1 Problems NS1

**Topic of this homework:** Solution method for the diffusion equation; History; Differential equation system classification

Deliverable: Answers to problems

**Problem # 1: A two-port network application for the Laplace transform**

![Figure 1: This three-element electrical circuit is a system that acts to low-pass filter the signal voltage $V_1(\omega)$, to produce signal $V_2(\omega)$. It is convenient to define the dimensionless ratio $s/s_c = RC$ in terms of a time constant $\tau = RC$ and cutoff frequency $s_c = 1/\tau$.](image)

- 1.1: Find the $2 \times 2$ ABCD matrix representation of Fig. 1. Express the results in terms of the dimensionless ratio $s/s_c$ where $s_c = 1/\tau$ is the cutoff frequency and $\tau = RC$ is the time constant.

**ANS:**

- 1.2: Find the eigenvalues of the $2 \times 2$ matrix. As summarized in Allen (2021) (Appendix B.3.1), the eigenvalues $\lambda_{\pm}$ of a $2 \times 2$ matrix

$$T = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

are

$$\lambda_{\pm} = \frac{1}{2} \left[ (A + D) \pm \sqrt{(A - D)^2 + 4BC} \right].$$

**ANS:**
1.3: Assuming that $I_2 = 0$, find the transfer function $H(s) \equiv V_2/V_1$.

ANS:

1.4: Find the pole and residue of $H(s)$?

ANS:

1.5: Find $h(t)$, the inverse Laplace transform of $H(s)$.

ANS:

1.6: Assuming that $V_2 = 0$, find $Y_{12}(s) \equiv I_2/V_1$.

ANS:

1.7: Find the input impedance to the right-hand side of the system, $Z_{22}(s) \equiv V_2/I_2$ for two cases:

1. $I_1 = 0$

ANS:
2. $V_1 = 0$

ANS:

– 1.8: Find the determinant of the ABCD matrix.

ANS:

**History**

**Problem #2:** Write a sentence or two about each person.

– 2.1: Provide a brief definition of the following properties:

1. Ramon y Cajal. **ANS:**

2. Charles Scott Sherrington. **ANS:**

3. Rafael Lorente de No. **ANS:**

5. McCulloch and Pitts. **ANS:**

6. Albert Einstein. **ANS:**

7. Hodgkin and Huxley. **ANS:**

8. Hermann Helmholtz. **ANS:**
System Classification

Problem # 3: Answer the following system classification questions about physical systems, in terms of the system postulates.

– 3.1: Provide a brief definition of the following properties:

L/NL : linear(L)/nonlinear(NL): ANS:

TI/TV : time-invariant(TI)/time varying(TV): ANS:

P/A : passive(P)/active(A): ANS:

C/NC : causal(C)/non-causal(NC): ANS:

Re/Clx : real(Re)/complex(Clx): ANS:
3.2: Along the rows of the table, classify the following systems: In terms of a table having 5 columns, labeled with the abbreviations: L/NL, TI/TV, P/A, C/NC, Re/C lx:

<table>
<thead>
<tr>
<th>#</th>
<th>Case</th>
<th>Definition</th>
<th>L/NL</th>
<th>TI/TV</th>
<th>P/A</th>
<th>C/NC</th>
<th>Re/C lx</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Conduction</td>
<td>$i(t) = g_m E(t)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Diffusion</td>
<td>$i(t) = D \frac{d[Na]}{dt}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Switch</td>
<td>$v(t) = \begin{cases} 0 &amp; t \leq 0 \ V_h &amp; t &gt; 0 \end{cases}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Channel</td>
<td>$i(t) = g_m(v(t))$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Membrane</td>
<td>$I_{out} = g_m(V_{in})$</td>
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<td></td>
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<td></td>
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</tr>
<tr>
<td>6</td>
<td>Nerve cell</td>
<td>Hogkin-Huxley Eqs.</td>
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<tr>
<td>7</td>
<td>Nerve cell</td>
<td>Physical nerve cells</td>
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</tr>
<tr>
<td>8</td>
<td>Neural spike</td>
<td>$v(t, x) = \delta(t - x/c_o)$</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Trans. Line</td>
<td>ABCD matrix</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>