Topic of this homework: Linear Algebra (Inverse of matrix, Gaussian elimination, computing determinants, Cramer's law.)
Deliverables: Your best attempt at the questions.

1 Gaussian Elimination

Read Lec 15 of the class notes (p. 77+)

Derive the solution set for each of the following systems using Gauss elimination and augmented matrix format. Document each step(e.g. 1nd row → 2nd row → + 5 times 1st row), and classify the result(e.g. unique solution, the system is inconsistent, 3 parameter family of solutions, etc.)

$$\begin{array}{rrrcr} x_1 & +x_2 & -2x_3 & = & 3 \\ x_1 & -x_2 & -3x_3 & = & 1 \\ x_1 & -3x_2 & -4x_3 & = & -1 \end{array}$$

2 Determinants

Evaluate the determinant of given matrix using a cofactor expansion about the first and last rows, and also about the last column.

$$\begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix}$$

3 Vandermonde determinant

Show that for the real numbers $x_1, x_2 \cdots x_n$. Hint, use gaussian elimination of the matrix, to render the matrix upper-diagonal.

$$\begin{vmatrix} 1 & x_1 & \cdots & x_1^{n-1} \\ 1 & x_2 & \cdots & x_2^{n-1} \\ & & \cdots & \\ 1 & x_n & \cdots & x_n^{n-1} \end{vmatrix} = \prod_{i<j} (x_j - x_i).$$

4 Inverse of matrix

Compute the inverse of the given matrix. If it doesn't exist, explain why.

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 3 \\ 0 & 3 & -1 \end{pmatrix}$$

5 Cramer's rule

Solve for x_1, x_2 by Cramer's rule.

(a) $x_1 + 4x_2 = 0$, $3x_1 - x_2 = 6$

(d) $x_1 + 2x_2 + 3x_3 = 9$, $x_1 + 4x_2 = 6$, $x_1 - 5x_3 = 2$