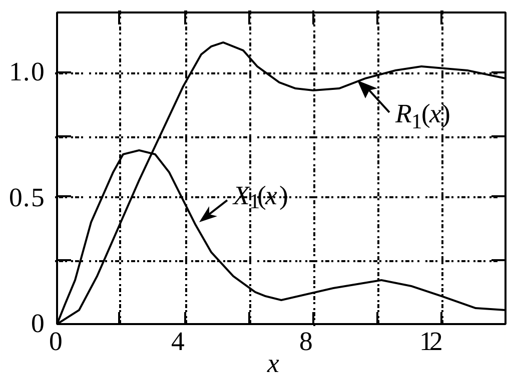


Figure 7.5.2 Radiation resistance and reactance for a plane circular piston of radius *a* radiating sound of wave number *k*(*x* = 2*ka*).

For $ka << 1$ (a small compared to l)

We use the series expansion for *J* and *H* to get

$$R_r \approx \frac{1}{2} r_0 c S (ka)^2$$
$$X_r \approx \frac{8}{3p} r_0 c S (ka)$$
$$m_r \approx \frac{8a}{3p} r_0 S$$

Physically, this means that the piston appears to be loaded with a cylindrical volume of fluid mass with cross sectional area *S* (the same as the piston surface) and a depth of the fluid mass cylinder reaching to $z = \frac{8a}{3p} \simeq 0.85a$ .We see how the fluid changes the resonant frequency.

For $ka >> 1$ (a large compared to l)
Again we use the series expansion for J and H at large ka.

$$R_r \cong r_0 cS(1)$$
$$X_r \cong \frac{2}{p\,ka} r_0 cS \;\Rightarrow\; 0 \text{ for } ka \text{ very large}$$
$$m_r \cong \frac{2r_0 a}{k^2} \quad \Rightarrow \quad 0 \text{ for } ka \text{ very large}$$

Thus, at large ka $\left(\text{or } \frac{a}{l}\right)$ the radiation impedance is almost real with no mass loading
So, the power is given by

$$\Pi_r = \frac{1}{2}U_0^2 R_r \cong \frac{1}{2} r_0 cSU_0^2 .$$

This is the same power as a plane wave propagating through area S with characteristic impedance $r_0 c$.

*************************** Example 7.6 ***************************

Calculate R_r, X_r and m_r in air and water for a circular piston source

Take a = 3 cm, f = 10 kHz, and U_0 = 1 m/s

<u>Air</u> $2ka = \frac{4p\,10^4}{343} 0.03 = 11$

<u>Water</u> $2ka = \frac{4p\,10^4}{1481} 0.03 = 2.55$

Parameter	Air	Water
$\pi a^2 r_0 c$	1.173	4,128
$2ka$	11	2.55
R_1	1.0321	0.6375
R_r	1.21	2,632 $\left(\frac{N-s}{m}\right)$
$\Pi_r = \frac{1}{2}U_0^2 R_r$	0.61	1,316 (W)
X_1	0.1464	0.693
X_r	0.172	2,861
m_r	2.73×10^{-6}	4.56×10^{-2} (kg)

Physically it takes a lot more work done against R_r in water than in air. Hence, the more power expended in water than in air by the transducer.

****

(7.6) Fundamental Properties of Transducers

(a) Directional Factor and Beam Pattern

The general expression for beam pattern, $b(q,f)$, represents the variation of intensity level with angle. The expression is given by

$$
\begin{aligned}
b(q,f) &= 10\log\left(\frac{I(r,q,f)}{I_{ax}(r)}\right) \\
&= 20\log\left(\frac{P(r,q,f)}{p_{ax}(r)}\right) \\
&= 20\log\left(H(q,f)\right)
\end{aligned}
$$

Note: is generally a function of both q and f since it may not have circular symmetry.

(b) Beam Width

Discussed previously – must specify criterion: half-power, quarter power, 0.1 etc. Remember in some cases, for a specified distance from source, can use a linear measure.

(c) Source Level

Source Level = $SL(P_{ref})$ = 20 log $\left(\frac{P_e(1)}{P_{ref}}\right)$ where $P_e(1)$ is effective axial pressure at r = 1 m, <u>extrapolated back from far field</u>. Must specify reference pressure

P_{ref} = 1 μPa, 20 μPa, 1 μbar

(d) Directivity

$$
\text{directivity} = \text{D} = \frac{I_{ax}(r)\begin{pmatrix}\text{nonspherical}\\ \text{directional}\\ \text{source}\end{pmatrix}}{I_S(r)(\text{spherical})} = \frac{P^2_{ax}(r)}{P^2_S(r)}
$$

for a simple source having the same acoustic power as the directive source. For the simple source

$$
\Pi_s = \frac{1}{2r_0c}4p\,r^2P^2_S(r)
$$

and there is no preferred direction. For the directional source

$$
\Pi_{directional} = \frac{1}{2r_0c}\int_{4p}P^2(r,q,f)r^2d\Omega
$$

where $d\Omega = \sin(q)\,dq\,df$ and the integration is over 4π steradians (f varies from 0 to 2π and q from 0 to π)

$$P(r,q,f) = P_{ax}(r)\,H(q,f)$$

$$\Pi_{directional} = \frac{1}{2 r_0 c} r^2 P_{ax}^2(r) \int_{4p} H^2(q,f)\,d\Omega$$

Since the acoustic powers are the same, we can solve for $P_{ax}^2(r)$ and $P_S^2(r)$ in terms of the acoustic power giving

$$\therefore D = \frac{P_{ax}^2(r)}{P_S^2} = \frac{\dfrac{2\Pi r_0 c}{r^2 \int_{4p} H^2(q,f)\,d\Omega}}{\dfrac{2\Pi r_0 c}{4p r^2}} = \frac{4p}{\int_{4p} H^2(q,f)\,d\Omega}$$

The larger the ratio a/λ the more directional the beam since $\int_{4p} H^2(q,f)\,d\Omega$ is smaller.

(e) Directivity Index

Directivity Index = DI = 10 log D

Directivity of baffled piston source

$$D = \frac{4p}{\int_{4p} H^2(q,f)\,d\Omega} = \frac{4p}{\int_0^{p/2} \left[\dfrac{2J_1(ka\sin q)}{ka\sin q}\right]^2 2p\,\sin q\,dq}$$

$$D = \frac{(ka)^2}{1 - \dfrac{J_1(2ka)}{ka}}$$ again Bessel functions are tabulated in appendix A6

for ka >> 1

$$D \cong (ka)^2$$

which is greater for greater a/l, or the beam is more highly directive when the frequency is increased

*************************** Example 7.7 ***************************

Examples of directivity calculations.

Take $a/\lambda = \dfrac{10}{2\pi} > 1 \quad ka = 10 \quad D \cong 100$

$a/\lambda = \dfrac{15}{2\pi} > 2 \quad ka = 15 \quad D \cong 225$

Thus the source starts to become quite directive even for a/λ not much greater than 1.

****

<u>Reversible Transducers</u>

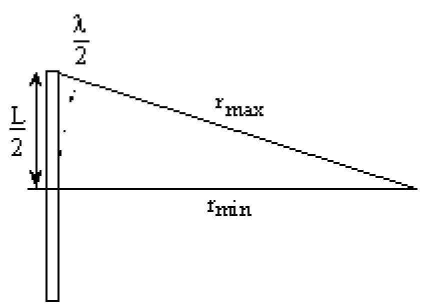
Ex. speakers $\Rightarrow$ intercom
ultrasonic transducers

Transmit and receiving patterns are the same.

$$H(\theta,\phi)_{receive} = H(\theta,\phi)_{transmit}$$

(f) <u>Rough Estimates of Radiation Patterns</u>

<u>Extent of Near Field</u>

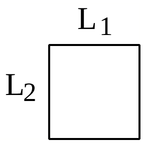


The extent of the near field for a source is defined by the two distances r_{min} (the distance from the field point to the closest element of the source) and r_{max} (the distance from the field point to the furthest element of the source).
When the field point is moved towards the source such that $r_{max} - r_{min} \approx \lambda/2$ then the axial pressure is sufficiently shifted from the far field axial response to alter the far field axial pressure amplitude. The above relation can be shown through geometry to imply the extent of the near field response is quantified by

$$\frac{r_{min}}{L} \cong \frac{1}{4}\frac{L}{\lambda} \qquad \text{or} \qquad r_{min} = \frac{L^2}{4\lambda} = \frac{\left(\frac{L}{2}\right)^2}{\lambda}$$

where L is the greatest extent of the source.

For a rectangular source:



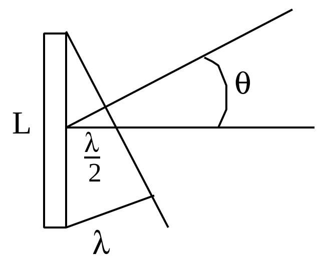
$L_2 > L_1$

$$r_{\min} = \frac{L_2^2}{4l}$$

<u>First angular null</u>

$$\sin q_1 \cong \frac{l}{L}$$

q_1 decreases with L
decreases with frequency



phase cancellation
(ON Transmit and ON Receive)

<u>Estimate of Directivity</u>

$$D \approx \frac{4p}{\Omega_{effective}}$$ $\therefore$ determine solid angle for main lobe–for side lobes low

Text pressure down 0.5 (–6 dB) as a good estimate

<u>Piston like source</u>

$$D \cong \frac{1}{4} k^2 L_1 L_2$$

increases with <u>dimensions</u> and <u>frequency</u>