1 Problems NS3

Topic of this homework: Neuron and synapse terminology; Postulates of systems; Analysis of a diffusion transmission line.

History

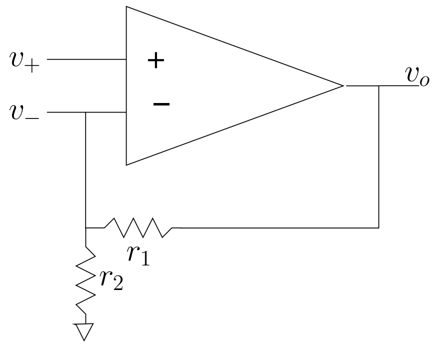
Problem *# 1: Provide a brief discussion of the following individuals:*

– *1.1: Albert Einstein*

– *1.2: Hodgkin and Huxley*

– *1.3: Hermann Helmholtz*

Problem *# 2: Set up the equations and solve for various properties of the OpAmp.*



– *2.1: In qualitative terms, what is the ratio of the input to output impedance.*

– *2.2: Describe the purpose and setup for the* space-clamp *circuit.*

– *2.3: Find the formula for the transfer function* $H = V_o/V_+$.

Problem *# 3: Thermodynamics of the cell membrane. Set up the equations to estimate the equilibrium sodium and potassium concentrations.*

– 3.1: Define the three membrane currents that re the most important to action potentials (spikes).

$$J_{\tt disp} = C_o \frac{d}{dt} v(t)$$
$$J_c(s) = q\mu_{\tt Na}[\texttt{Na}^+]E = q\mu_{\tt Na}[\texttt{Na}^+]\frac{dV}{dx}$$
$$J_d = -qD_{\tt Na}\frac{d}{dx}[\texttt{Na}^+]$$

– 3.2: What is the relation between the conduction and diffusion currents under equilibrium conditions?

– 3.3: Derive the relation between the voltage and $\lfloor \texttt{Na}^+ \rfloor$ *when the system is in the equilibrium condition?*

– 3.4: Integrate the differential equation and derive the relation between the $\texttt{Na}^+$ *concentrations on the two sides of the membrane and the voltage across the membrane* $V = V_o - V_i$*. Hint: see pp. 57-59.*

Problem *# 4: Analyize a* Δ *long patch of membrane. Set up the equations to estimate the properties of a mylinated nerve fiber.*

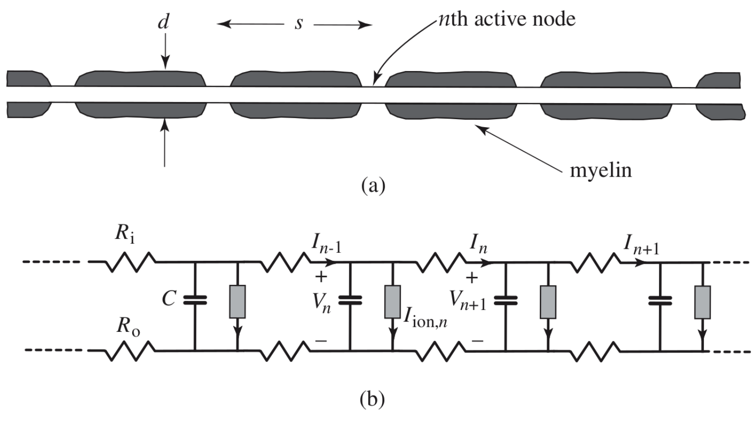


Figure 1: *Diagram of the Frog axon showing the physical and electrical circuit (Scott, p. 142).*

– 4.1: Assume the following

$$\lambda_o f_{\max} = v_e = 23 \text{ [m/s]}$$
$$= \lambda_o/2 \text{ [mm]}$$

From a previous homework we assumed that $\Delta = \lambda/2$ and $\tau = RC$. Here $\Delta = s$ is taken to be the distance between nodes. Find $f_{\max}$.

Standard frog axon

Distance between nodes $(s) = 2$ mm.

Outside fiber diameter $(d) = 14$ μm.

Internal resistance/length $(r_\mathrm{i}) = 140$–145 megohm/cm.

External resistance/length $(r_\mathrm{o}) \ll r_\mathrm{i}$.

Capacity of myelin/length $(c_\mathrm{m}) = 10$–16 pF/cm.

Capacity of active node $(C_\mathrm{n}) = 0.6$–1.5 pF.

Experimental impulse speed $v_\mathrm{e} = 23$ m/s.

Figure 2: *Parameters measured for the Frog axon (Scott, p. 142)*

– 4.2: Find the time constant ($\tau = RC$) and the cutoff frequency $f_c = 1/2\pi\tau$. Compare f_c to f_max.

– 4.3: The two figures below show the parameters $n(t), m(t), h(t)$ used in Hodkins–Huxley's model of spike generation. Their equation for the current is :

$$I(t) = C_m \frac{d}{dt} V + g_\mathrm{K} n^4(t)(V - V_\mathrm{K}) g_\mathrm{Na} m^3(t) h(t)(V - V_\mathrm{Na}).$$

The generally accepted but unproven form of the spike voltage is the solution to the linear wave equation

$$V(t) = S(x - v_e t),$$

where x is the axial distance down the fiber and c_s is the spike velocity. A number of models have attempted to replace the phenomenological H–H model with a physical model into closer agreement with the data, however H–H is still the generally accepted explaination.

Explain in your own words why the nonlinear model would have the same properity as the wave equation, where the spikes travel at the fixed velocity v_e (see the above equation).