Rhode's 9 dB: What does it mean?

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Find the relation between Rhode's BM displacement formula and the pressure transfer function:

Definitions Define:

$$H(x, \omega) \equiv \frac{\Delta P_{bm}(x, \omega)}{P_{ec}}$$

and

$$G(x, \omega) \equiv \frac{\xi_{bm}}{\xi_{stapes}}.$$

Rhode (as well as others) have measured G. These data show that the slope of G with respect to frequency is about 9 dB/oct. It is useful to understand why G has this particular slope.

From the WKB method, the spatial pressure distribution of a tone stimulus in the base of the cochlea is given by

$$\begin{aligned} \frac{\Delta P_{bm}(x, \omega)}{P_{bm}(0, \omega)} &= \sqrt{\frac{Z_c(x)}{Z_c(0)}} \; e^{-i\omega \int_{\xi=0}^{x} d\xi/c(\xi)} && (1) \\ &= e^{-ax/2} \; e^{-i\omega\tau(x,\omega)}, && (2) \end{aligned}$$

where the local wave speed is $c(x) = \sqrt{K_p(x)A(x)/\rho}$ and the local characteristic impedance is $Z_c(x) = \sqrt{\rho K_p(x)/A(x)}$. The effective scala area is $A(x)$ and ρ is the scala fluid density.

The definition of the partion impedance Z_p

$$Z_p(x, \omega) \equiv \frac{\Delta P_{bm}}{i\omega\xi_{bm}}$$

and the definition of the cochlear input impedance at place x is

$$z_c(x, \omega) \equiv \frac{P_{sv}}{i\omega\xi_{st}}.$$

Under the constraint that the frequency is less than the CF, $\Delta P_{bm} = P_{sv}(x, \omega)$ since $P_{st} \approx 0$.

Thus it follows that for frequencies below the characteristic frequency at place x:

$$G(x, \omega) = H(x, \omega)\frac{z_c}{Z_p(x, \omega)}$$

In the base the partition impedance $Z_p \approx K_p(x)/i\omega$ is dominated by the BM stiffness, resulting in

$$G = \frac{i\omega z_c}{K_p(x)} e^{-ax/2 - i\omega\tau(x)}.$$

Conclusion: Based on the above formula, if the cochlear input impedance is independent of frequency, then the BM gain function should be 6 dB/Oct. Rhode's 9 dB/Oct observation requires that the cochlear input impedance ($z_c(x, \omega)$) must be 3 dB/oct, which means that

$$|z_c(\omega)| \propto \sqrt{\omega}$$

over the range of Rhode's experimental data (1–10 kHz).

This prediction may be verified with other data from the literature. For example, from Lynch (JASA, 72(1) July 1982, page 114) Fig. 1, I find

$$|z_c| \approx 1.8 \times 10^6 \sqrt{f/600},$$

which varies as the square root of frequency from 0.6-10.0 kHz. Below 600 Hz the stiffness of the round window dominates. Figure 15 gives a slightly different frequency range, from 100 to 1000 Hz, and then constant impedance above 1000 Hz for 3 ears, and above 3 kHz for one other. Figure 13 gives some idea of the scatter and frequency range of the punitive $\sqrt{2}$ frequency range, with some ears going up to 10 kHz.

Thus both Rhode's data and Lynch's data indicate that perhaps the cochlear input impedance varies as the root of frequency over a significant frequency range, starting around 0.6 and going as high as 10 kHz.

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