1 Problems NS1

Topic of this homework: Solution method for the diffusion equation; History; Differential equation system classification

Deliverable: Answers to problems

Problem *# 1: A two-port network application for the Laplace transform*

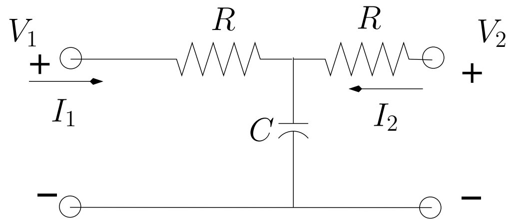

Figure 1: *This three-element electrical circuit is a system that acts to low-pass filter the signal voltage $V_1(\omega)$, to produce signal $V_2(\omega)$. It is convenient to define the dimensionless ratio $s/s_c = RCs$ in terms of a time constant $\tau = RC$ and cutoff frequency $s_c = 1/\tau$.*

– 1.1: Find the 2×2 ABCD matrix representation of Fig. 1. Express the results in terms of the dimentionless ratio s/s_c where $s_c = 1/\tau$ is the cutoff frequency and $\tau = RC$ is the time constant.

ANS:

– 1.2: Find the eigenvalues of the 2×2 matrix. As summarized in Allen (2021) (Appendix B.3.1), the eigenvalues $\lambda_\pm$ of a 2×2 matrix

$\mathcal{T} = \begin{bmatrix} \mathcal{A} & \mathcal{B} \\ \mathcal{C} & \mathcal{D} \end{bmatrix}$ are $\lambda_\pm = \frac{1}{2}\begin{bmatrix} (\mathcal{A}+\mathcal{D}) - \sqrt{(\mathcal{A}-\mathcal{D})^2 + 4\mathcal{B}\mathcal{C}} \\ (\mathcal{A}+\mathcal{D}) + \sqrt{(\mathcal{A}-\mathcal{D})^2 + 4\mathcal{B}\mathcal{C}} \end{bmatrix}$.

ANS:

– *1.3: Assuming that* $I_2 = 0$*, find the transfer function* $H(s) \equiv V_2/V_1$.

ANS:

– *1.4: Find the pole and* residue *of* $H(s)$*?*

ANS:

– *1.5: Find* $h(t)$*, the inverse Laplace transform of* $H(s)$.

ANS:

– *1.6: Assuming that* $V_2 = 0$*, find* $Y_{12}(s) \equiv I_2/V_1$.

ANS:

– *1.7: Find the input impedance to the* right-hand side *of the system,* $Z_{22}(s) \equiv V_2/I_2$ *for two cases:*

1. $I_1 = 0$

   ANS:

2. $V_1 = 0$
   ANS:

– *1.8: Find the determinant of the ABCD matrix.*

ANS:

History

Problem *# 2: Write a sentence or two about each person.*

– *2.1: Provide a brief definition of the following properties:*

1. Ramon y Cajal. ANS:

2. Charles Scott Sherrington. ANS:

3. Rafael Lorente de No. ANS:

4. Minsky and Papert (1969). ANS:

5. McCulloch and Pitts. ANS:

6. Albert Einstein. ANS:

7. Hodgkin and Huxley. ANS:

8. Hermann Helmholtz. ANS:

System Classification

Problem *# 3: Answer the following system classification questions about physical systems, in terms of the system postulates.*

– *3.1: Provide a brief definition of the following properties:*

L/NL : linear(L)/nonlinear(NL): ANS:

TI/TV : time-invariant(TI)/time varying(TV): ANS:

P/A : passive(P)/active(A): ANS:

C/NC : causal(C)/non-causal(NC): ANS:

Re/Clx : real(Re)/complex(Clx): ANS:

– *3.2: Along the rows of the table, classify the following* systems*: In terms of a table having 5 columns, labeled with the abbreviations: L/NL, TI/TV, P/A, C/NC, Re/Clx:*

					Category		
#	Case:	Definition	L/NL	TI/TV	P/A	C/NC	Re/Clx
1	Conduction	$\boldsymbol{i}(t) = g_m \boldsymbol{E}(t)$	____	____	____	____	____
2	Diffusion	$i(t) = D\frac{d[Na]}{dx}$	____	____	____	____	____
3	Switch	$v(t) \equiv \begin{cases} 0 & t \le 0 \\ V_0 & t > 0. \end{cases}$	____	____	____	____	____
5	Channel	$i(t) = g_m(v(t))$	____	____	____	____	____
6	Membrane	$I_{out} = g_m(V_{in})$	____	____	____	____	____
7	Nerve cell	Hogkin-Huxley Eqs.	____	____	____	____	____
8	Nerve cell	Physical nerve cells	____	____	____	____	____
9	Neural spike	$v(t,x) = \delta(t - x/c_o)$	____	____	____	____	____
10	Trans. Line	ABCD matrix	____	____	____	____	____