sensitive, whereas the Nyquist samples are position insensitive. For nonuniform sampling, as used in VQ, we may find $\widetilde{P}_x(\nu)$ directly from the sample "correlated" points x_n which define the aliasing errors. Because the samples are not computed on a uniform grid, direct evaluation of the samples by DFT is seriously inaccurate.

4.1 Magnitude of the aliasing error

As an example, we look at the scalar Lloyd-Max quantizer, and study the nature of the aliasing errors in the $\widetilde{CF}$ domain. The vector case, in the CF domain seems to be a trivial extension of this case.

For a Gaussian PDF, the optimum quantizers from 2 through 16 and 32 have been tabulated. Thus we may easily compute the $\widetilde{CF}$ for each of these cases. We assume that the $\widetilde{p}_x(\zeta)$ has probability weights given by the area defined by the quantizer region for x_n. Thus we may find $\widetilde{P}_x(\nu)$ from the Lloyd-Max tables by Fourier transform of the following $\widetilde{p}_x(\zeta)$

$$\widetilde{p}_{LM}(\zeta) = \sum_{l=1}^{L} \delta(\zeta - \zeta_l) \int_{\zeta=b_l}^{B_l} p_x(\zeta) d\zeta, \tag{26}$$

where ζ_l are the table quantizer levels corresponding to the L regions $[b_l, B_l]$. In Fig. 3 we see a plot of $\widetilde{P}_x(\nu)$ for nine different L values between 2 and 32, as obtained from the tables.

The nature of the aliasing error is clear from these $\widetilde{P}_x(\nu)$ plots. At high frequencies, the aliasing error is large, but for a limited range of ν values, the CF is close to Gaussian. From the CF point of view, the nonuniform quantizer does the best it can in removing low frequency errors, at the expense of high frequency aliasing error. This type of noise spectrum is called *noise shaping* in the $\Sigma\Delta$ field. Thus the Loyd-Max quantizer uses noise shaping. I dont think this observation is well known.

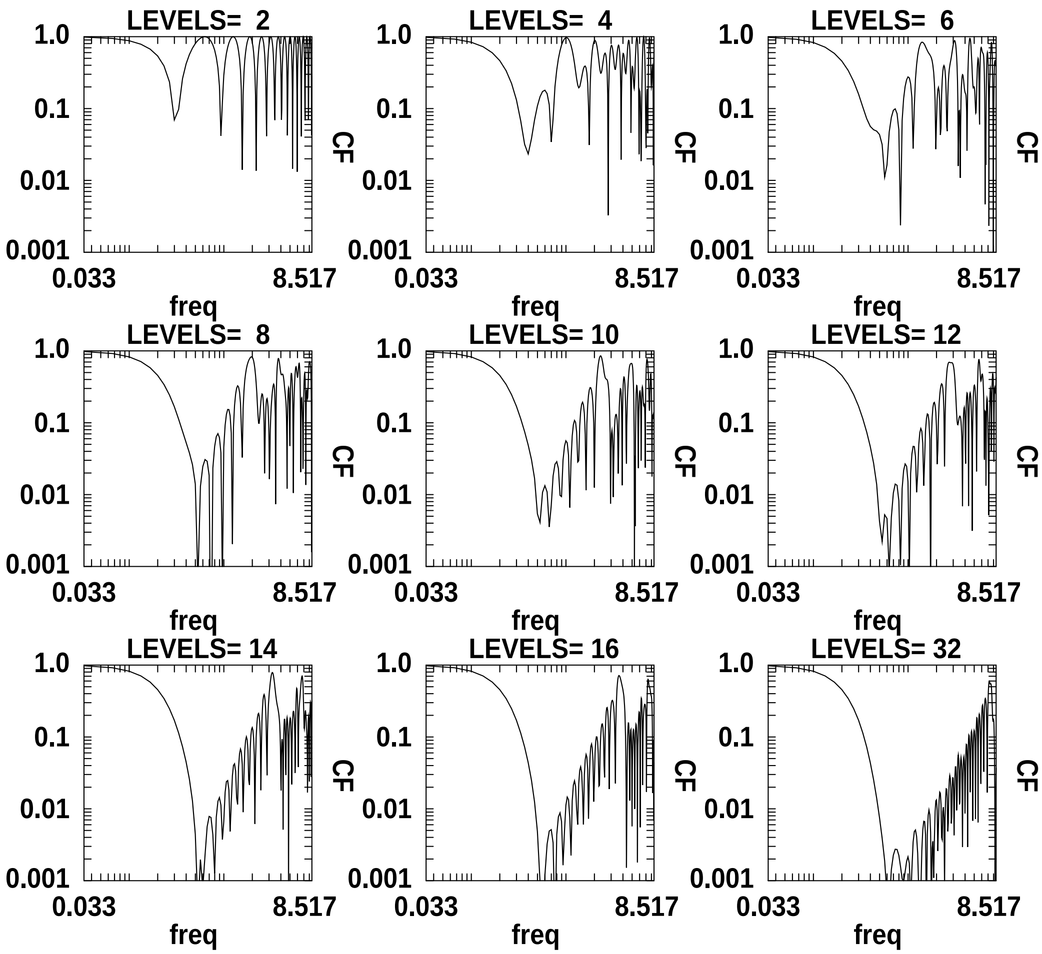

Figure 3: *This montage shows the $\widetilde{CF}$ for the nonuniformly spaced Gaussian Lloyd-Max quantizer, computed using the formula of Eq. 26. The quantizer levels run from 2 to 32 for these examples. The spectrum is* noise shaped*, a jargon term of $\Sigma\Delta$ conversion, indicating that the error is pushed to high frequencies. This type of error is useful in that the best approximation to the desired signal may be obtained simply by lowpass filtering.*