

Chapter 8 – Absorption and Attenuation of Sound

(8.1) We derived the wave equation in Chapter 5 assuming no losses of acoustic energy, which as we know is not a realistic case. In any real acoustic wave we will have losses, acoustic energy will be dissipated and converted into thermal energy.

Attenuation includes:

- Absorption – conversion of acoustical (mech.) energy to heat
- Scattering – redirection of energy due to inhomogenities in medium
[used for medical and NDE imaging]

There can also be refraction (redirection) of energy at an interface, which will not be considered here to be a part of attenuation.

Absorption

- (A) Classical –
 - (1) Viscous → frictional losses associated with relative motion (related to viscosity)
 - (2) Heat Conduction – heat flow from region of condensation (higher temp.) to region of rarefaction (lower temp.) (related to thermal conductivity)
- (B) Molecular Exchanges → Relaxational Absorption
 - (1) Stored potential energy – structural rearrangement
 - (2) Internal rotational and vibrational energies (polyatomic molecules)
 - (3) Energies of association and dissociation of different ionic species

There is a time constant associated with these phenomena.

Scattering

Energy incident on small regions of the material with different properties from the bulk of the material is scattered in various directions. In some instances, depending upon size versus the wavelength, the energy may be scattered equally in all directions. In general, however, the scattered wave may have an amplitude that is dependent upon direction. A clear example of a scattering medium would be water with small air bubbles in it.

It should be noted that reflection and backscattering (the energy scatter back toward the source) are used in imaging with ultrasound. Thus scattering is critical to the use of ultrasound for nondestructive imaging in industry and to medical imaging.

The first two absorption processes (viscosity and thermal conductivity) follow classical absorption processes, which we will consider in a general form.

(8.2-8.5) Classical absorption

Previously we considered the classical wave equation describing acoustic waves by

$$\nabla^2 p - \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} = 0.$$

Relaxation absorption

Thermal Relaxation: Translational energy going into other internal modes such as rotation or vibration. Associated with temperature change that affects translational energy. Applies for diatomic and polyatomic gases.

Structural Relaxation: Change in state or structure. Associated with a change in volume (density). Applies to water.

Chemical Relaxation: Change in ion or complex chemical equilibria. Applies to sea water where high attenuation is due almost solely to the salt MgSO_4 .

Each of these is characterized by a relaxation time, \mathbf{t} . There is a change or relaxation of some acoustic property with frequency. The relaxation frequency $\omega_R = 1/\mathbf{t}$ and $f_R = 1/(2\pi\mathbf{t})$.

Let's consider the wave equation with a relaxation term:

$$\nabla^2 p - \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} = -\mathbf{t} \frac{\partial}{\partial t} (\nabla^2 p).$$

No classical absorption implies that $\mathbf{t} = 0$ (in which case we get the classical wave equation back). \mathbf{t} 1) includes the classical absorption mechanism, 2) is a constant, 3) has units of time, 4) goes to zero for zero absorption.

Let's try a solution to the wave equation:

$$\tilde{p}(\vec{r}, t) = \tilde{f}(\vec{r}) e^{j\omega t}$$

giving

$$\frac{\partial^2 \tilde{p}}{\partial t^2} = -\omega^2 \tilde{f}(\vec{r}) e^{j\omega t}$$

$$\nabla^2 \tilde{p} = \nabla^2 \tilde{f}(\vec{r}) e^{j\omega t}$$

and

$$\frac{\partial}{\partial t} \nabla^2 \tilde{p} = j\omega \nabla^2 \tilde{f}(\vec{r}) e^{j\omega t}.$$

Plugging into the above relaxation wave equation:

$$\nabla^2 \tilde{f}(\vec{r}) \cancel{e^{j\omega t}} + \frac{\omega^2}{c^2} \tilde{f}(\vec{r}) \cancel{e^{j\omega t}} = -\mathbf{t} \left(j\omega \nabla^2 \tilde{f}(\vec{r}) \cancel{e^{j\omega t}} \right).$$

Grouping terms gives

$$\nabla^2 \tilde{f}(\vec{r})[1 + j\omega t] + \frac{\mathbf{w}^2}{c^2} \tilde{f}(\vec{r}) = 0$$

or

$$\nabla^2 \tilde{f}(\vec{r}) + \frac{\mathbf{w}^2}{c^2 [1 + j\omega t]} \tilde{f}(\vec{r}) = 0.$$

If we define

$$\tilde{k} = \frac{\mathbf{w}}{c\sqrt{1 + j\omega t}}$$

a complex wavenumber then we have the good ole Helmholtz equation back

$$\nabla^2 \tilde{f}(\vec{r}) + \tilde{k}^2 \tilde{f}(\vec{r}) = 0.$$

For a simple 1-D solution

$$\tilde{f}(\vec{r}) = Ae^{-j\tilde{k}x}$$

(3-D would be $\tilde{f}(\vec{r}) = Ae^{-j\tilde{k}\cdot\vec{r}}$)

So our general solution (1-D) is

$$\tilde{p}(x, t) = Ae^{j(\mathbf{w} - \tilde{k}x)}$$

Since \tilde{k} is complex we can set $\tilde{k} = k_r - j\mathbf{a} = \frac{\mathbf{w}}{c\sqrt{1 + j\omega t}}$ which gives

$$\tilde{p}(x, t) = Ae^{j(\mathbf{w}t - [k_r - j\mathbf{a}]x)}$$

$$\tilde{p}(x, t) = Ae^{-\mathbf{a}x} e^{j(\mathbf{w} - k_r x)}$$

where the $e^{-\mathbf{a}x}$ acts as a damping term (just like for the SHO). The expression for the acoustic pressure in a medium with attenuation is expressed as follows, where \mathbf{a} is taken to be the attenuation coefficient. In cases where there is negligible or no scattering \mathbf{a} is used to represent the absorption coefficient. The intensity (assuming plane wave) is given by

$$I = \frac{(Ae^{-\mathbf{a}x})^2}{2\rho_0 c}$$

$$I = I_0 e^{-2\mathbf{a}x} \quad 2\mathbf{a} \text{ because it is related to } p^2$$

There are two commonly used expressions for the attenuation coefficient. When the expression for acoustic pressure showing the exponential decrease in amplitude is used,

$$p(x) = p(0)e^{-\mathbf{a}x}$$

then \mathbf{a} is expressed in Nepers per meter (Np/m). The Neper is often used in the specification of \mathbf{a} even though the actual units are m^{-1} in order to distinguish from the expression of \mathbf{a} in dB/m. The two expressions of \mathbf{a} are related as follows.

The decibel is ten times the logarithm (base 10) of the ratio of intensities.

$$\begin{aligned}
 10 \log \left(\frac{I(0)}{I(x)} \right) &= 10 \log \left(\frac{p_{(0)}^2}{p_{(x)}^2} \right) \\
 &= 20 \log \frac{p(0)}{p(x)} \\
 &= 20 \log \frac{p(0)}{p(0)e^{-ax}} \\
 &= 20 ax \log(e) \\
 &= \underbrace{8.686 a x}_{\text{dB/m}}
 \end{aligned}$$

Here ax is specified in Nepers (no units) where the Neper is the natural logarithm of the ratio of pressures as follows.

$$\ln \left(\frac{p(0)}{p(x)} \right) = ax \quad \text{nepers (no units)}$$

In summary, the attenuation (or absorption) coefficient may be expressed in either Np/m or dB/m and they are related as follows.

$$a \left(\frac{\text{dB}}{\text{m}} \right) = 8.686 a \left(\frac{\text{Np}}{\text{m}} \right)$$

Remember that whenever you want to compute the pressure (or some other first order quantity associated with the wave), you must use the expression of a in Np/m in the exponential. Also remember that the actual units for a are m^{-1} .

It can be shown (Homework assignment) that the form for the absorption and wavenumber based on the relaxation are

$$\begin{aligned}
 k_r &= \frac{1}{\sqrt{2}} \frac{w}{c} \left[\frac{\sqrt{1+(wt)^2} + 1}{1+(wt)^2} \right]^{1/2} \\
 a &= \frac{1}{\sqrt{2}} \frac{w}{c} \left[\frac{\sqrt{1+(wt)^2} - 1}{1+(wt)^2} \right]^{1/2} .
 \end{aligned}$$

Note as $t \rightarrow 0$, $a \rightarrow 0$ and $k = \frac{w}{c}$.

We can also look at the speed, we have a

Complex speed $\tilde{c} = \frac{\mathbf{w}}{\tilde{k}}$

Real phase speed $c_p = \frac{\mathbf{w}}{k} = \sqrt{2}c \left[\frac{1 + (\mathbf{w}t)^2}{\sqrt{1 + (\mathbf{w}t)^2} + 1} \right]^{1/2}$

Specific acoustic impedance is complex

$$\begin{aligned} \tilde{z} &= \frac{\tilde{p}}{\tilde{u}} = \mathbf{r}_0 \tilde{c} = \mathbf{r}_0 \frac{\mathbf{w}}{\tilde{k}} \quad \text{for plane wave} \\ &= \mathbf{r}_0 c_p \frac{1}{1 - j \frac{\mathbf{a}}{k_r}} \approx \mathbf{r}_0 c_p \quad \text{for } \frac{\mathbf{a}}{k} \ll 1, \text{ usually true} \end{aligned}$$

Let's examine a couple of cases:

$T \gg t$ ($\mathbf{w}t \ll 1$) When the period of the wave is much greater than the relaxation time, the pressure variation is slow enough that the material has time to adjust to the pressure change and the energy in the wave is largely recovered for each cycle.

The trivial approximation for $\mathbf{w}t \ll 1$ gives

$$k_r = \frac{1}{\sqrt{2}} \frac{\mathbf{w}}{c} \left[\frac{\sqrt{1 + (\mathbf{w}t)^2} + 1}{1 + (\mathbf{w}t)^2} \right]^{1/2} \xrightarrow{\mathbf{w}t \ll 1} k_r = \frac{\mathbf{w}}{c}$$

$$\mathbf{a} = \frac{1}{\sqrt{2}} \frac{\mathbf{w}}{c} \left[\frac{\sqrt{1 + (\mathbf{w}t)^2} - 1}{1 + (\mathbf{w}t)^2} \right]^{1/2} \xrightarrow{\mathbf{w}t \ll 1} \mathbf{a} = 0$$

$$c_p = \frac{\mathbf{w}}{k} = \sqrt{2}c \left[\frac{1 + (\mathbf{w}t)^2}{\sqrt{1 + (\mathbf{w}t)^2} + 1} \right]^{1/2} \xrightarrow{\mathbf{w}t \ll 1} c_p = c$$

Actually, an approximation is used where 1st order terms are kept for low frequencies $\mathbf{w}t \ll 1$. Using the binomial expansion

$$(1 + \mathbf{e})^n \approx 1 + n\mathbf{e} \quad \text{if } \mathbf{e} \ll 1$$

then

$$(1+(wt)^2)^{1/2} \approx 1 + \frac{1}{2}(wt)^2 \quad \text{and} \quad (1+(wt)^2)^{-1/2} \approx 1 - \frac{1}{2}(wt)^2.$$

So,

$$\begin{aligned} a &= \frac{1}{\sqrt{2}} \frac{w}{c} \left[\frac{\sqrt{1+(wt)^2} - 1}{1+(wt)^2} \right]^{1/2} = \frac{1}{\sqrt{2}} \frac{w}{c} \left[\sqrt{1+(wt)^2} - 1 \right]^{1/2} [1+(wt)^2]^{-1/2} \\ a &\approx \frac{1}{\sqrt{2}} \frac{w}{c} \left[\chi + \frac{1}{2}(wt)^2 - \chi \right]^{1/2} \left[1 - \frac{1}{2}(wt)^2 \right] = \frac{1}{\sqrt{2}} \frac{w}{c} \frac{1}{\sqrt{2}} wt \left[1 - \frac{1}{2}(wt)^2 \right] \end{aligned}$$

keeping only the first order terms gives

$$a \approx \frac{1}{2} \frac{w^2 t}{c}.$$

So, for classical attenuation, it is proportional to the frequency squared at low frequencies.

As a consequence, in experimental measurements of absorption, data are usually plotted as $\frac{a}{f^2}$

against f so that at any departure from a horizontal line signals a deviation from the classical prediction. Several measured values are shown in the following figure.

Table 8.5.1

<i>All Data for T = 20°C and p₀ = 1 atm</i>	α/f^2 (Np · s ² /m)			
	Shear Viscosity	Thermal Conductivity	Classical	Observed
<i>Gases</i>	<i>Multiply all values by 10⁻¹¹</i>			
Argon	1.08	0.77	1.85	1.87
Helium	0.31	0.22	0.53	0.54
Oxygen	1.14	0.47	1.61	1.92
Nitrogen	0.96	0.39	1.35	1.64
Air (dry)	0.99	0.38	1.37	α/f peaks at 40 Hz
Carbon dioxide	1.09	0.31	1.40	α/f peaks at 30 kHz
<i>Liquids</i>	<i>Multiply all values by 10⁻¹²</i>			
Glycerin	3000.0	—	3000.0	3000.0
Mercury	—	6.0	6.0	5.0
Acetone	6.5	0.5	7.0	30.0
Water	8.1	—	8.1	25.0
Seawater	8.1	—	8.1	α/f peaks at 1.2 kHz and 136 kHz

Likewise for the phase speed (keeping 2nd order terms only):

$$c_p = \sqrt{2}c \left[\frac{1 + (\omega t)^2}{\sqrt{1 + (\omega t)^2} + 1} \right]^{1/2} \approx \sqrt{2}c \left[\frac{1 + (\omega t)^2}{1 + \frac{1}{2}(\omega t)^2 + 1} \right]^{1/2}$$

$$c_p \approx \sqrt{2}c \left[\frac{\frac{1}{2} + \frac{1 + (\omega t)^2}{2}}{1 + \frac{1}{4}(\omega t)^2} \right]^{1/2} \approx c \left[1 + \frac{1}{2}(\omega t)^2 \right] \left[1 - \frac{1}{8}(\omega t)^2 \right]$$

$$c_p \approx c \left[1 + \frac{1}{2}(\omega t)^2 - \frac{1}{8}(\omega t)^2 \right] = c \left[1 + \frac{3}{8}(\omega t)^2 \right]$$

The dispersion is of order $(\omega t)^2$ so it is only slight at low frequencies and the phase speed is virtually identical with c .

$T \ll t$ ($\omega t \gg 1$) When the period of the wave is much less than the relaxation time, the pressure changes too fast for much change of energy or change of state per cycle and the material exhibits a different value of the property from that above. Note that the loss per cycle is low but that there are many more cycles.

$$k_r = \frac{1}{\sqrt{2}} \frac{\omega}{c} \left[\frac{\sqrt{1 + (\omega t)^2} + 1}{1 + (\omega t)^2} \right]^{1/2} \xrightarrow{\omega t \gg 1} k_r = \frac{1}{c} \sqrt{\frac{\omega}{2t}}$$

$$\mathbf{a} = \frac{1}{\sqrt{2}} \frac{\omega}{c} \left[\frac{\sqrt{1 + (\omega t)^2} - 1}{1 + (\omega t)^2} \right]^{1/2} \xrightarrow{\omega t \gg 1} \mathbf{a} = \frac{1}{c} \sqrt{\frac{\omega}{2t}}$$

$$c_p = \frac{\omega}{k} = \sqrt{2}c \left[\frac{1 + (\omega t)^2}{\sqrt{1 + (\omega t)^2} + 1} \right]^{1/2} \xrightarrow{\omega t \gg 1} c_p = c\sqrt{2\omega t}$$

$T = 2\pi t$ ($\omega t = 1$) At this frequency the loss per cycle is a maximum.

$$k_r = \frac{1}{\sqrt{2}} \frac{\omega}{c} \left[\frac{\sqrt{1 + (\omega t)^2} + 1}{1 + (\omega t)^2} \right]^{1/2} \xrightarrow{\omega t = 1} k_r = 0.777 \frac{\omega}{c}$$

$$a = \frac{1}{\sqrt{2}} \frac{w}{c} \left[\frac{\sqrt{1+(wt)^2} - 1}{1+(wt)^2} \right]^{1/2} \xrightarrow{wt=1} a = 0.322 \frac{w}{c}$$

$$c_p = \frac{w}{k} = \sqrt{2}c \left[\frac{1+(wt)^2}{\sqrt{1+(wt)^2} + 1} \right]^{1/2} \xrightarrow{wt \gg 1} c_p = 1.28c$$

The absorption per wavelength is modeled by the following expression:

$$aI = a \frac{c}{f} = 2(aI)_{\max} \frac{wt}{1+(wt)^2}$$

From this relation it can be seen that $\alpha\lambda = (\alpha\lambda)_{\max}$ at the relaxation frequency, $\omega_R = 1/\tau$, where the absorption per wavelength is a maximum. Well below (above) ω_R the absorption per wavelength increases (decreases) linearly with frequency. What is the frequency behavior of the absorption coefficient?

***** Example 7.1 *****

Ultrasound is propagated through soft tissues and through bone in order to image different structures. If the frequency-dependent attenuation is approximately 5.75×10^{-8} Np/cm/Hz for soft tissue, how much loss in dB occurs for a propagation depth of 1 cm (for 1 MHz, 10 MHz, 100 MHz).

Typically, the easiest way to calculate the losses in dB is to convert the attenuation into dB scale. Using

$$a \left(\frac{\text{dB}}{\text{m}} \right) = 8.686 a \left(\frac{\text{Np}}{\text{m}} \right)$$

gives

$$a = 8.686 \times 5.75 \times 10^{-8} \text{dBcm}^{-1}\text{Hz}^{-1} \times \frac{1 \times 10^6 \text{Hz}}{\text{MHz}}$$

$$a \approx 0.5 \text{dBcm}^{-1}\text{MHz}^{-1}.$$

So, if we look at the propagation depth for imaging of 1 cm (2 cm for travel distance there and back) gives

$$\text{dB}_{\text{loss}}(1\text{MHz}, 1\text{cm}) = 0.5 \text{dBMHz}^{-1}\text{cm}^{-1} \times 1\text{MHz} \times 2\text{cm} = 1\text{dB}$$

$$\text{dB}_{\text{loss}}(10\text{MHz}, 1\text{cm}) = 0.5 \text{dBMHz}^{-1}\text{cm}^{-1} \times 10\text{MHz} \times 2\text{cm} = 10\text{dB}$$

$$\text{dB}_{\text{loss}}(100\text{MHz}, 1\text{cm}) = 0.5 \text{dBMHz}^{-1}\text{cm}^{-1} \times 100\text{MHz} \times 2\text{cm} = 100\text{dB}$$

So we have huge attenuation losses as we increase frequency of imaging in tissues. Typically in ultrasound imaging, the frequencies range from 1 to 12 MHz. Bone typically has a larger attenuation than soft tissue and the frequency-dependent attenuation is proportional to f^2 , so losses in bone tend to be much larger than soft tissues.

The classical absorption coefficient combines thermal and viscous losses.....

$$a_{classical} = \frac{w^2}{2r_0c^3} \left[\frac{4}{3}h + (g-1) \frac{k}{C_p} \right]$$

where k = thermal conductivity

C_p = heat capacity at constant pressure

g = C_p/C_v

h = shear viscosity coefficient

Note: varies as f^2

The relaxation times for the viscous and thermal losses are related by

$$\frac{t_s}{t_k} = \frac{4hC_p}{3k}$$

where t_s is the viscous relaxation time and t_k is the thermal relaxation time.

Scattering

Ex. water droplets in air
gas bubbles in liquid
cell nuclei in soft tissue

We can define the scattering in terms of a scattering cross section σ (m^2). The scattering cross section is the fraction of energy that the scatterer extracts from a sound beam of $1 m^2$ cross section. Near resonance for a bubble in water the scattering cross section can be many times the scatterer's geometric cross section. The following relations apply.

$$I = I_0 e^{-Nsx} \text{ and } P = P_0 e^{-\frac{Ns}{2}x},$$

where N is the number of scatterers per unit volume, x is the distance traveled into the medium and I_o and P_o are the initial values of the intensity and pressure, respectively, at $x = 0$. Thus, the portion of the attenuation coefficient due to scattering is

$$\mathbf{a} = \frac{N\mathbf{s}}{2}$$

In tissue mimicking acoustic phantoms, small particles (scatterers) are used to adjust the attenuation of the materials.