3.7.1 System postulates

Solutions of differential equations, such as the wave equation, are conveniently described in terms of mathematical properties, which we present here in 11 system postulates (see Appendix F, p. 251, for greater detail):

(P1) *Causality* (noncausal/acausal): Causal systems respond when acted upon. All physical systems obey causality. An example of a causal system is an integrator, which has a response of a step function. Filters are also examples of causal systems. Signals represent acausal responses. They do not have a clear beginning or end, such as the sound of the wind or traffic noise. A causal linear system is typically complex analytic and is naturally represented in the complex s plane via Laplace transforms. A nonlinear system may be causal but not complex analytic.

(P2) *Linearity* (nonlinear): Linear systems obey superposition. Let two signals $x(t)$ and $y(t)$ be the inputs to a linear system, producing outputs $x'(t)$ and $y'(t)$. When the inputs are presented together as $ax(t) + by(t)$ with weights $a, b \in \mathbb{C}$, the output is $ax'(t) + by'(t)$. If either a or b is zero, the corresponding signal is removed from the output.

Nonlinear systems mix the two inputs, thereby producing signals that are not present in the input. For example, if the inputs to a nonlinear system are two sine waves, the output contains distortion components that have frequencies not present at the input. One example of a nonlinear system is one that multiplies the two inputs. A second is a diode, which rectifies a signal, letting current flow in only one direction. Most physical systems have some degree of nonlinear response, but this is not always desired. Other systems are designed to be nonlinear, such as the diode example.

(P3) *Passive* (active): An active system has a power source, such as a battery, while a passive system has no power source. Although you may consider a transistor amplifier to be active, it is so only when connected to a power source. Brune impedances satisfy the positive-real condition (Eq. 3.2.2.18, p. 85).

(P4) *Real* (complex) time response: All physical systems are Real in, Real out. They do not naturally have complex responses (real and imaginary parts). While a Fourier transform takes real inputs and produces complex outputs, this is not an example of a complex time response. This postulate is a characterization of the input signal, not its Fourier transform.

(P5) *Time-invariant* (time varying): For a system to be a time-varying system, the output must depend on when the input signal starts or stops. If the output, relative to the input, is independent of the starting time, then the system is said to be *time-invariant*.

(P6) *Reciprocal* (non- or antireciprocal): In many ways this is the most difficult postulate to understand. It is best characterized by the ABCD matrix (see p. 105). If $\Delta_T = 1$, the system is said to be *reciprocal*. If $\Delta_T = -1$, it is said to be *antireciprocal*. The impedance matrix is reciprocal when $z_{12} = z_{21}$ and antireciprocal when $z_{12} = -z_{21}$. Dynamic loudspeakers are antireciprocal and must be modeled by a gyrator, which may be thought of as a transformer that swaps the force and flow variables. For example, the input impedance of a gyrator terminated by an inductor is a capacitor. This property is best explained by Fig. 3.7 (p. 108). For an extended discussion on reciprocity, see page 255.

(P7) *Reversibility* (nonreversible): If swapping the input and output of a system leaves the system invariant, it is said to be reversible. When $A = D$, the system is reversible. Note the distinction between reversible and reciprocal.

(P8) *Space-invariant* (space-variant): If a system operates independently as a function of where it physically is in space, then it is space-invariant. When the parameters that characterize the system depend on position, it is space-variant.

(P9) *Deterministic* (random): Given the wave equation along with the boundary conditions, the system's solution may be deterministic, or not, depending on its extent. Consider a radar or sonar wave propagating out into uncharted territory. When the wave hits an object, the reflection can return waves that are not predicted due to unknown objects. This is an example where the boundary condition is not known in advance.

(P10) *Quasistatic* ($ka < 1$): Quasistatics follows the Nyquist sampling theorem for systems that have dimensions that are small compared to the local wavelength (Nyquist, 1924). This assumption fails when the frequency is raised (the wavelength becomes short). Thus this is also known as the *long-wavelength* approximation. Quasistatics is typically stated as $ka < 1$, where $k = 2\pi/\lambda = \omega/c_o$ and a is the smallest dimension of the system. See page 201 for a detailed discussion of the role of quasistatics in acoustic horn wave propagation.

Postulate P10 is closely related to the Feynman lecture *The "underlying unity" of nature,* where Feynman asks (Feynman, 1970b, Ch. 12-7): "Why do we need to treat the fields as smooth?" His answer is related to the wavelength of the probing signal relative to the dimensions of the object being probed. This raises the fundamental question: Are Maxwell's equations a band-limited approximation to reality? Today we have no definite answer to this question.

The following quote seems relevant:[19]

> *The Lorentz force formula and Maxwell's equations are two distinct physical laws, yet the two methods yield the same results.*
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> *Why the two results coincide was not known. In other words, the flux rule consists of two physically different laws in classical theories. Interestingly, this problem was also a motivation behind the development of the theory of relativity by Albert Einstein. In 1905, Einstein wrote in the opening paragraph of his first paper on relativity theory, "It is known that Maxwell's electrodynamics—as usually understood at the present time—when applied to moving bodies, leads to asymmetries which do not appear to be inherent in the phenomena." But Einstein's argument moved away from this problem and formulated special theory of relativity, thus the problem was not solved.*
>
> *Richard Feynman once described this situation in his famous lecture (The Feynman Lectures on Physics, Vol. II, 1964), "we know of no other place in physics where such a simple and accurate general principle requires for its real understanding an analysis in terms of two different phenomena. Usually such a beautiful generalization is found to stem from a single deep underlying principle. . . . We have to understand the "rule" as the combined effects of two quite separate phenomena."*

[19] `https://www.sciencedaily.com/releases/2017/09/170926085958.htm`