ECE 493 **HW0 – Version 1.00 January 1, 2018** **Spring 2018**

Univ. of Illinois **Due TR, Jan. 23, 2018** **Prof. Allen**

Topic of this homework: Evaluation exercises; Not graded.
Deliverables: You best attempt at the questions. If you don't know, just say so. I'm looking for your collective baseline. It is not in your best interest to answer questions you don't understand (e.g., don't copy stuff from Wikipedia).

1. (a) In a few sentences explain how an abacus works. Given a few examples that shows 1, 10, 11, 100, etc. Lets define a short bead as one of the group of five, and a long bead as one of the two. Normally the abacus is held with the short beads (group of 5) near you, and the long beads (group of 2) away from you.
   (b) Is an abacus base 10, or base $16 = 2^4$? Justify your answer.
   (c) Explain why the acabus is relevant to mathematics.

2. (a) Integrate the equation
   $$F = ma$$
   where F is the force, acceleration $a = dv/dt$ is the rate of change of the velocity $v(t)$, and m is the mass.

   Solution:
   $$F(t) = m\frac{dv}{dt}$$
   it follows that
   $$\int dv = \frac{1}{m}\int F(t)dt.$$
   Justify the above procedures and clean up the argument.
   (b) If $F(t) = \delta(t)$ then the velocity is a step function at t=0
   $$v(t) = \frac{1}{m}\int \delta(t)/m dt = u(t).$$
   (c) What is the displacement $x(t)$ if $v \equiv dx/dt$?
   (d) What if $F(t) = \sin(cx - t)$? Find
   $$\frac{\partial F}{\partial x} =?$$

3. Given the differential equation $\ddot{x} + b(x)\dot{x} + 1 = 0$
   (a) What is the order
   (b) If $b(x) = 0$, the equation homogeneous or inhomogeneous? Explain.

4. Expand the following functions in a power series and state the *Radius of convergence* (ROC):
   (a) $e^x =$
   (b) $xe^x =$
   (c) $1/(1 - x) =$
   (d) $1/(1 - jx) =$
   (e) $x + x^2 =$
   (f) Can you expand $1/(1 - x)$ about $x = 1$?

5. If $F = ma$ as above but also $F = kx$, then what happens? Describe what kind of system this is, and what happens.

6. Starting from Euler's formula $e^{ix} = \cos(x) + i\sin(x)$, expand each of the three terms in a Taylor series, and prove this relationship.

7. Find the *radius of convergence* (ROC) x_0 of $f(x) := \sum_{n=0}^{\infty} nx^n$. Explain.

8. An electrical (mechanical) circuit consists of an inductor (mass) *in series* with a capacitor (spring). A sine wave $\sin(\omega t)$ is turned on, and after a "long time," the voltage (force) and current (velocity) are measured. The impedance $Z(\omega)$ is defined as the *steady-state* voltage (force) $V(\omega)$ over the steady-state current (velocity) $I(\omega)$. The impedance of the inductor is $i\omega L$, and of the capacitor, $1/i\omega C$.

   (a) When two impedances are in series, the total impedance Z_{total} is the sum of the two impedances. What is the formula for the total impedance?

   (b) When the two impedances are in parallel, the voltages (forces) are the same, but the currents (velocities) add. What is the formula for the total impedance?

   (c) Find and discuss the *resonant frequency*.

   (d) How many resonant frequencies are there?

9. A pure delay of T seconds may be expressed as $\delta(t - T)$, where $\delta(t)$ is called a delta function.

   - What is the formula for the Fourier Transform of $f(t)$?
   - What is the Fourier Transform of the pure delay function $f(t) = \delta(t - T)$?
   - Define the *angle* of the Fourier Transform of a delay of T?

10. Plot the function $y = x^{-p}$ for the case of $p \to \infty$.

11. (a) Compute the *determinant* of D if

$$D = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}?$$

   i. Find the *eigenvalues* of D.

   ii. What is the definition of the *eigenvectors*? Do you know how to find them?

   (b) Find the determinant of the matrix

$$\begin{bmatrix} 1 & 2 & 1 \\ 3 & 4 & 2 \\ 1 & 1 & 1 \end{bmatrix}.$$

   i. What is the "best" way of computing this determinant? "A computer." is not the answer I'm looking for.

   ii. Write out the matlab code that verifies your answer.