Wave Transmittance

2-port wave networks

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ECE-537

Basic definitions for 2-ports

- Pressure and Velocity at an interface

$$x_k \equiv x(k\Delta_x)$$

At every frequency ω and place x:

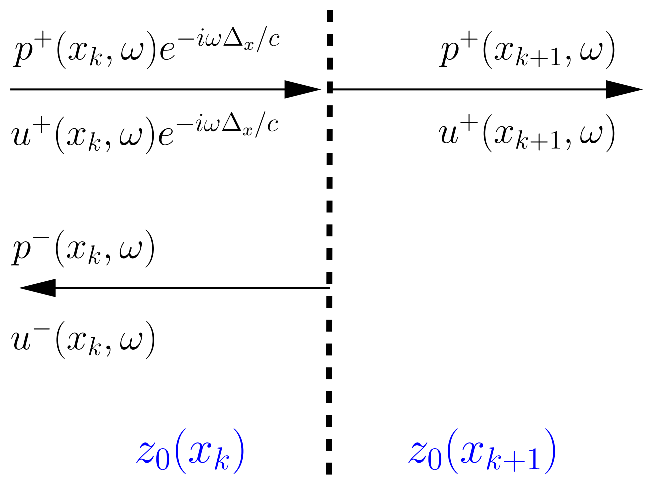
$$p(x_k,\omega) = p^+(x_k,\omega)e^{-i\omega\Delta_x/c} + p^-(x_k,\omega)$$

$$u(x_k,\omega) = u^+(x_k,\omega)e^{-i\omega\Delta_x/c} - u^-(x_k,\omega)$$

Solving for the out-going waves:

$$p^+(x_{k+1},\omega) = p^+(x_k,\omega)e^{-i\omega\Delta_x/c} + p^-(x_k,\omega)$$

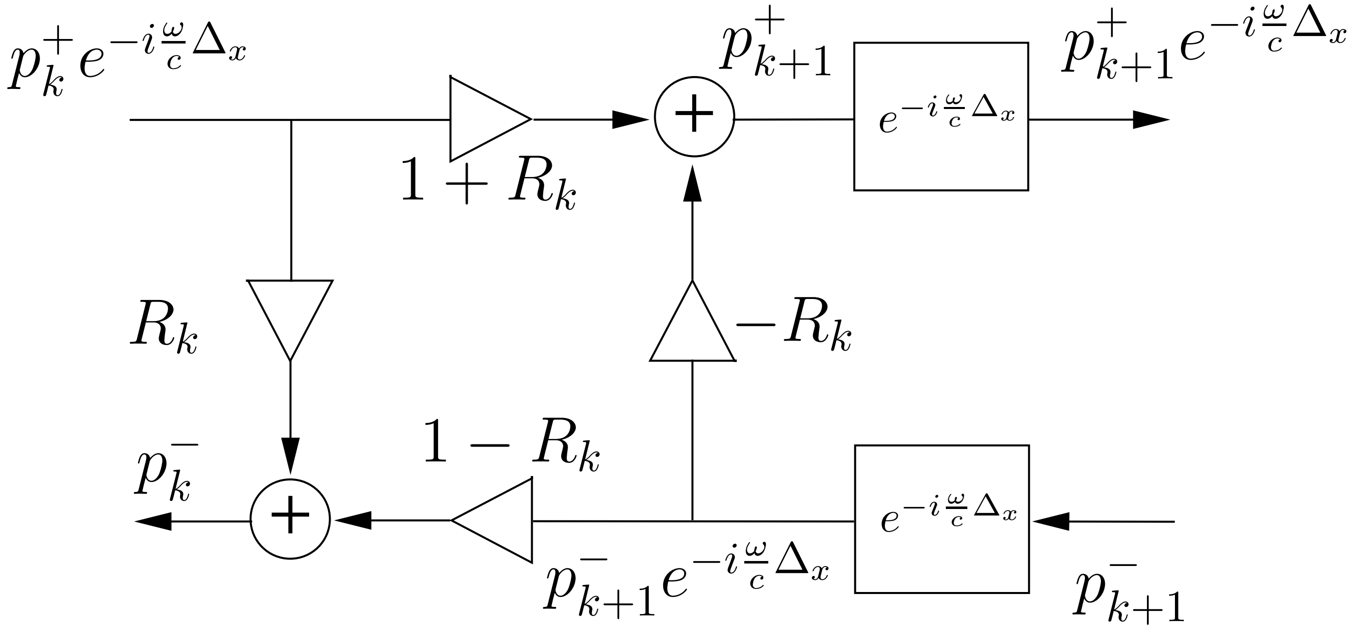
$$u^+(x_{k+1},\omega) = u^+(x_k,\omega)e^{-i\omega\Delta_x/c} - u^-(x_k,\omega)$$



- so at the interfact $\frac{p^+(x_{k+1},\omega)}{u^+(x_{k+1},\omega)} = \frac{p^+(x_k,\omega)e^{-i\omega\Delta_x/c}+p^-(x_k,\omega)}{u^+(x_k,\omega)e^{-i\omega\Delta_x/c}-u^-(x_k,\omega)}$
- or $\frac{z_0(x_{k+1})}{z_0(x_k)} = \frac{1+R(x_k)}{1-R(x_k)}$
- finally $R_k \equiv R(x_k) = \frac{z_0(x_{k+1})-z_0(x_k)}{z_0(x_{k+1})+z_0(x_k)}$

2-Port Wave modeling

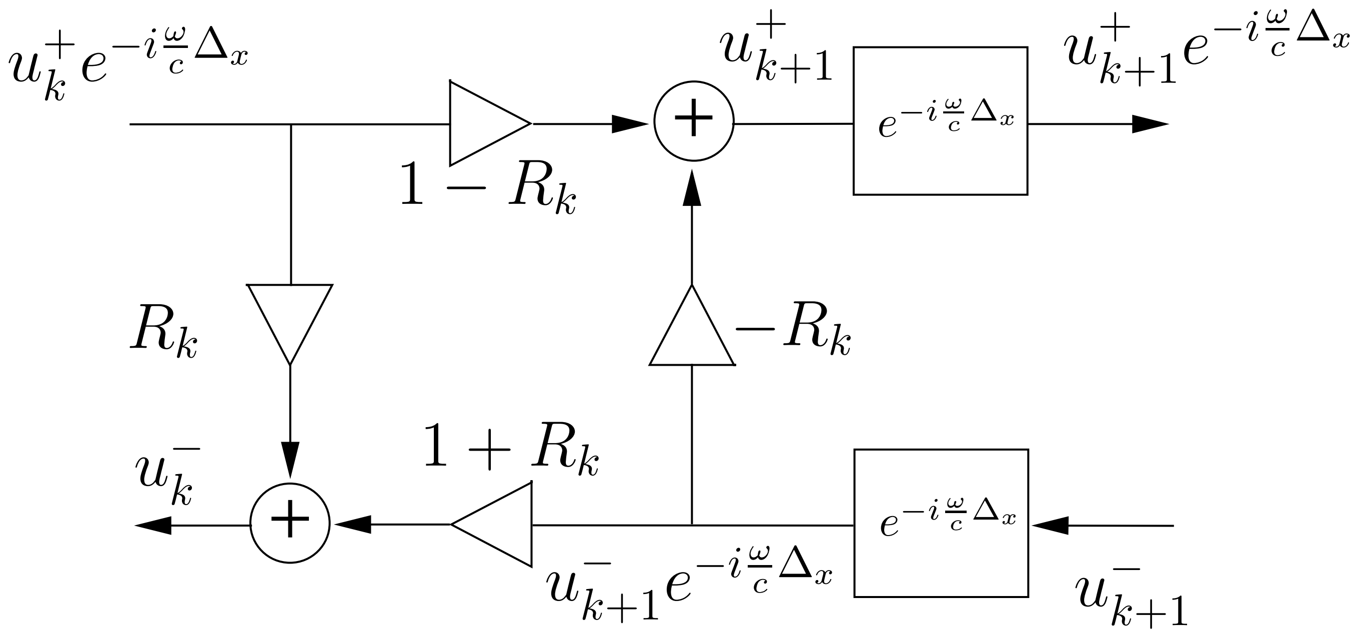
- Basic pressure wave section at x_k



- $R_k \equiv \frac{z_0(k+1)-z_0(k)}{z_0(k+1)+z_0(k)}$
- $p^+(x_{k+1}, t) \equiv (1+R_k)p^+(x_k, t-\Delta_x/c)$

2-Port Wave modeling

- Basic volume-velocity wave section at x_k



- $R_k \equiv \frac{z_0(k+1)-z_0(k)}{z_0(k+1)+z_0(k)}$
- $u^+(x_{k+1}, t) \equiv (1 - R_k)u^+(x_k, t - \Delta_x/c)$

2-Ports Summary

- The reflected and transmitted waves are determined by
  - Continuity of the pressure and velocity at each impedance discontinuity
  - The forward Pressure wave is given by

$$p^+_{k+1}(\omega) = (1 + R_k)p^+_k(\omega)e^{-i\omega L_k/c}$$

  - The forward volume-velocity wave is given by

$$u^+_{k+1}(\omega) = (1 - R_k)u^+_k(\omega)e^{-i\omega L_k/c}$$

- In practice you do not need both pressure and velocity.
  - If you need u, use z_0 to get it from p:

$$u^+_k = p^+_k / z_0(k)$$