1 Problems NS2

Topic of this homework: Neuron and synapse terminology; Postulates of systems; Analysis of a diffusion transmission line.

Problem *# 1: Labeled sketch of the neuron and synapes.*

- *1.1: Sketch and fully label the drawing of the neuron Fig. 2.1*

- *1.2: Sketch and fully label the Synapes Fig. 2.2*

Problem *# 2: Using the system properties of networks, discuss the following properties*

- *2.1: Causality*

- *2.2: Reciprocity*

- *2.3: Positive-real impedance*

- *2.4: Explain why the postulates are important for the case of neurons and cells.*

- *2.5: In mathematical terms, define negative and positive feedback*

- *2.6: In a few paragraphs discuss the McCullock-Pitts model of a Neuron*

Problem *# 3: Analysis of the diffusion equation:*
In the previous homework we analyzed the circuit of Fig. 1.

By cascading many of these cells together we may find the solution of the diffusion equation

$$\frac{\partial^2}{\partial x^2}v(x,t) = D_o\frac{\partial}{\partial t}v(x,t) \leftrightarrow sD_oV(s,x).$$

where D_o is the the diffusion constant. In this problem we will find the solution to a system composed of many of these cells cascaded together. The diffusion equation is used to model neural spike propagation. However the equation must be made nonlinear to emulate neural spikes. Here we study the linear fission equation.

From the previous homework for the diffusion line we found:

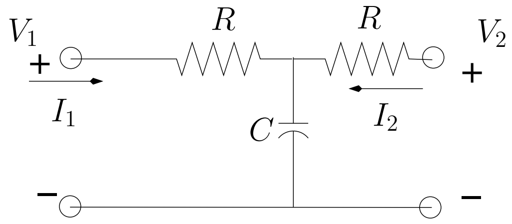

Figure 1: *This three-element electrical circuit that is a lumped element version of the diffusion equation. In the previous homework we defined the normalized frequency as $s/s_c = RCs$ in terms of a time constant $\tau = RC$ and cutoff frequency $s_c = 1/\tau$.*

1. Find the 2×2 ABCD matrix representation of Fig. 1. Express the results in terms of the dimensionless ratio s/s_c where $s_c = 1/\tau$ is the cutoff frequency and $\tau = RC$ is the time constant.

$$\begin{aligned}
\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} &= \begin{bmatrix} (1+RCs) & R \\ sC & 1 \end{bmatrix} \begin{bmatrix} 1 & R \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \\
&= \begin{bmatrix} (1+RCs) & R(2+RCs) \\ sC & (1+RCs) \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \\
&= \begin{bmatrix} (1+\frac{s}{s_c}) & R(2+\frac{s}{s_c}) \\ sC & (1+\frac{s}{s_c}) \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \\
&= \mathcal{T} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}
\end{aligned}$$

We wish to cascade N cell we need to compute $\mathcal{T}^N$, which may be done using an eigenmatrix expansion. If we define the matrix

$$\mathcal{T} = \begin{bmatrix} \mathcal{A} & \mathcal{B} \\ \mathcal{C} & \mathcal{D} \end{bmatrix}.$$

In terms of eigenvectors $\boldsymbol{E}$ and eigenvalues Λ,

$$\mathcal{T}\boldsymbol{E} = \boldsymbol{E}\Lambda.$$

Post multiplying by $\boldsymbol{E}^{-1}$ gives

$$\mathcal{T} = \boldsymbol{E}\Lambda\boldsymbol{E}^{-1}.$$

Reversible systems: When $\mathcal{A} = \mathcal{D}$ the transmission matrix is said to be reversible, and the properties greatly simplify. In this case the eigenmatrix $\boldsymbol{E}$ and eigenvalue matrix are:

$$\boldsymbol{E} = \begin{bmatrix} -\sqrt{\frac{\mathcal{B}}{\mathcal{C}}} & +\sqrt{\frac{\mathcal{B}}{\mathcal{C}}} \\ 1 & 1 \end{bmatrix} \qquad \Lambda = \begin{bmatrix} \mathcal{A} - \sqrt{\mathcal{B}\mathcal{C}} & 0 \\ 0 & \mathcal{A} + \sqrt{\mathcal{B}\mathcal{C}} \end{bmatrix}$$

2. Write out the eigenmatrix equation for the diffusion line as the product of $N = 2$ cells.

3. Find the eigenvalues of the 2×2 diffusion matrix.

4. Find the eigenvector matrix of the transmission matrix.

5. Finally find $\mathcal{T}^N$.