Linear predictive speech coding

Extracting poles from the waveform

Jont Allen

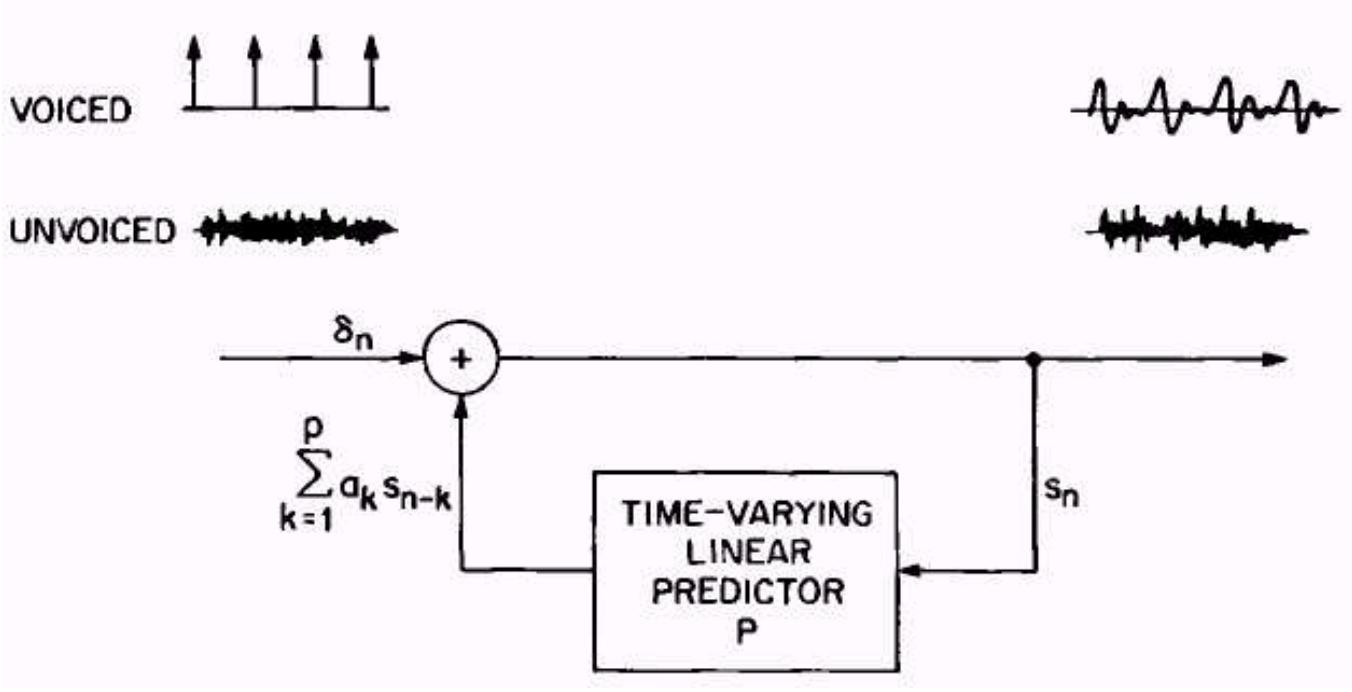
ECE-437

Basic Assumptions

- Speech is generated by glottal pulses of air . . .
- spectrally shaped by the vocal tract transfer function
- The vocal tract is a transmission line, with an output at the mouth
- When the input and output of a tube are at the ends, the spectrum is “all-pole”
  - The voiced speech transfer function is an all-pole response

Model of speech generation Atal 71

- Basic model of speech generation:



Concepts: Pros and Cons

- Atal (1971): *... traditional Fourier analysis methods require a relatively long speech segment to provide adequate spectral resolution. As a result, rapidly changing speech events cannot be accurately followed.*
  - This is very true, but a heavy price is paid, when the speech is noisy.
- Atal (1971): *Although pitch-synchronous* analysis by synthesis *techniques can provide a partial solution . . . , such techniques are extremely cumbersome and time consuming even for modern digital computers and are therefore unsuitable for automatic processing of large amounts of speech data.*
  - Today pitch-synchronous methods are widely used, due to their much higher quality.

Basic covariance method

- The linear prediction estimate is defined in terms of unknown coefficients a_k

$$\hat{s}_n \equiv \sum_{k=1}^{p} a_k s_{n-k} \equiv \mathbf{S}_n \cdot \mathbf{A}$$

- The *prediction error* is defined as

$$e_n \equiv s_n - \hat{s}_n$$

- and the total error E_T is minimized to find $\mathbf{A}$

$$E_T \equiv \sum_{n=1}^{N} e_n^2 = \sum_{n=1}^{N} \left(s_n - \sum_{k=1}^{p} a_k s_{n-k} \right)^2$$

Finding the prediction coefficients

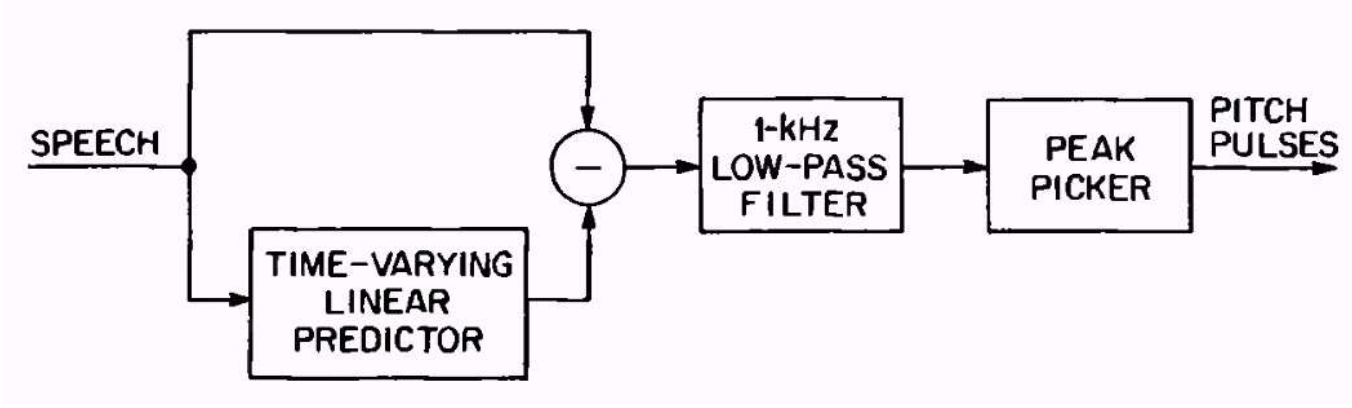
- The *normal equations* for prediction coefficients $\mathbf{A}$

$$-\frac{1}{2}\frac{\partial E_t}{\partial a_i}\bigg|_{i=1,\ldots,p} = \underbrace{\sum_{n=1}^{N} s_n s_{n-i}}_{r_i \equiv \mathbf{S}_i \cdot \mathbf{S}_0} - \underbrace{\sum_{k=1}^{p} a_k \sum_{n=1}^{N} s_{n-k} s_{n-i}}_{\mathbf{R}_i \cdot \mathbf{A}} = 0,$$

$$\begin{bmatrix} R_{11} & R_{12} & \cdots & R_{1p} \\ R_{21} & & & \\ \vdots & & \ddots & \\ R_{p1} & & & R_{pp} \end{bmatrix} \mathbf{A} = \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_p \end{bmatrix}$$

Model of error signal coding

- Once $\mathbf{A(t)}$ is determined, every 5-10 ms, the error e_n is computed



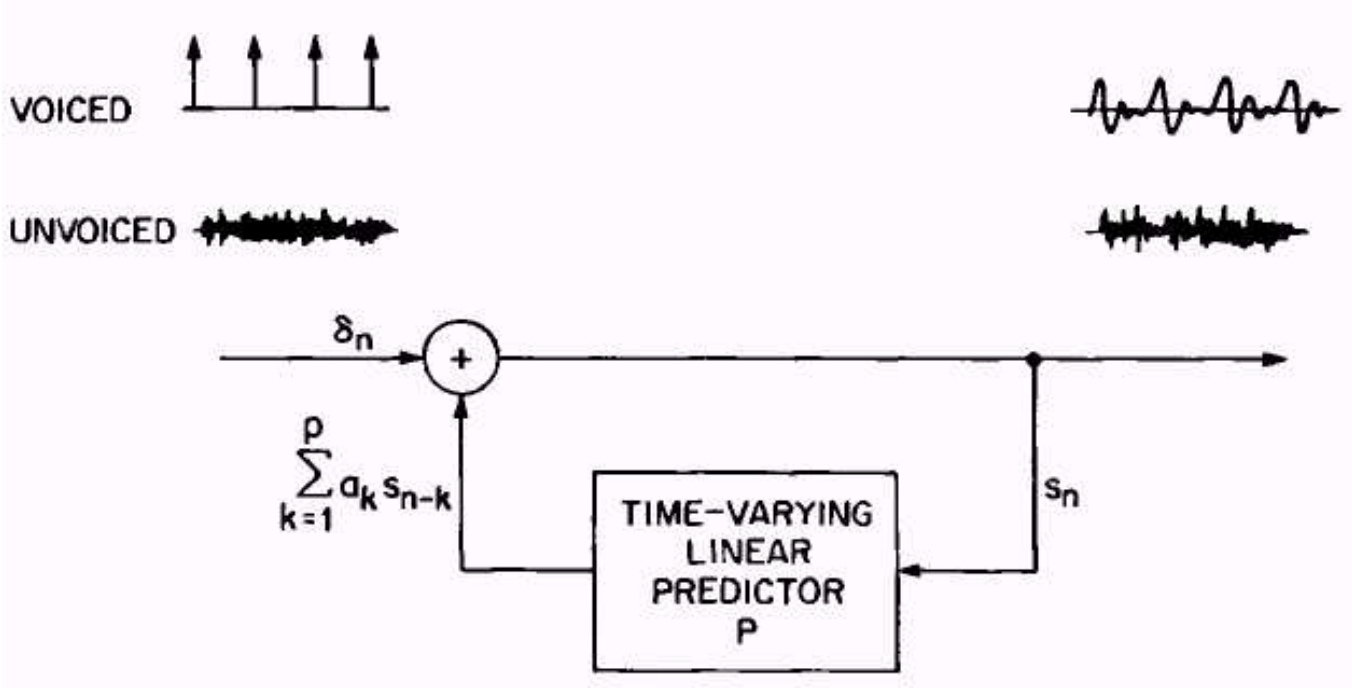
- The error is computed by convolution of s_n with $[1, -A]$

$$e_n = s_n - \sum_{k=1}^{p} a_k s_{n-k} = [1, -A] \star \mathbf{S}_n$$

- e_n contains everything that was not captured by $\mathbf{A(t)}$

Model of speech generation Atal 71

- The speech may be regenerated exactly from e_n



- Or, the speech may be regenerated from the encoded error

Other definitions

- Normal-equations may also be defined in terms of *windowed speech* samples
- or *windowed error*
- All these options, and many more, have been extensively explored, and in great depth

Finding the formants

- The roots of $[1, -A] \equiv [1, , -a_1, -a_2, \ldots, -a_p]$ are estimates of the formants of the speech
- Ideally speaking, the formants are the poles of the vocal tract
- In practice, these estimates are not very robust