# The intensity JND comes from Poisson neural noise

## Implications for image coding

**J. B. Allen**[1]
**AT&T Labs–Research**
**Florham Park, NJ 07932-0971**

# CHRONOLOGICAL DEVELOPMENT

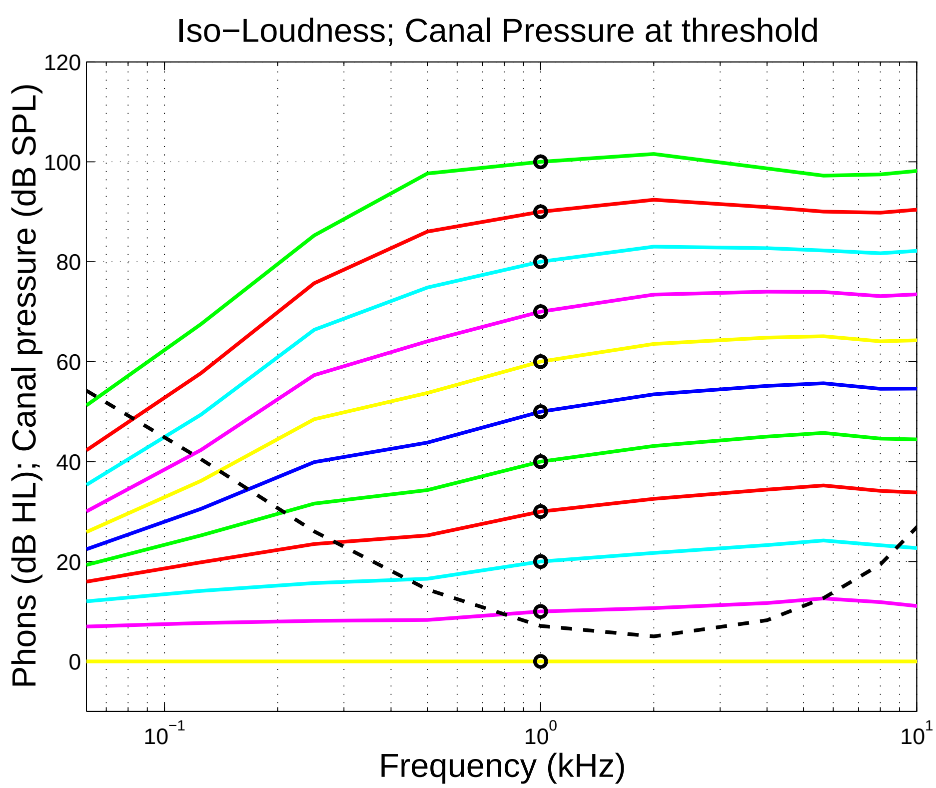
| YEAR | CONCEPT | REFERENCE |
|---|---|---|
| 1846 | **JND** | Weber |
| 1860 | **Counting JNDs** | Fechner |
| 1923 | **Hearing Threshold** | Fletcher and Wegel |
| 1927 | **Decision theory model** | Thurstone |
| 1928 | **Near-miss to Weber's law** | Riesz |
| 1933 | **Masking and loudness** | Fletcher and Munson |
| | $N_{\text{JND}}(\mathcal{L}, I, f)$ | Riesz |
| 1947 | *Wide–band JND ($J = 0.1$)* | G. A. Miller |
| | *Tones vs. NB noise maskers* | Egan and Hake |
| 1966 | **Signal detection theory** | Green and Swets |
| 1997 | **Loudness and the JND** | Allen and Neely |

# Psychophysics of hearing

- Threshold of a tone,
- JND
  - The threshold is the first JND
  - Counting JNDs
- Loudness
  - Cochlear compression
    - The outer hair cell
  - Additivity of loudness
    - One vs. two ears
- Relating Loudness to the JND

# Threshold and superthreshold contours

- Ear canal pressure at threshold (dB SPL)
- Equal loudness curves (dB HL)




- The threshold is the first JND

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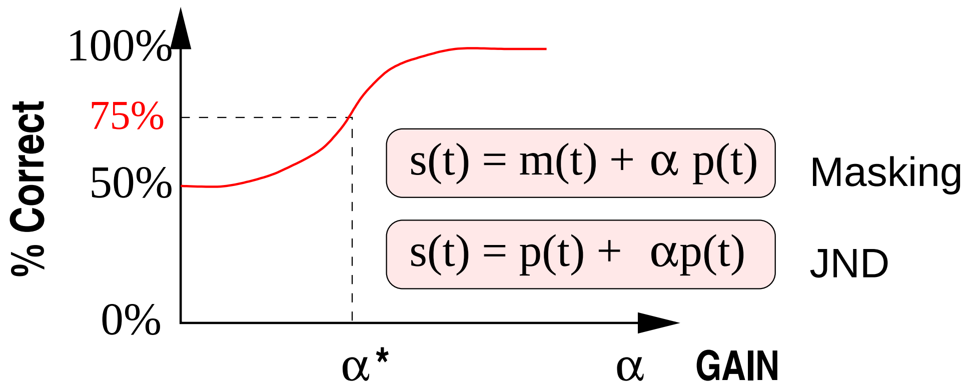
# THE JND

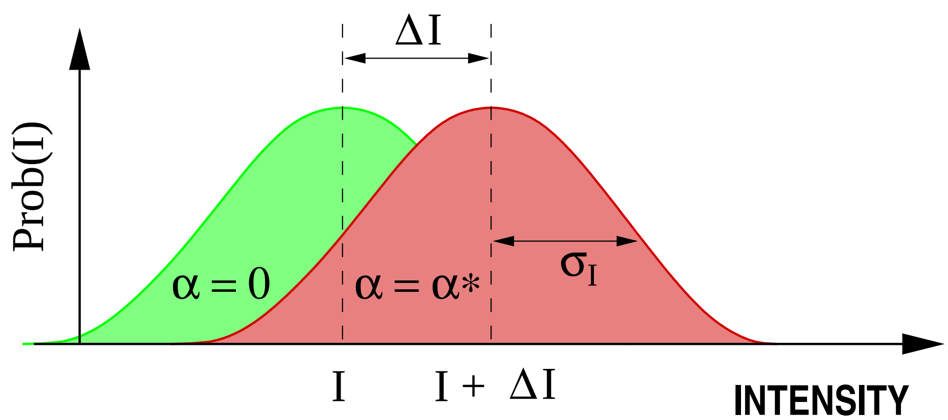
# The JND reflects internal noise

- **1846** Weber proposes that the just-noticeable discrimination (i.e., the JND) is proportional to the magnitude of a stimulus
  - **Examples:**
    - $\Delta$weight $\propto$ weight
    - $\Delta B \propto B$ ($B$ is the light intensity)
    - $\Delta I \propto I$ ($I$ is the sound intensity)
- 1927 Thurstone's model
  - *The law of comparitive judgment*
  - $\Delta$weight, $\Delta B$, $\Delta I \propto$ perceptual noise

# SIGNAL DETECTION THEORY

- The SIGNAL DETECTION MODEL of masking introduced into psychophysics by L. L. Thurstone 1927 and David Green 1965:




$$\alpha = \alpha^* \quad \text{when} \quad \Delta I = \sigma_I$$

- Signal Detection Theory is used to define the Just Noticeable Difference (JND) in intensity between two otherwise identical signals: The JND is the relative signal level where the level difference is identified 75% of the time.

$$\Delta I \propto \sigma_I$$

# SNR of floating point

- Perceptual noise is analogous to floating point (actually $\mu$-law)

  i.e.: $\Delta I \propto$ RMS-error $\sigma \propto$ mean

- $\Delta I / I = \sigma_I / I$ is a NOISE/SIGNAL RATIO
- $J \equiv \Delta I / I$ = constant is called Weber's law
- **PROBLEM:** Weber formulated his problem in the physical domain, while the noise is internal

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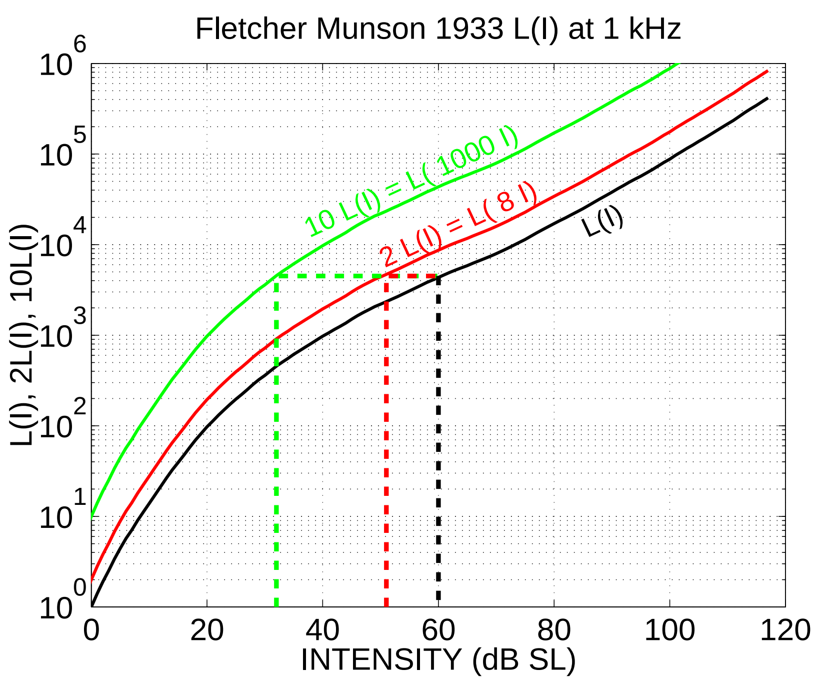
# LOUDNESS

# LOUDNESS–LEVEL AND LOUDNESS

- 1933 Fletcher and Munson's loudness growth data based on loudness additivity is now called Stevens' Law

$$\mathcal{L}(I) = I^{0.3}$$



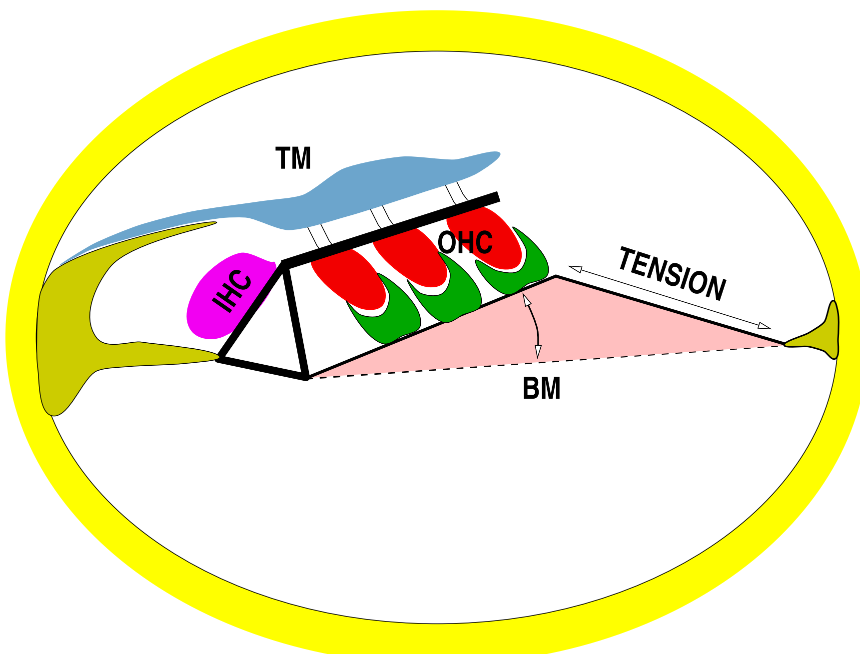

**Loudness vs. intensity for 1, 2, and 10 equally loud components**

- Brightness has the same exponent as loudness

$$\mathcal{B}(I) = I^{0.3}$$

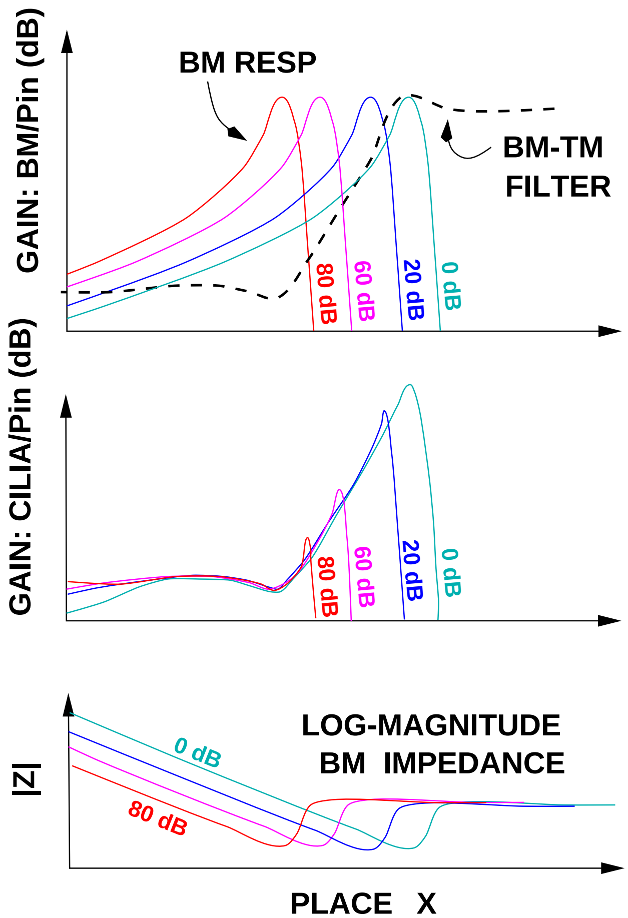
# How does the OHC compress the dynamic range?

- Series of events:
  - Intensity $I \uparrow \Rightarrow$
    OHC hyper-polarization $\Rightarrow$
    OHC stiffness $K_{\text{OHC}} \downarrow \Rightarrow$
    BM stiffness $K_{\text{BM}} \downarrow \Rightarrow$
    characteristic frequency $f_{\text{CF}} \downarrow \Rightarrow$
    characteristic place $x_{\text{CF}} \rightarrow$ base



# Effect of shifting EP on IHC tuning

- Small changes in the BM stiffness will have a large effect on the IHC tuning when the TM is assumed to act as a high–pass filter




Cartoon showing low–pass BM excitation patterns and high–pass tectorial membrane transduction filter, as a function of place for one stimulus frequency, at levels 0, 20, 40 and 80 dB SL.

The neural response defined as the product of the BM excitation pattern and the TM transduction filter responses.

The log–magnitude BM impedance at 0, 20, 40, and 80 dB SL assuming the BM stiffness changes with level. This figure shows that the EP shifts toward the base as the stiffness is reduced.

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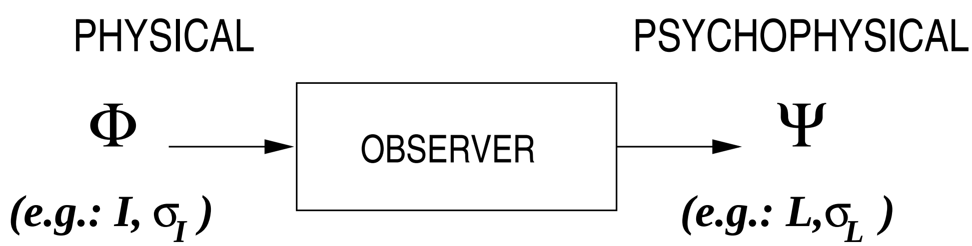
# MODELING THE JND

# The intensity JND reflects loudness uncertainty

- Perception is stochastic (Thurstone, 1927):

  Each time you hear (see) the same short tone (light) pulse, you hear (see) it with a different loudness (brightness)

- The intensity $\text{JND}_I$ $(\Delta I)$ is a measure of this internal perceptual fluctuation (noise) given by $\sigma_{\mathcal{L}}$



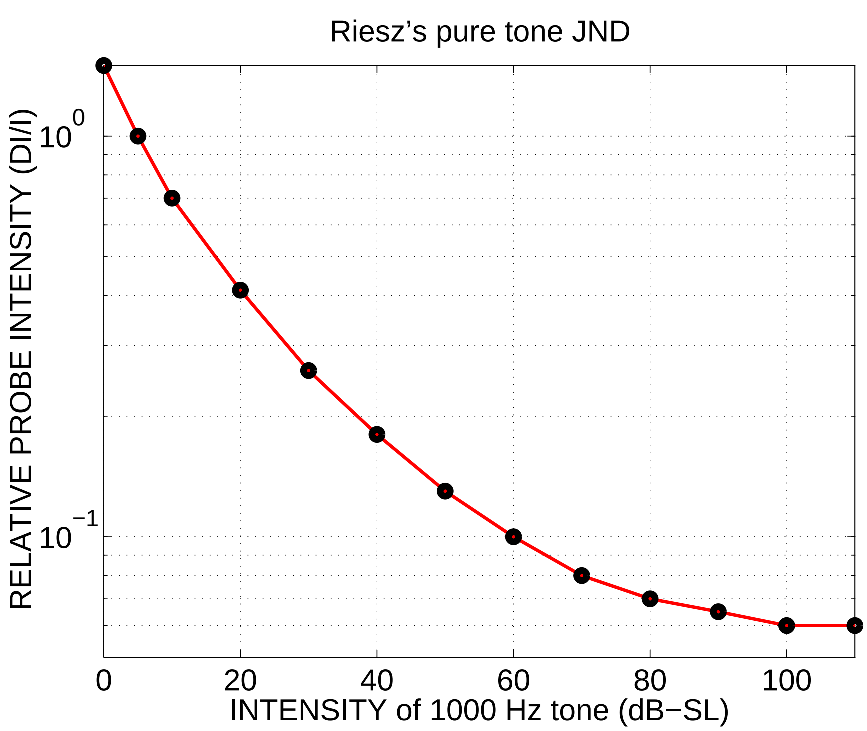

- Namely

$$\Delta\mathcal{L} \propto \sigma_{\mathcal{L}}(\mathcal{L}),$$

  the loudness JND is proportional to the internal “loudness noise

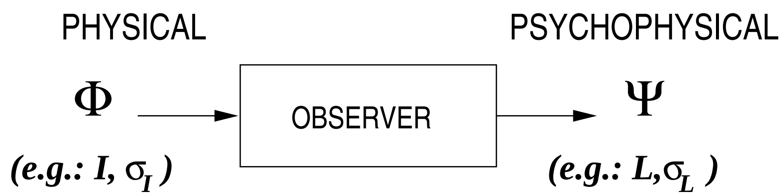
# PURE-TONE INTENSITY DISCRIMINATION

- Weber's "law" says that $\Delta I \propto I$
  - Weber's Law holds for floating point conversion
  - For fixed point, $\sigma_I = \Delta I$ is a constant
  - Is the ear a fix or floating point converter?
- **1928** Riesz establishes the near-miss to Weber's law for tones
  - Riesz used two beating tones 3 Hz apart for this measurement (i.e., 1000 Hz masker and a low-level 1003 Hz probe)



# FECHNER'S THEORY OF THE JND

- Fechner is called the father of psychophysics.




- **1860** Fechner's idea was that the loudness $\mathcal{L}(I)$ is proportional to the number of JND steps $N_{\text{JND}}$, which is given by:

$$N_{\text{JND}} \equiv \int \frac{d\mathcal{L}}{\Delta\mathcal{L}(\mathcal{L})} = \int \frac{dI}{\Delta I(I)}$$

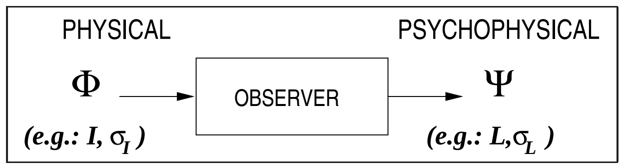
  – He assumed that the internal noise $\Delta\mathcal{L} = \sigma_{\mathcal{L}}$ is constant
  – He assumed that $\Delta I \propto I$, i.e. Weber's Law
    * These two assumptions give Fechner's "Law":

$$\mathcal{L}(I) \propto \log(I)$$

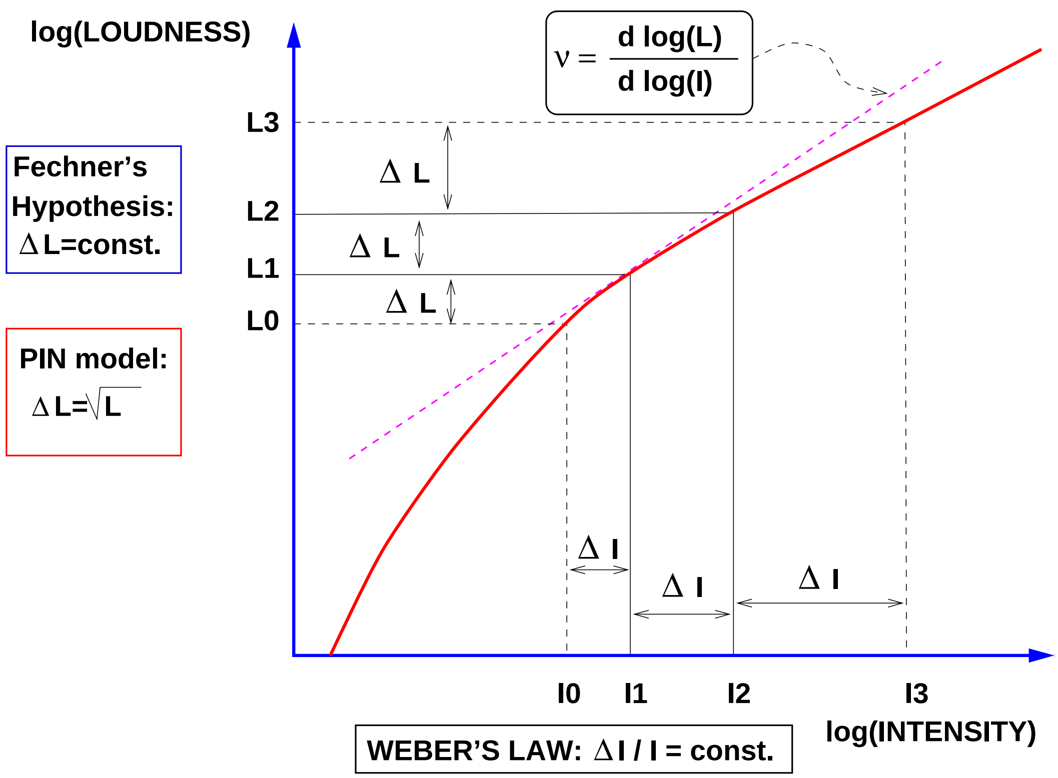
- Counting JNDs is a great conceptual start :○)
  – Both assumptions are wrong :○(
  – Fechner's "Law" is wrong

# BASIC MODEL OF OBSERVER

- How to find $\Delta\mathcal{L}(\mathcal{L})$ Allen and Neely 1997




- Transform from $\Delta I(I)$ to $\Delta\mathcal{L}(\mathcal{L})$




- Since the loudness is a *Power-Law* where $L(I) \propto I^{\nu}$:

$$\frac{I}{\Delta I(I)} = \nu(I)\frac{\mathcal{L}}{\Delta\mathcal{L}(\mathcal{L})},$$

which is the same as

$$\boxed{\text{SNR}_I = \nu\text{SNR}_{\mathcal{L}}.}$$

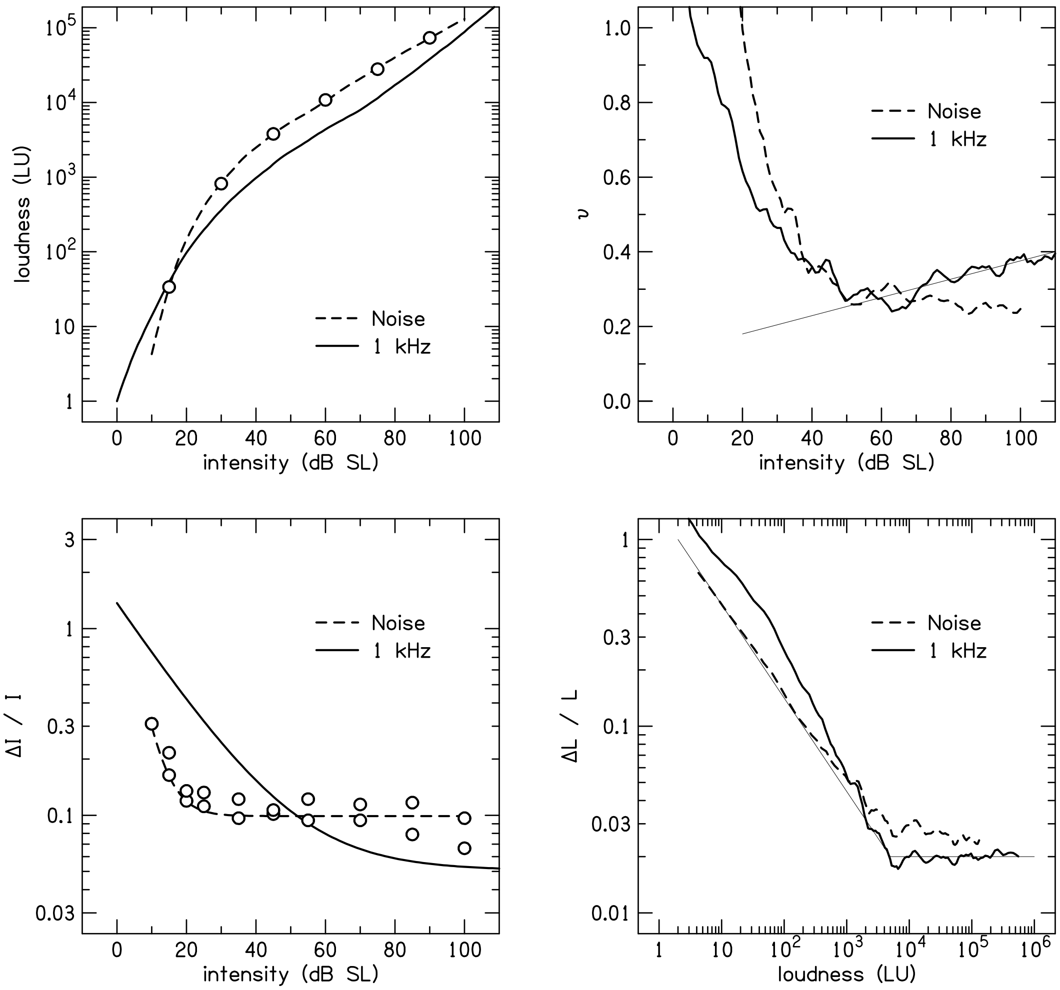
# TONES VERSUS NOISE

- The internal noise is estimated for the cases of tones and WB noise, and they are the same Allen and Neely1997
- The loudness SNR is the same for both tones and noise because the "near-miss to Stevens' Law" cancels the "near-miss to Weber's Law:"

$$\frac{1}{\text{SNR}_{\mathcal{L}}(\mathcal{L})} \equiv \frac{\Delta\mathcal{L}(\mathcal{L})}{\mathcal{L}} = \frac{\Delta I(I)}{\nu(I)I}$$

- Assuming we know $\text{SNR}_{\mathcal{L}}(\mathcal{L})$, given the loudness $\mathcal{L}$, we may calculate the internal noise $\sigma_{\mathcal{L}} = \Delta\mathcal{L}$ since

$$\sigma_{\mathcal{L}}(\mathcal{L}) = \frac{\mathcal{L}}{\text{SNR}_{\mathcal{L}}(\mathcal{L})}.$$



loudness (LU)
intensity (dB SL)
Noise
1 kHz
ν
intensity (dB SL)
Noise
1 kHz
ΔI / I
intensity (dB SL)
Noise
1 kHz
ΔL / L
loudness (LU)
Noise
1 kHz

# LUMINANCE JND

- Luminance JND vs. Riesz's auditory intensity data

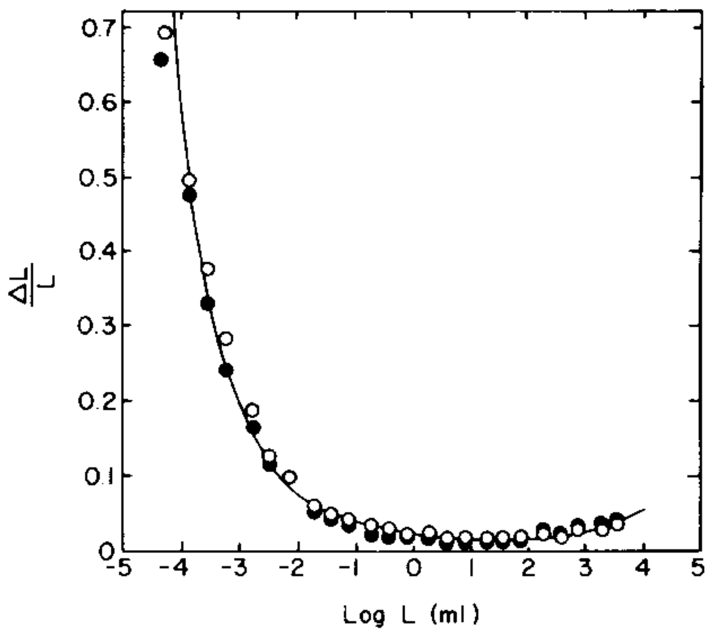
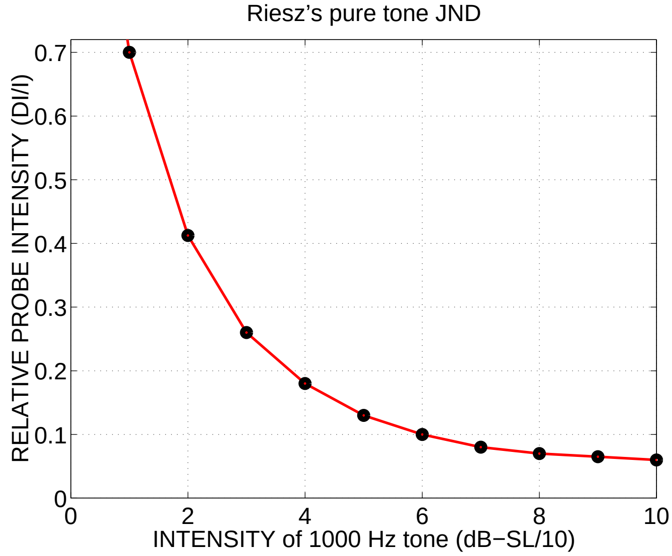



FIG. 1.3. Relation between $\Delta\phi/\phi$ and log luminance as shown by König (open circles) and Brodhun (solid circles). (From König & Brodhun, 1889; after Hecht, 1934, Fig. 27, p. 769.)



# RIESZ'S 1933 proportional jnd hypothesis

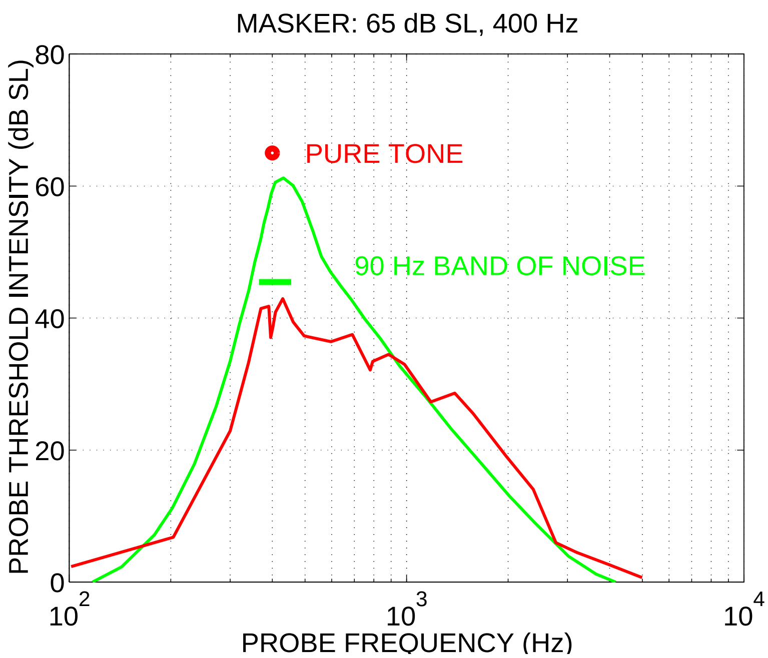
- The number of JNDs between iso-loudness contours $L_1$ and $L_2$ is
$$N_{12} = \int_{L_1}^{L_2} \frac{dI}{\Delta I(I)}.$$

- Riesz observed that for any $L_1, L_2, L_3$
$$\frac{N_{31}}{N_{21}}$$
is independent of frequency.

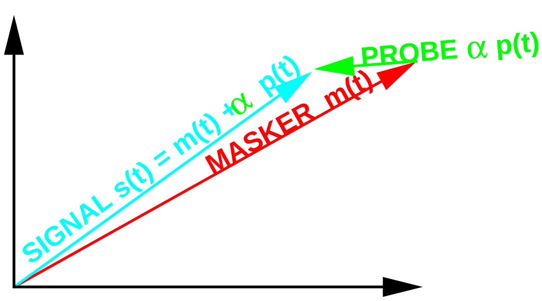
- This is the same as saying that $\Delta\mathcal{L}(\mathcal{L})$ is only a function of loudness $\mathcal{L}$, and is not a function of $I$ or frequency.

# Effect of correlations between a masker and probe

- **1950** Egan and Hake show the second asymmetry of masking




**Masking of equally-loud tone and narrow-band noise**




$$s(t) = m(t) + \alpha p(t)$$

$$\alpha_*^2 + 2\rho_e\alpha_* = \frac{1}{\nu \text{SNR}_{\mathcal{L}}(\mathcal{L})} \approx \frac{1}{10}$$

- When $\rho_e = 1$, $\alpha_* \approx 0.05$
- When $\rho_e = 0$, $\alpha_* \approx 0.32$