

# The EM Algorithm

## *Parameter modeling and extraction*

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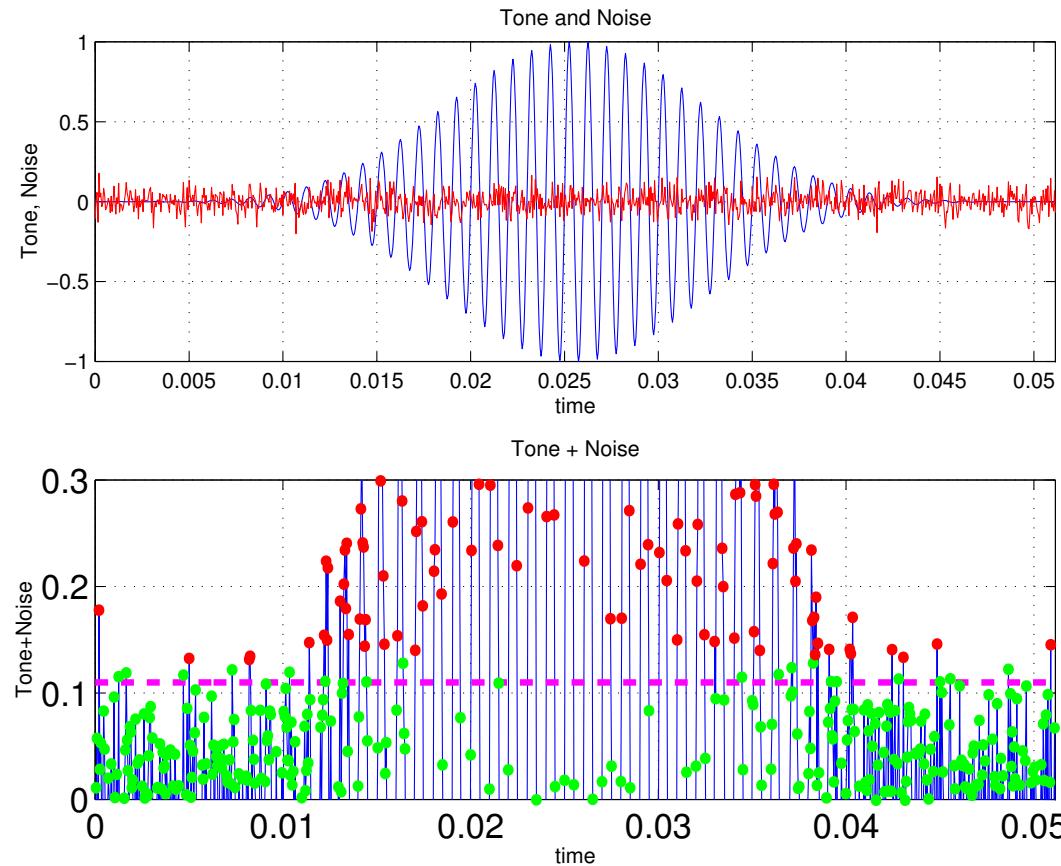
**ECE-537**

# The EM Algorithm

- This method is one of the most successful (and therefore important) methods used today
- It is a parametric model
- It is a nonlinear iterative method
- It is easy to program for simple problems
  - Summary: EM is a **must-learn** technique

# An example

- Let's take the case of a tone-burst in noise

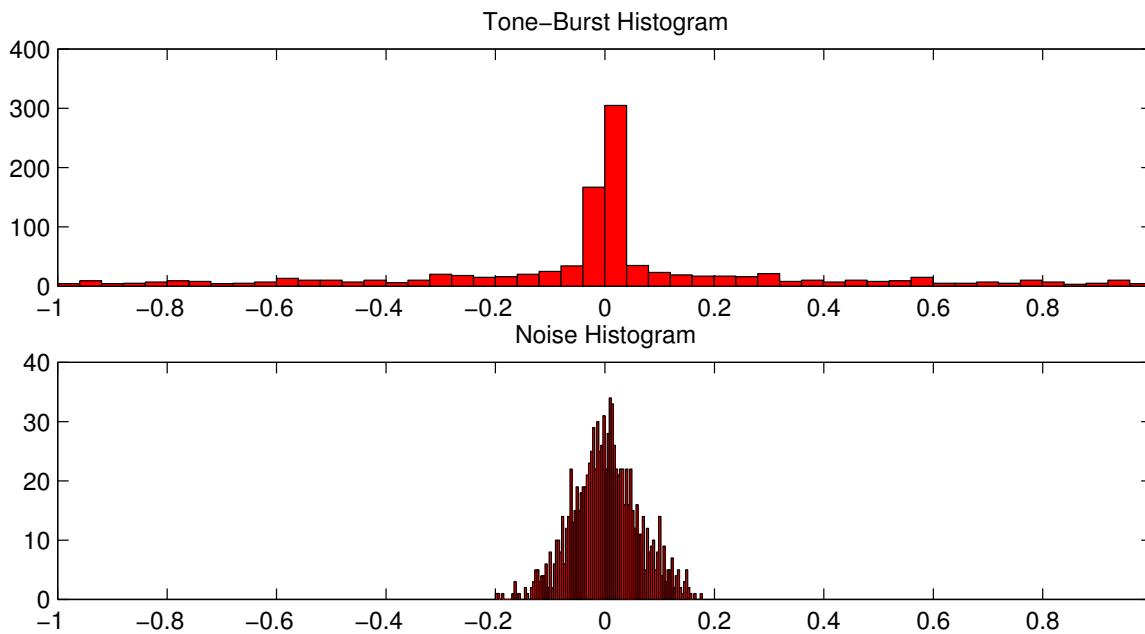


- A simple threshold is used to classify two regions
- How do we pick the threshold?

# Picking the Threshold

- We start with a parametric model of the two distributions

$$P(x|tone) = \mathcal{G}(\mu_t, \sigma_t), \quad P(x|noise) = \mathcal{G}(\mu_n, \sigma_n)$$



- Find the  $\hat{\sigma}_n \approx \min(\text{std}(x(t : t + M)))$ ,  $M$  point blocks
- Set threshold  $\Theta_n \equiv 3\hat{\sigma}_n$  (i.e., 0.13)

# More generally

- Estimate: the  $k^{th}$  model parameters  $k \geq 1$   
(guess the first time,  $k = 0$ )

$$\mu_{noise|signal}^{(k)} \equiv \text{mean} \left( x(i_{noise|signal}^{(k-1)}) \right)$$

$$\sigma_{noise|signal}^{(k)} \equiv \text{std} \left( x(i_{noise|signal}^{(k-1)}) \right)$$

- Maximize:  $\Theta$  is the threshold on the likelihood ratio, given the  $k^{th}$  parameter estimates:

$$i_{signal}^{(k)} = \text{find} \left( \frac{\mathcal{G}(x, \mu_{signal}^{(k)}, \sigma_{signal}^{(k)})}{\mathcal{G}(x, \mu_{noise}^{(k)}, \sigma_{noise}^{(k)})} \geq \Theta \right)$$

$$i_{noise}^{(k)} = \text{find} \left( \frac{\mathcal{G}(x, \mu_{signal}^{(k)}, \sigma_{signal}^{(k)})}{\mathcal{G}(x, \mu_{noise}^{(k)}, \sigma_{noise}^{(k)})} < \Theta \right)$$