ECE 493 **LA2 – Version 1.07 March 6, 2018** **Spring 2018**

Univ. of Illinois **Due Thur, Mar. 8** **Prof. Allen**

Topic of this homework: LinearAlgebra(Schwarz inequality for n dimensional space. Eigenvalues and eigenvectors)

Deliverables: Show your work. Numerical results are not sufficient expect when specifically requested.

1 Gaussian operations

Determine the sequence of P operations (as I discussed in class) that renders each matrix zero below the main diagonal. Hint ck the determinant

1. $A = \begin{bmatrix} 1 & 0 & 1 \\ 3 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix}$

2. Compute the determinant of (show your work) $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2.01 \\ 3 & 3 & 3.01 \end{bmatrix}$

3. Find a matrix that renders $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 2 & 2 \\ 1 & 0 & 3 \end{bmatrix}$ upper-diagonal (zeros below the diagonal).

2 Eigenvalues and eigenvectors

Write the Eigen matrix $E = [e_1 e_2 \cdots e_n]$

1. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

2. $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

3. $\begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & 5 \\ 0 & 0 & 3 \end{bmatrix}$

3 Inequalities

Prove the Schwarz inequality by following the argument below.

1. Let $V, U, E(a)$ be vectors in R^n and a is a real scalar. Taking the norm of the error $E(a)$ gives $||E(a)|| = ||V - aU||$. Find the minimum of E with respect to a. (See pages 187-188 of the class notes.)

2. Give a diagram showing what is happening when you minimize $E(a)$ with respect to a. Provide a diagram showing the minimum value of $E(a^*)$ where a^* is the value such that the gradient of E with respect to a is zero.

1. Prove the triangular inequality $||\vec{u}+\vec{v}|| \leq ||\vec{u}||+||\vec{v}||$. All of these are to be done for R^n where n is greater that 3. In other words, don't assume that $n = 2$ or 3.

2. What is the name of the triangular inequality when $n = 2$? Who first showed this?

4 Gram-Schmidt

Apply Gram-Schmidt method to vectors $v_1 = (1, 1, 0, 1)^T$, $v_2 = (1, -2, 0, 0)^T$ and $v_3 = (1, 0, -1, 2)^T$ to get three orthogonal (or orthonormal) vectors. As shown in class

$$e_K = v_K - \sum_{k=1}^{K-1} (v_K \cdot \hat{e}_k)\hat{e}_k \tag{1}$$

where $\hat{e}_k = e_k/||e_k||$.

5 Vector sums

Given $A\vec{x} = \vec{b}$, expand this as a sum of vectors of the form $x_1A_1 + x_2A_2 + \cdots = \vec{b}$. Give the formula for A_1, A_2, etc. What is x_1 here? What is $\vec{b}$?

6 Quadratic forms

- Expand the following as a *quadratic form*

$$f(x_1, x_2, x_3) = x^T \begin{bmatrix} 2 & 1 & -1 \\ 1 & 4 & 3 \\ -1 & 3 & 4 \end{bmatrix} x$$

- Now expand this in the form: $f = \lambda_1\tilde{x}_1^2 + \lambda_2\tilde{x}^2 + \lambda_3\tilde{x}^3$
- Is the matrix *positive definite*? Explain.
- Is this equivalent to *completing the squares*? Explain.