The motional impedance as a unique characteristic of the anti-reciprocal system

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Abstract

We investigate a classical notion named 'Motional impedance (Z_{mot})' introduced by Kennelly in 1912 which was explored by many other researchers early in the 20th century. The Z_{mot} refers to components of the input impedance that depends directly on changing output load conditions. Its unique characteristics can be explained by the anti-reciprocal nature of an electro-mechanical system along with a gyrator having the negative real parts (resistance). The shunt eddy current loss on the electrical side of the system is one possible source of this negative real parts of Z_{mot}. Note that the gyrator represents the anti-reciprocity. Unlike in general impedances, Z_{mot} is just a type of transfer function, not a minimum-phase nor a positive-Real (PR) function. By taking a Balance Armature Receiver (BAR) as a specific example of the electro-mechanical systems, this study puts BAR's physics on both the empirical and theoretical basis.

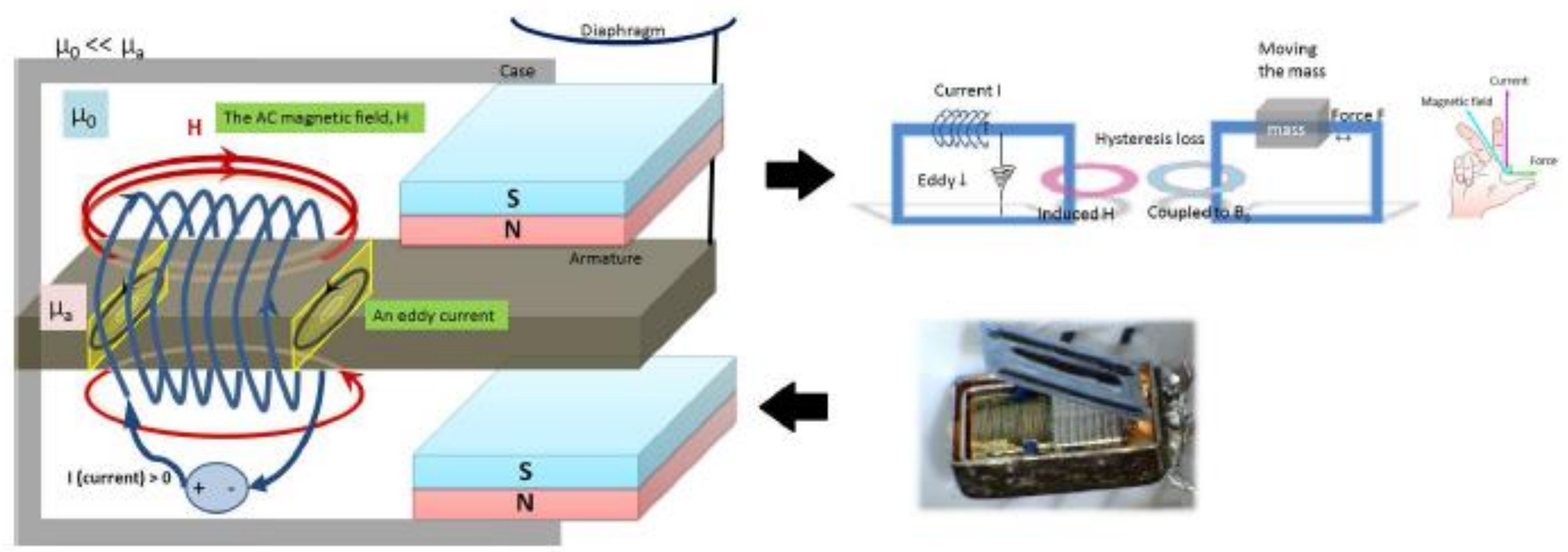


Figure 5: This figure shows the schematic modeling process of the BAR (Knowles ED7045) starting from the real picture of the device after detaching its front case (right bottom) to show its internal components. Core physical components of a BAR include a diaphragm, a coil, an armature, magnets, and case which give rise to current (I), magnetic field (H), eddy current, hysteresis loss, force (F), and etc. The left is the schematic representation of the BAR. The right top figure is the 3D model of BAR which consists of essential elements of the device; an inductor, a semi-inductor, a gyrator and mass. Note directions of the variables (F, H, and I) follow Fleming's right-hand rule.

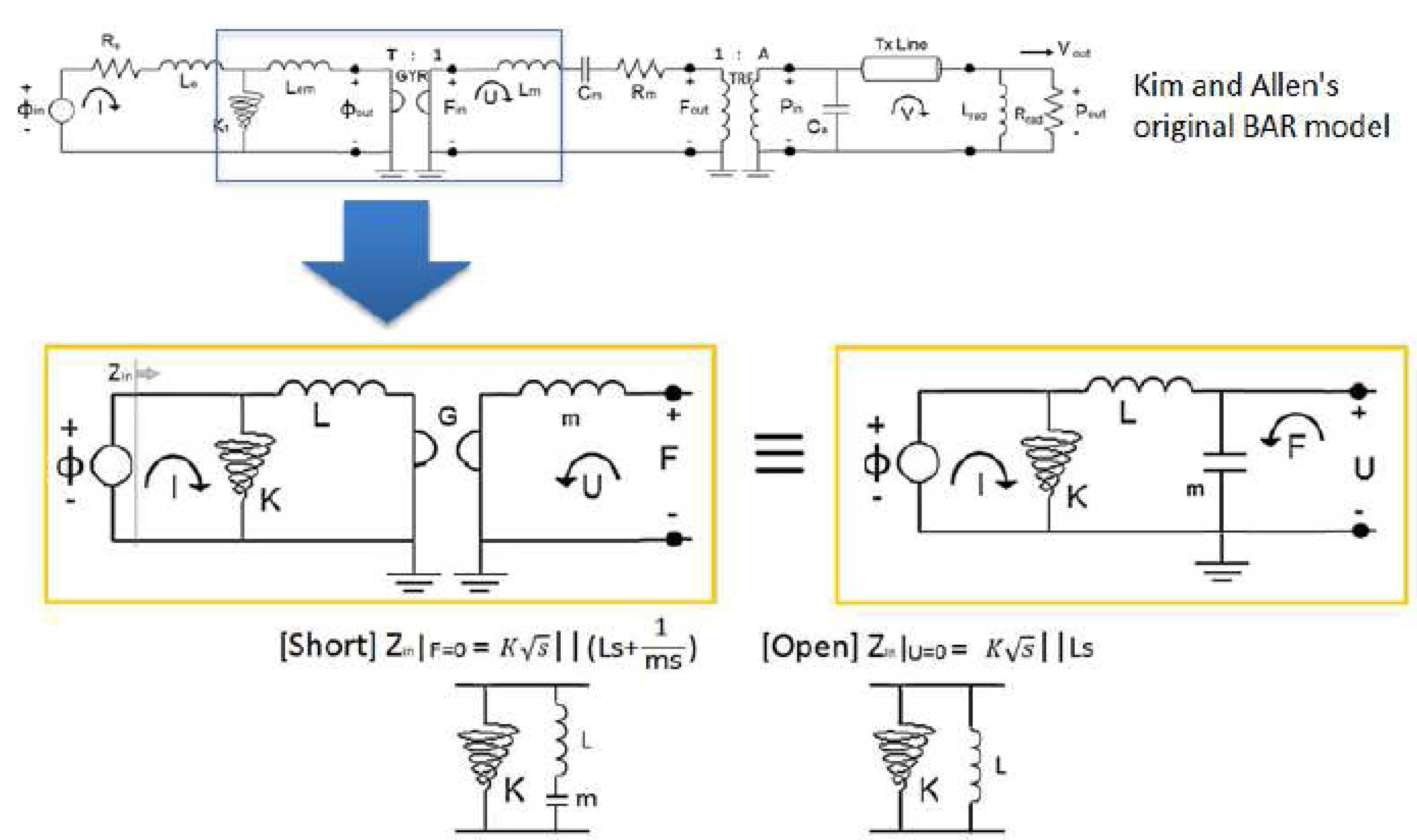


Figure 6: The top figure shows the Balanced Armature Receiver (BAR) circuit model from Kim and Allen [2013]. The electrical and the mechanical circuits are coupled by a gyrator (GYR, realizing an anti-reciprocal network), while a transformer (TRF) is used for the coupling of the mechanical and the acoustical circuits. The $K1$ is a semi-inductor representing 'electro-magnetic diffusion' due to 'skin effect'. And the $TX\,Line$ stands for a transmission line to involve delay in the system, breaking a quasistatic assumption in this electro-acoustic system. This non quasistatic element is the proper way of modeling this system. In this full model, the input and output potentials for each section are specified as voltage (Φ), force (F), and pressure (P). Current (I), particle velocity (U), and volume velocity (V) represent the flow for each of the three physical sections. The lower left circuit: A simple anti-reciprocal network with a semi-inductor presence. The lower right circuit: The dual representation of the left circuit (equivalent) by applying mobility analogy beyond the gyrator. Z_{mot} is reconsidered based on Eq. 11. The frequency dependent real parts (shunt loss) of the semi-inductor in $Z_{in}|_{F=0}$ (short) experience positive phase shift when the open condition impedance ($Z_{in}|_{U=0}$) is subtracted from it.

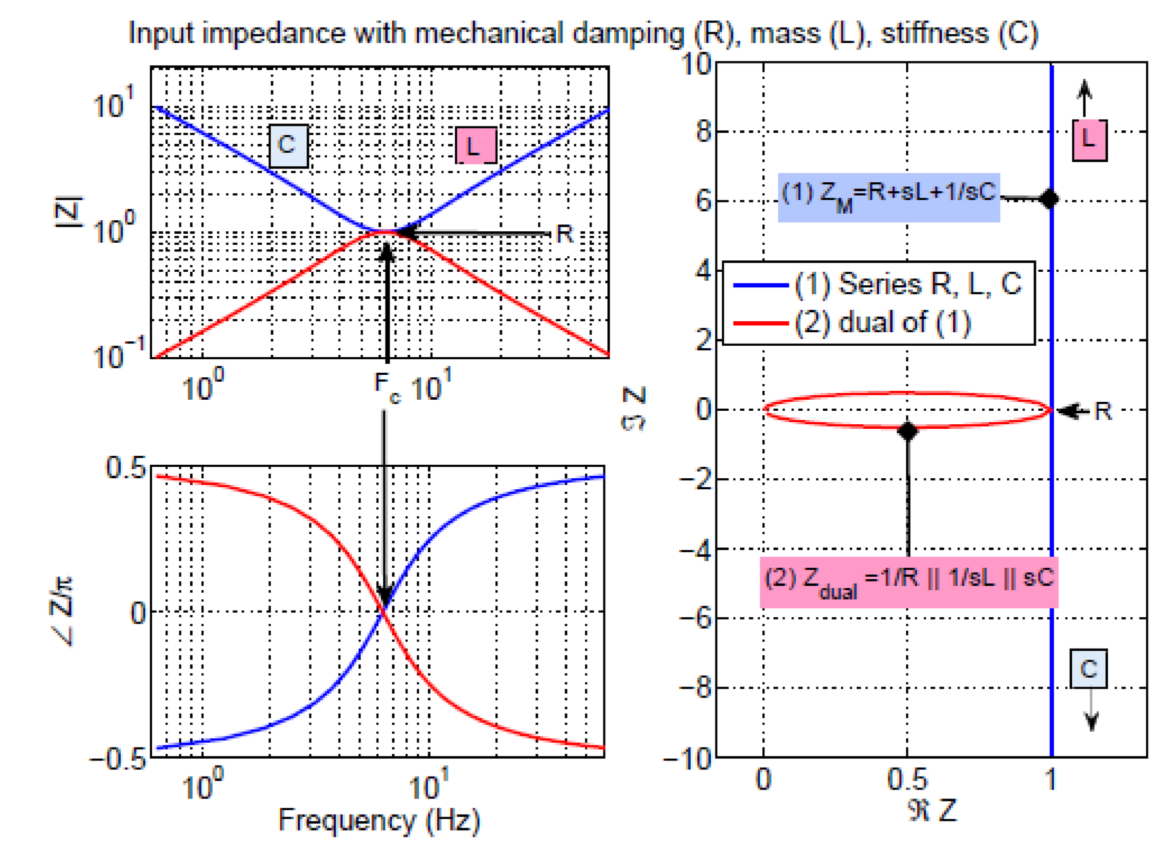
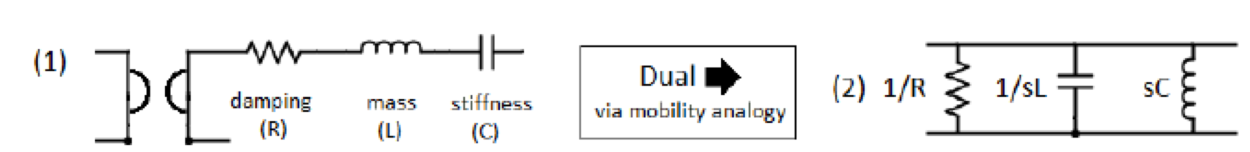


Figure 9: This figure explains the circular shape of Z_{mot} where the motion of the mechanical behavior (i.e., damping (loss), mass, and stiffness) projected to the electrical side defines Z_{mot}. When the mechanical behavior is seen on the electrical input side, due to the gyrator, the series mechanical network becomes a dual network based on the mobility analogy. The blue line shows input impedance based on the series relationship ((1) in Fig. 8 without considering the gyrator) while the red line represents the dual. The upper-left and lower-left plots show magnitude and phase of impedance and the right plot (polar plot) shows real and imaginary parts of the impedance. The red circle on the polar plot justifies the circular shape of Z_{mot}. F_c stands for the transition frequency between C (low-frequency) and L (high-frequency) for both original and dual of magnitude and phase plots. In polar plots, if $\Im Z \to +\infty$, Z is dominated by L, and in case of $\Im Z \to -\infty$, Z depends on C. Note that this figure only discusses the shape of typical Z_{mot}, not its negative real parts. For simplification, values for L, R, and C, are 1 in this simulation.



$$Z_M = R + \frac{1}{sC} + sL\Big|_{R,L,C=1} = 1 + \frac{1}{j\omega} + j\omega = \begin{cases} \infty & \omega \to \infty \\ 1 & \omega \to 1 \\ -\infty & \omega \to -\infty \end{cases},$$

$$Z_{dual} = \frac{1}{R}||sC||\frac{1}{sL}\Big|_{R,L,C=1} = \frac{1}{1 + j\omega + \frac{1}{j\omega}} = \begin{cases} 0 & \omega \to 0 \\ 1 & \omega \to 1 \\ 0 & \omega \to \infty \end{cases}.$$

Conclusions

The Z_{mot} reflects a purely mechanical behavior, a resonance, when it is looked at electrical side, thus it has a circular shape. And we have shown that the nature of anti-reciprocity makes a loop in the impedance of the electro-mechanic system by applying the mobility analogy with a gyrator which represents an anti-reciprocal characteristic.