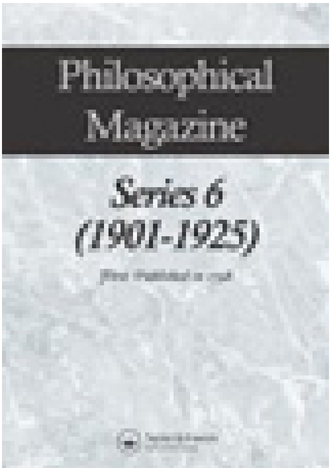

Philosophical Magazine Series 6
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/tphm17

XXX. On loaded lines in telephonic transmission

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Published online: 09 Dec 2010.

To cite this article: George A. Campbell (1903) XXX. On loaded lines in telephonic transmission , Philosophical Magazine Series 6, 5:27, 313-330, DOI: 10.1080/14786440309462928

To link to this article: http://dx.doi.org/10.1080/14786440309462928

Nitrate and Thiocyanate.—These salts were found to have an accelerating effect. With nitrate the reaction was somewhat irregular, and the gas evolved was found to contain nitric oxide. In the case of thiocyanate, hydrogen sulphide was produced in considerable quantity. The results, consequently, are of no value.

Finally, two experiments were made, with acetic acid, and with a mixture of acetic acid and potassium chloride; for according to the theory, presence of chloride should have an accelerating effect. The acetic acid solution was that of maximum conductivity, and the potassium chloride solution was 3·9 molar. The action was very slow and the curves obtained were irregular. In 30 hours 18 c.c. of hydrogen were evolved when acetic acid alone was used, and in the same time 22 c.c. of hydrogen with a mixture of 2 c.c. KCl and 60 c.c. of acetic acid.

These experiments must be regarded as rough preliminary observations, and we wish to return to the subject at a later date.

Summary.

1. The influence of chloride, bromide, nitrate, acetate, chlorate, and thiocyanate, in varying concentration, on an aluminium anode in sulphuric acid was investigated.
2. A theory to explain the results was brought forward and tested experimentally.
3. The essential peculiarities of an aluminium anode were reproduced by means of a platinum anode and a film of aluminium hydroxide.
4. Some measurements were made to determine the influence of chloride and of bromide, on the reaction between aluminium and sulphuric acid.

Chemical Laboratory,
University of Edinburgh.
October 1902.

XXX. *On Loaded Lines in Telephonic Transmission.* *By* George A. Campbell*.

[Plates V. & VI.]

The loaded line discussed in this paper is an electrical circuit of two long parallel conducting wires having self-induction coils inserted at regular intervals. An elementary mathematical treatment adapted to engineering

* Communicated by Prof. Trowbridge.

applications will be given; a second paper will present an engineering study and an account of experimental methods and results.

Vaschy*, Heaviside†, and others have either suggested or unsuccessfully tested the insertion of self-induction coils on actual lines. Heaviside stated in 1893 that there was "no direct evidence of the beneficial action of inductance brought in in this way," and no progress was made till 1899, when the subject was investigated independently by Professor M. I. Pupin‡ and myself. It has been shown that the loaded line affords a practical method of improving the transmission efficiency of long lines employed for telephonic, telegraphic, or other electrical purposes.

An interesting contribution to the general properties of this structure has been made by Mr. Charles Godfrey§ in a paper on wave propagation along a periodically loaded string, and I am indebted to that article for equation (18) which furnishes a complete solution of the propagation.

This study has been made with special reference to telephonic applications, and I have limited the mathematical treatment to the forced harmonic steady state, as that furnishes all the theoretical information which we are in position to use in telephony, and practical applications generally, provided only a sufficient frequency range is considered. The range which it is necessary to consider in telephony might be determined by constructing a network which would transmit uniformly all frequencies between certain limits, and then experimentally determining the interval which is just sufficient to preserve the full character of speech. Practical cable transmission shows that speech remains intelligible even when the superior limit is comparatively low. Cable quality is, however, not desirable, and for unimpaired articulation it appears from the tests which have been made that the limit lies well above two thousand cycles per second. Efficient, clear transmission requires a low and constant attenuation, and constant velocity, throughout the telephonic frequency interval, and constant line impedance of negligible reactance is desirable. With an open wire line of heavy copper wire (resistance R, inductance L, capacity C) this is approximately attained, the attenuation-coefficient,

* *La Lumière Electrique*, January 12, 1889.

† Electromagnetic Theory, i. p. 445 (1893).

‡ Trans. Am. Math. Soc. p. 259, July 1900; Trans. Am. Inst. Elec. Eng. xvii. May 1900.

§ Phil. Mag. xvi. p. 356 (1898).

velocity, and line impedance being respectively—

$$a=\frac{R}{2}\sqrt{\frac{C}{L}},$$

$$v=\frac{1}{\sqrt{LC}},$$

$$k=\sqrt{\frac{L}{C}}.$$

For a loaded line these formulæ apply approximately and the problem taken up in this paper is the determination of the correction factors. This is direct; the approximation of loaded to uniform line shows only indirectly the performance of the loaded line.

I.

A summary must be given of the general transmission formulæ upon uniform lines which will be required. An harmonic electromagnetic steady state is resolvable into a wave propagation with definite velocity and attenuation, but no distortion, throughout each uniform interval of the line, the wave suffering reflexion at points of non-uniformity. This is mathematically an exact and simple analysis of the steady state, but it conforms only approximately to the physical action. It neglects the diffusion or distortion resulting from dissipation which is, in the steady state, not in evidence, except indirectly, as a variation with the frequency of the velocity and attenuation. This variation of the velocity and attenuation furnishes sufficient measure of the distortion at the head of an advancing wave for most practical applications. Except for this head distortion an harmonic steady state is established by pure wave propagation—the line presents a definite line impedance which determines the initial current at the impressed force; the electromagnetic wave originating at the impressed force travels with a definite attenuation and a definite velocity along the line and divides upon reaching a point of non-uniformity into a reflected wave and a transmitted wave. Repeated reflexions establish the steady state.

The equation of a simple current wave upon a uniform line is

$$i=\frac{Ee^{\iota pt}}{k}e^{-\gamma x}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where the line impedance k, the propagation coefficient γ, the attenuation coefficient a, and the velocity of propagation v, are given by

$$k=\sqrt{J_1J_2}, \qquad (2)$$

$$\gamma=a+\frac{p\iota}{v}=\sqrt{\frac{J_1}{J_2}}, \qquad (3)$$

where, if R, L, C, S are the effective loop-line resistance, inductance, capacity, and leakage conductance at a frequency $p/2\pi$, the series and shunt impedances are

$$J_1=R+Lp\iota, \qquad (4)$$

$$J_2=(S+Cp\iota)^{-1}; \qquad (5)$$

consequently

$$a=\sqrt{\tfrac{1}{2}\sqrt{(R^2+L^2p^2)(S^2+C^2p^2)}+\tfrac{1}{2}(RS-LCp^2)} \qquad (6)$$

$$=\frac{R}{2}\sqrt{\frac{C}{L}}\left(1+\frac{SL}{CR}\right) \text{ for large L and small S} \qquad (7)$$

$$v=\frac{2a}{CR+LS}. \qquad (8)$$

For transmission from one construction of impedance k_1 to a second of impedance k_2 the current reflexion and transmission coefficients are respectively

$$\alpha=\frac{k_1-k_2}{k_1+k_2}, \qquad (9)$$

$$1+\alpha=\frac{2k_1}{k_1+k_2}. \qquad (10)$$

If upon a line of length l with terminal sets of impedances J_s, J_r, at the sending and receiving ends, there is an impressed force $Ee^{\iota pt}$, the current at distance x from the sending set is

$$i=\frac{Ee^{\iota pt}}{k+J_s}\cdot\frac{e^{-\gamma x}+\alpha_r e^{\gamma x-2\gamma l}}{1-\alpha_r\alpha_s e^{-2\gamma l}}, \qquad (11)$$

where the reflexion coefficients are

$$\alpha_r=\frac{k-J_r}{k+J_r}, \qquad \alpha_s=\frac{k-J_s}{k+J_s}.$$

If the line attenuation is large and the sending and receiving sets are similar, the receiving current is approximately

$$i_r=\frac{Ee^{\iota pt}}{2J_s}\cdot\frac{4kJ_s}{(k+J_s)^2}e^{-\gamma l}, \qquad (12)$$

If terminal transformers are added and the transformer impedances are J_1 for the set side winding, J_2 for the line side winding, and J_{12} between the two windings, then (12) becomes

$$i_r=\frac{Ee^{\iota pt}}{2J_s}\left(\frac{4kJ_s}{(k+J_s)^2}\right)\left(\frac{J_{12}(k+J_s)}{(k+J_2)(J_1+J_s)-J_{12}^2}\right)^2 e^{-\gamma l}, \quad (13)$$

where the four factors are, respectively, the value of the current for a circuit consisting of the two sets alone, the effect of terminal reflexion, the effect of transformers, and the effect of transmission over the line. For transformers of high inductance, negligible resistance, and negligible magnetic leakage (13) becomes

$$i_r=\frac{Ee^{\iota pt}}{2J_s}\cdot\frac{4k\frac{J_1}{J_2}J_s}{\left(k\frac{J_1}{J_2}+J_s\right)^2}\cdot e^{-\gamma l}, \quad . \quad . \quad . \quad (13a)$$

and the transformers are equivalent to a change in the line impedance from k to $k\,J_1/J_2$.

The impedance of a circuit consisting of length l of uniform line $(k\gamma)$ closed through an impedance J_0 at the further end is by formula (11)

$$J=k\,.\frac{1-\alpha e^{-2\gamma l}}{1+\alpha e^{-2\gamma l}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

where $\alpha=(k-J_0)/(k+J_0)$ is the reflexion coefficient from line to terminal impedance.

Diagram I. (Pl. V.) shows the value of the reflexion factor in equation (12). The factor involves the two impedances symmetrically, is a function of their ratio only, and becomes unity if they are equal. Let the absolute value and angle of the impedance ratio be r, θ, and of the reflexion factor be e^{-b}, ϕ, then

$$\frac{k}{J_s}=r\text{ cis }\theta,$$

$$\frac{4kJ_s}{(k+J_s)^2}=e^{-b}\text{ cis }(-\phi),$$

$$b=\log\left\{\frac{1}{4}\left(r+\frac{1}{r}\right)+\frac{1}{2}\cos\theta\right\}, \quad . \quad . \quad . \quad (15)$$

$$\phi=\tan^{-1}\frac{\left(r-\frac{1}{r}\right)\sin\theta}{2+\left(r+\frac{1}{r}\right)\cos\theta}, \quad . \quad . \quad . \quad . \quad . \quad (16)$$

and (12) may be written

$$i_r = \frac{Ee^{ipt}}{2J_s} e^{-(b+al)} \operatorname{cis} - \left(\phi + \frac{p}{v} l\right), \quad . \quad . \quad . \quad (17)$$

where the second and third factors are, respectively, the effective attenuation and the phase lag due to the line. As b can be negative, reflexion may augment the receiving current, but in general the effect is a loss which may be comparable with the attenuation loss. Thus for $k/J_s=10$, $b=1{\cdot}11$, and the range of "easy commercial" telephonic transmission, which requires an effective attenuation coefficient of 3·2 with present instruments, would be reduced 1·11/3·2, or 35 per cent.

By Diagram I. (Pl. V.) the transformer efficiency in equation (13a) is a maximum and completely offsets the reflexion loss when $J_2/J_1 = |k/J_s| = r$. It follows that, by introducing transformers into a line at every point of non-uniformity due to apparatus or a change in line construction, reflexion losses may be entirely eliminated and the effective attenuation made as small as or smaller than the real line attenuation.

The effect of loading a line uniformly is shown by Diagram II. (Pl. V.), which gives the attenuation coefficient for lines having $R=2$, $C=1$, and different values of L. Also the velocity, for with these values of the constants the velocity and attenuation coefficient are numerically equal. With no inductance the attenuation curve is a parabola. Any increase in inductance reduces the attenuation and makes it more nearly uniform, and by a sufficient increase in the inductance the attenuation can be reduced to any desired value, but for this it is essential that the leakage be null.

The effect of leakage is added in Diagram III. (Pl. V.), which is plotted for $K=1$, $L=1$, $C=1+\mu$, $S=1-\mu$, but gives the attenuation coefficient and velocity curves for any uniform line by a change in scales only.

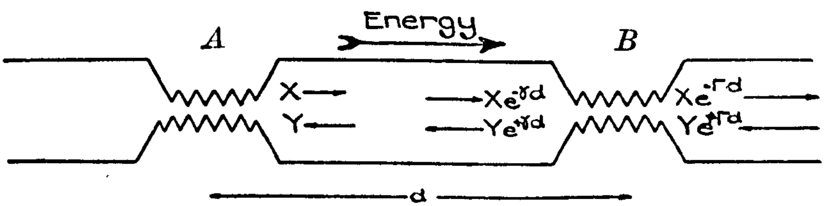
II.

An infinite loaded line will be considered first in order to treat propagation and terminal conditions separately. We might follow in detail the repeated division of the wave by reflexion at loads and the interference of the resulting wavelets which mutually annul each other, with the exception of a group suffering reflexion and transmission in a certain ratio which gives rise to a wave of small attenuation and negligible distortion, although the individual wavelets may be enormously attenuated and distorted by the length of their course back and forth upon the line. The following theorem will, however, lead directly to the solution and avoid the infinite summation.

Upon an infinite line of periodic recurrent structure a steady forced harmonic disturbance falls off exponentially from one periodic interval to the next. The theorem is proven by the consideration that as the line is infinite, there are identical circuits beyond points separated by a periodic interval, and the relative effect upon the disturbance of advancing an interval must be the same for all portions of the line.

Consider a uniform line (k, γ) with loading coils A, B, &c.. at the interval d of impedance Hd, or H per unit length of

Fig. 1.



line, and we will designate by (K, Γ) the impedance of the loaded line at the middle of a load and the propagation coefficient of the loaded line. Then if X, Y are the direct and reflected current waves at the further side of the loading coil (A), at the next coil (B) the direct and reflected waves are $Xe^{-\gamma d}$, $Ye^{+\gamma d}$ on the sending side, and $Xe^{-\Gamma d}$, $Ye^{+\Gamma d}$ on the further side. At a coil the reflexion coefficient is

$$\frac{k-(\mathrm{H}d+k)}{k+\mathrm{H}d+k}=-\frac{\mathrm{H}d}{2k+\mathrm{H}d}, \quad \text{and} \quad \frac{2k}{2k+\mathrm{H}d}$$

is the transmission coefficient; the equations of condition at B are therefore

$$Ye^{\gamma d}=-\frac{\mathrm{H}d}{2k+\mathrm{H}d}Xe^{-\gamma d}+\frac{2k}{2k+\mathrm{H}d}Ye^{-\Gamma d},$$

$$Xe^{-\Gamma d}=-\frac{\mathrm{H}d}{2k+\mathrm{H}d}Ye^{-\Gamma d}+\frac{2k}{2k+\mathrm{H}d}Xe^{-\gamma d}.$$

Eliminating X and Y, we have

$$\cosh(\Gamma d)=\cosh\gamma d+\frac{\mathrm{H}d}{2k}\sinh\gamma d, \quad . \quad . \quad . \quad (18)$$

which completely determines the propagation coefficient of the loaded line, including the attenuation coefficient and velocity of propagation*.

To determine the line impedance (K), observe that the impedance is periodic and apply formula (14) to the circuit, beginning at the middle of one coil and extending to the middle of the next coil:—

$$K=\frac{Hd}{2}+k\cdot\frac{1-\dfrac{k-\left(\dfrac{Hd}{2}+K\right)}{k+\dfrac{Hd}{2}+K}e^{-2\gamma d}}{1+\dfrac{k-\left(\dfrac{Hd}{2}+K\right)}{k+\dfrac{Hd}{2}+K}e^{-2\gamma d}},$$

or

$$K=\sqrt{k^2+\frac{H^2d^2}{4}+Hdk\coth\gamma d}$$

$$=k\sqrt{\left(1+\frac{Hd}{2k}\tanh\frac{\gamma d}{2}\right)\left(1+\frac{Hd}{2k}\coth\frac{\gamma d}{2}\right)}, \quad . \ (19)$$

which completely determines the loaded line impedance at the middle of a load. The impedance at any other point might be found by the same method.

Substituting the values of Γ and K given by (18) and (19) in equation (11) we have the formula for the current at any load, or substituting in (12), (13), (13 a), or (17) we have the value of the receiving current, but the substitution can best be made after numerical values are obtained.

The method which has been employed in deducing (18)

* For a simpler proof of equation (18), short circuit the loaded line at the middle of the coil B and consider the ratio of the current at A to the current at B. As section A–B may then be considered, either, (1) as a uniform line of constants k, γ, and length d, terminating in an impedance $Hd/2$, or, (2) as a portion of a uniform line of constants K, Γ, and length d, terminating in a short circuit, two values for the ratio of the current at A to the current at B may be obtained, and the two equated give a relation between the two sets of line constants. The two expressions are found on making the proper substitutions in (11) to be identically the right and left hand members of equation (18).

In this proof it is to be noticed that the loaded line is short-circuited at the middle of a load in order that it shall act like a short-circuited uniform line with the constants K, Γ; if the line is not short-circuited at the middle of a load (or at the middle of a line section) a wave traverses on reflexion a line which is not throughout of uniform periodic structure. For the suggestion leading to this proof I am indebted to Dr. A. E. Kennelly. (July 1902.)

and (19) is quite general and has been applied to a variety of cases of interest, such as artificial lines with mutual induction between loads, S. P. Thompson's compensated cable, and periodic lines of two or more different intervals, but these constructions lie outside the scope of this paper.

III.

To give a precise and comprehensive idea of the performance of the loaded line, formulæ (18) and (19) must be reduced to diagrams giving the attenuation coefficient (A), velocity (V), and line impedance (K). The diagrams can best be constructed for the correction factors α, η, κ, defined by the equations:—

$$A=\alpha\frac{R+R'}{2}\sqrt{\frac{C}{L+L'}}, \quad . \quad . \quad . \quad . \quad (20)$$

$$V=\eta\frac{1}{\sqrt{(L+L')C}}, \quad . \quad . \quad . \quad . \quad . \quad (21)$$

$$K=\kappa\sqrt{\frac{L+L'}{C}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

where R, L, C, R′, L′, are the line resistance, inductance, and capacity, and the load resistance and inductance, all per unit of length. We can reduce the number of independent variables from 7 to 4 by introducing in place of R, L, C, R′, L′, d, p, the new variables ω, δ, ρ, λ defined by

$$\omega=\frac{1}{2}pd\sqrt{(L+L')C}, \quad . \quad . \quad . \quad . \quad . \quad (23)$$

$$\delta=\frac{R+R'}{2}d\sqrt{\frac{C}{L+L'}}, \quad . \quad . \quad . \quad . \quad (24)$$

$$\rho=\frac{R'}{R}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

$$\lambda=\frac{L'}{L}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

For a discussion of practical applications we may assume that there is no leakage, no line inductance, and that δ is small. While leakage, on a heavily loaded line, seriously increases the attenuation, the effect of small leakage will be given with sufficient accuracy by the correction (7) for uniform lines. The practical effect of distributing a small portion of the total inductance along the line must be to make the line act a trifle more like a uniform line. Its general effect as shown by Godfrey's results will be discussed later.

In cable circuits the inductance is quite negligible, and a moderately loaded open wire circuit would have several times more load than line inductance. δ is the attenuation coefficient for a periodic interval, and will be small if the line is to be efficient.

Substituting (20) to (26) in (18) and (19), and expanding

$$\alpha\delta+\frac{2\omega}{\eta}\iota=\cosh^{-1}\sum_{0}^{\infty}\frac{(2n+1)(1+n\rho)-2\omega^2}{(2n+1)\,!}\left(\frac{4\,\delta\omega\iota}{1+\rho}\right)^n, \quad (27)$$

$$\sqrt{\frac{\sum_{n=0}^{n=\infty}\frac{[2(2n+1)(2\omega^2+2n\omega^2\rho-n)-4\omega^4-n\rho(4n^2-1)(\rho n-\rho+2)]}{(2n+1)\,!}\left(\frac{4\,\delta\omega\iota}{1+\rho}\right)^n}{4\omega^2\sum_{n=0}^{n=\infty}\frac{1}{(2n+1)\,!}\left(\frac{4\,\delta\omega\iota}{1+\rho}\right)^n}} \quad . \; . \; . \; . \; . \; . \; . \quad (28)$$

for all values of the variables. As the series converge rapidly these formulæ may be used for general computation by noticing that

$$\cosh^{-1}(x+y\iota)=\cosh^{-1}\frac{\sqrt{(x+1)^2+y^2}+\sqrt{(x-1)^2+y^2}}{2}$$
$$+\iota\cos^{-1}\frac{2x}{\sqrt{(x+1)^2+y^2}+\sqrt{(x-1)^2+y^2}}. \quad (29)$$

In the limiting case $\delta\doteq 0$, the formulæ become :—

$$\omega\lessgtr 1\left\{\begin{aligned}\alpha&=\frac{3+3\rho-2\omega^2}{3(1+\rho)\sqrt{1-\omega^2}}, \quad . \; . \; . \; . \; . \quad (30)\\ \eta&=\frac{\omega}{\sin^{-1}\omega}, \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (31)\\ \kappa&=\sqrt{1-\omega^2}, \quad . \; . \; . \; . \; . \; . \; . \quad (32)\end{aligned}\right.$$

$$\omega > 1\left\{\begin{aligned}Ad&=2\cosh^{-1}\omega, \quad . \; . \; . \; . \; . \; . \; . \; . \quad (33)\\ \eta&=\frac{2\omega}{\pi}, \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (34)\\ \kappa&=\iota\sqrt{\omega^2-1}. \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (35)\end{aligned}\right.$$

These simple formulæ, which furnish the practical information we require, are reduced to curves in diagrams IV., V., VI. (Pl. V.), which also give a few curves for $\delta=0{\cdot}1$, to show the close approximation with which the (α, η, κ) curves for $\delta=0$ will apply to any practical loaded line, for δ would, perhaps, never approach 0·1 in actual lines, but have a value nearer 0·01.

Inspection of the diagram shows that at $\omega=1$ the character of the propagation changes; that at this point reflexion, *per se*, introduces attenuation. With perfect conductivity ($\delta=0$) there is no attenuation below $\omega=1$, but there is attenuation above this value, and this is due, necessarily, to reflexion. Below $\omega=1$ the attenuation is proportional to the resistance so long as the resistance remains small, while above $\omega=1$ the attenuation is almost independent of the resistances (or may even decrease with an increase in resistance), but increases rapidly with the value of ω. Consequent on this change in the character of the propagation there is an accompanying change in the line impedance from pure resistance below $\omega=1$ to pure reactance for higher values of ω. The velocity curve also changes its direction and character abruptly at $\omega=1$; there are two coils per actual wave-length for $\omega=1$, *i. e.*, the disturbance is in opposite phases at consecutive coils. The critical value $\omega=1$ furnishes the first essential condition for an efficient loaded line, viz.:—

$$\tfrac{1}{2}pd\sqrt{LC}<1, \quad . \quad . \quad . \quad . \quad . \quad . \qquad (36)$$

i. e., there must be more than two coils per actual wave-length, or approximately, π coils per wave-length with the load uniformly distributed. This has long been known for loaded strings*.

I have made use of these results by employing artificial loaded lines for cutting out harmonics in generator currents. The harmonics may all be cut down as far as desired by the use of a sufficient number of sections, while the attenuation of the fundamental can be reduced at pleasure by decreasing the resistance. The line does not require tuning, and with a small value of ρ the action would be quite independent of the frequency throughout a considerable range. The curves for $\rho=\alpha$ correspond to the case of an artificial line. Combining condensers and inductances, we may make a system which will not only cut out higher frequencies, but also all frequencies below a certain limit, as suggested at the beginning of this article. This system will be an inversion of a model of J. H. Vincent's†.

The velocity and impedance are approximately independent of the resistances. The attenuation below $\omega=1$ is not only approximately proportional to the total resistance, but the curves also show that the attenuation is reduced by trans-

* Routh, Advanced Rigid Dynamics, p. 260, § 411; Rayleigh, Theory of Sound, i., § 148, pp. 233, 234.

† Phil. Mag. xlvi. p. 557 (1898).

ferring resistance from the coil to the line, and that the loaded line attenuation may be less than that for a uniform line of the same total resistance, inductance, and capacity. Inspection of the formulæ shows that this applies to any line of high inductance for which

$$\rho<\tfrac{1}{3},$$

$$\omega<\frac{\sqrt{3(1-3\rho)(1+\rho)}}{2},$$

and that the maximum reduction is for

$$\rho=0,$$

$$\omega=\frac{1}{\sqrt{2}},$$

$$\alpha=\tfrac{2}{3}\sqrt{2},$$

when it amounts to 5·7 per cent.

IV.

To determine the loading for maximum efficiency, the total weight of copper in cable and load being given, we may make use of the following formulæ for the weights of loading coils and cable conductors per unit length of line, the formulæ applying to coils of similar proportions and cables of similar cross-section:—

$$d f \mathrm{W}=w'\left(\frac{\mathrm{L}'}{\mathrm{R}'}\right)^{\frac{3}{2}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (37)$$

$$(1-f)\mathrm{W}=\frac{w}{\mathrm{RC}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (38)$$

where W is the total weight of copper per unit length; w, w' are constants made a minimum by suitable proportions of coils and cables, respectively; d is the spacing of the coils; and f is the proportion of the total copper in the coils. We will assume that the line is to be of high efficiency so that formulæ (30), (31), (32) apply, and suitable terminal apparatus or terminal transformers will be employed to practically annul terminal reflexion. The problem reduces to securing minimum attenuation at the frequency of transmission. Taking the product of equations indicated by (20), $(23)^{\frac14}$, (30), $(37)^{\frac14}$, $(38)^{\frac12}$, and substituting $\mathrm{R}'=\rho\mathrm{R}$, we obtain for the attenuation

$$\mathrm{A}=\frac{3+3\rho-2\omega^2}{6\cdot 2^{\frac14}\omega^{\frac14}\rho^{\frac38}f^{\frac14}(1-f)^{\frac12}(1-\omega^2)^{\frac12}}\cdot\frac{w'^{\frac14}w^{\frac12}p^{\frac14}}{\mathrm{W}^{\frac34}} \quad . \quad . \quad (39)$$

which takes its minimum value

$$A = 1{\cdot}639\,(w'^2 w^5 p^2 W^{-7})^{\frac{1}{8}} \quad . \quad . \quad . \quad . \quad . \qquad (40)$$

for

$$f = \tfrac{2}{7} = {\cdot}2857,$$
$$\rho = \tfrac{3}{7} = {\cdot}4286,$$
$$\omega = \sqrt{\tfrac{3}{7}} = {\cdot}6547,$$

and therefore :—

For a given total weight of copper in cable and load the attenuation of a given frequency is made a minimum by placing 2/7 of the copper in coils having a resistance equal to 3/7 of the line resistance and spacing them $\pi\sqrt{7/3} = 4{\cdot}80$ *per uniform line wave-length,* i.e., $\pi/\sin^{-1}\sqrt{3/7} = 4{\cdot}401$ *per actual wave-length on the loaded line.* These proportions make

$$\alpha = 1{\cdot}058,$$
$$\eta = {\cdot}9171,$$
$$d = 2{\cdot}028\left(\frac{w'^2 W}{w^3 p^6}\right)^{\frac{1}{8}}, \quad . \quad . \quad . \quad . \quad . \quad . \qquad (41)$$
$$\frac{L'}{R} = 0{\cdot}695\left(\frac{W^3}{p^2 w w'^2}\right)^{\frac{1}{4}}, \quad . \quad . \quad . \quad . \quad . \qquad (42)$$
$$K = 0{\cdot}488\left(\frac{w^3}{w^2 \omega^8 p^2 W}\right)^{\frac{1}{8}}, \quad . \quad . \quad . \quad . \quad . \qquad (43)$$
$$Wl = 1{\cdot}759\left(\frac{w^5 w'^2 p^2 l^{15}}{(Al)^8}\right)^{\frac{1}{7}}, \quad . \quad . \quad . \qquad (44)$$

The last formula, giving the total weight of copper in a line of length l and attenuation e^{-Al}, shows the relative importance of the different factors. The weight increases as the 2·14 power of the range. Open wire circuits also increase in weight somewhat faster than the square of the length, but cable weights vary as the cube of the range. Loading, therefore, presents the greatest possibilities upon long cable circuits. The weight is evidently comparatively independent of the frequency. The attenuation coefficient comes in approximately inversely as the first power. Of the two specific weights w, that for the cable is far more important than that for the coils w. Thus the total weight Wl will be doubled by changing w to 2·64 w, or w' to 11·3 w'. This shows the comparative importance of the coil weight.

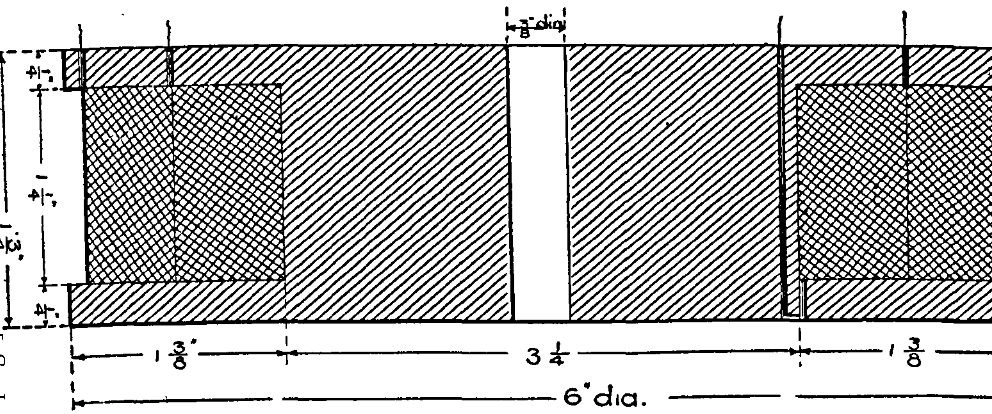
In practical engineering, costs must be substituted for theoretical copper weights, (37) (38) being replaced by the actual relation between gross costs and effective time constants at telephonic frequencies, and a variety of practical requirements are involved which will materially modify the above results. In special cases it may be necessary to connect a loaded line of high impedance directly to an unloaded line or terminal apparatus designed for present lines. Diagram I. (Pl. V.) will give the reflexion loss and reduction in range. From the formulæ already deduced, the proportions may be determined for maximum efficiency with a given total weight of copper and given terminal conditions.

Experimental Work.

In January 1899, I was assigned the problem of investigating the possibilities of improving the efficiency of cables for telephonic service. After considering some other methods I concluded that the loaded line presented the greatest promise, and, as I felt that more progress would be made by experimental tests than by mathematical work, I immediately planned to have made an artificial line with 100 loading coils on a twenty-mile cable circuit. Before this line was completed, becoming more confident of the success of loading, an experiment on an actual cable was planned, and for these tests three reels of 100-pair telephone cable, commonly known as "Conference Standard" cable, were brought to the laboratory. Each reel contained about 600 feet of cable, so that the entire circuit, when connected back and forth, formed a metallic cable pair thirty-five miles in length, with a resistance of about 87 ohms per mile and a mutual capacity of about ·057 microfarad per mile. For a laboratory test a circuit thirty-five miles in length could not be stretched out to its full length, and we actually used the cable on the reels with the circuit looped back and forth, fifty times through the first cable, then into the second and third cables. The equivalence of a looped circuit of this kind to a straight-away circuit had been shown, provided the circuit was balanced as all telephone circuits must be balanced in order to eliminate cross talk and noise. This point was also carefully tested during the investigation.

For the loading of this cable 300 coils were manufactured. A cross-section of this loading coil, known as the T-14 coil, is shown in fig. 2. On a wooden spool a primary of 578 turns of No. 20 single cotton-covered wire was wound, and a secondary of 465 turns of No. 20 single cotton-covered wire. The turns were so chosen as to give the primary and secondary the same inductance, and they also had approximately the same resistance. The cable circuit, as has been explained, consisted of 300 lengths connected in series. Between each length and the succeeding length a T-14 coil was inserted, the primary in one wire of the pair and the secondary in the other wire of the pair, the connexions being

Fig. 2.



so made as to put the coil into the cable inductively. Each coil added about ·11 henry and 12 ohms to the circuit. To ensure the reliability of the test it was necessary to so place the loading coils that the mutual induction should be negligible. Accordingly they were spread over all the space available, and tests showed that any effect of mutual induction between coils was quite negligible.

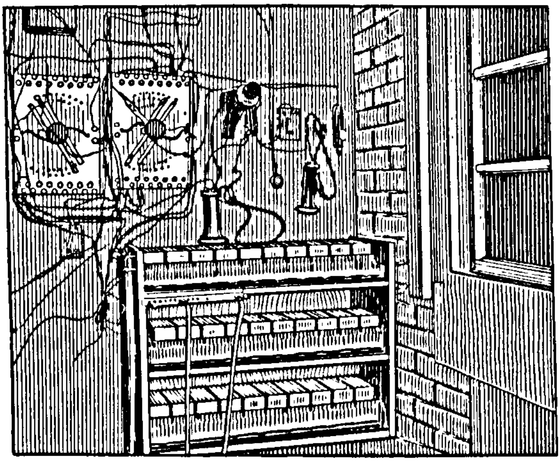
The experimental line is shown in the photograph on Plate VI. The three reels of cable are all visible; one is in plain sight, another is at the extreme left, and the third can be distinguished at some distance to the right. The reel at the extreme right had no connexion with this experiment. The cables were brought out to pot-heads, and each wire terminated in a screw-cup. In this way any combination of connexions could be made. Two of the pot-heads of the middle reel of cable are in plain sight in the photograph. The coils were placed upon shelves—being placed horizontally and on edge on alternate shelves. With this arrangement there was practically no mutual induction except between one coil and the adjacent coil, or two on either side. The four coil terminals were carried underneath the shelf to a distributing board between the pot-heads, and this enabled us to connect in the coils in any desired manner.

An artificial section was also made and loaded with 100 T-14 coils, and this is seen in the right-hand half of the photograph. The coils upon the shelves are plainly visible.

The artificial cable, which consisted of mica condensers and German-silver resistances, does not show conspicuously.

One of the transmitting stations is shown at the left of the photograph. The receiving station is shown in the second photograph. At this station there were switches and an artificial cable, in addition to the telephone set. The artificial cable, known as the "cable standard," consisted of mica condensers and German-silver resistances. The photograph (fig. 3)

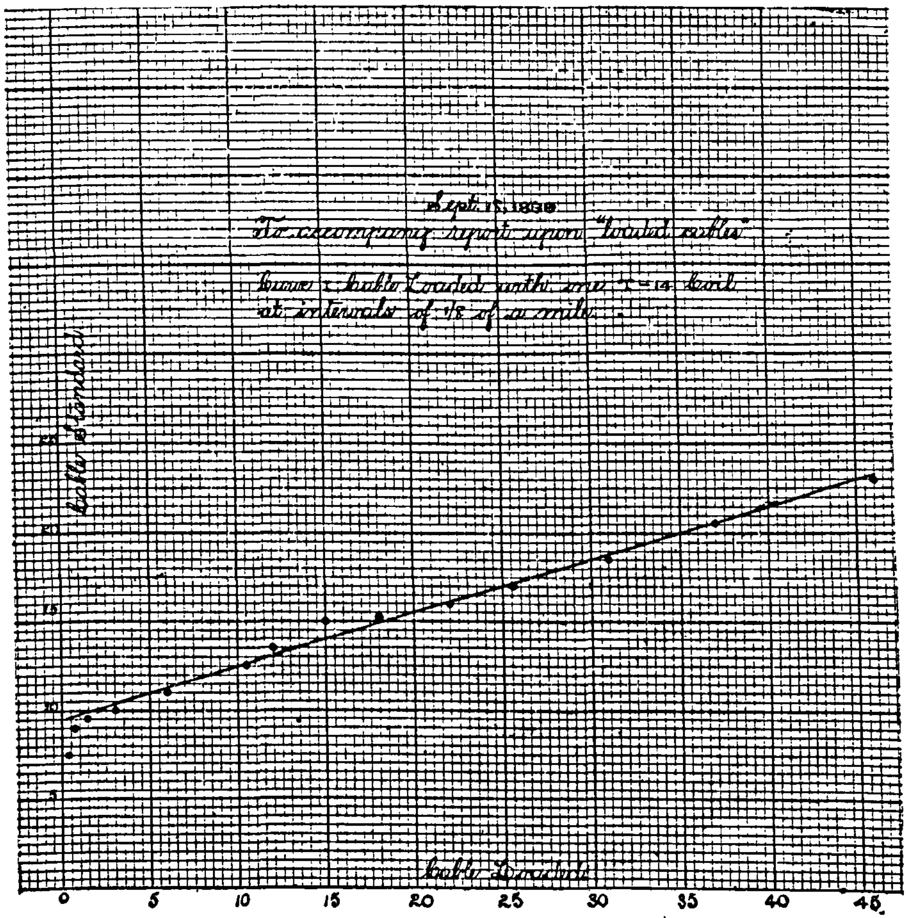
Fig. 3.



shows the thirty boxes, each containing ten condensers, and, somewhat indistinctly, the German-silver wires. It also shows a jack strip and two cords and plugs, by means of which any length of cable up to thirty miles, by steps of one mile, could be put in circuit. For lengths greater than thirty miles another artificial cable was added to this.

The manner of making the test was as follows:—The cable was connected back and forth without the loading coils and the artificial section was also connected up without the loading coils, and then the whole of this circuit was compared with the cable standard. It was found that the entire cable and artificial section were equivalent to forty-six miles of the cable standard, and shorter lengths were equivalent to corresponding lengths of the cable standard. Next, the coils were introduced into the cable circuit and into the artificial section, making a cable circuit forty-six miles in length with four hundred equally spaced loading coils. The transmission was greatly improved and found equivalent to the transmission over twenty-three miles of cable standard, and in addition the articulation was clearer and sharper. In addition to this the side tone at the transmitting end was reduced so as to be hardly noticeable, so that the result of loading was to increase the receiving-end current while decreasing the sending-end current. Tests were made upon different lengths of the loaded cable and the results of one set of tests are shown in the accompanying diagram (fig. 4). The results were

Fig. 4.



represented fairly well by a straight line which corresponds to the approximate formula (17). This line shows that the initial loss was equivalent to nine miles of standard cable, but on account of the greatly reduced attenuation, the loaded line is better than the unloaded line for all distances greater than fifteen miles. If the line representing the experimental results is to be parallel to itself, lowered nine miles we have the curve for a line with terminal transformers, and for this case the loading is shown by the experiment to nearly treble the distance over which transmission of a given volume is obtained, and it actually accomplishes more than that, for the quality is improved. The tests upon which this diagram is based were made with telephonic transmission and ear estimates at the other end of the line and are, of course, more or less qualitative, since the difference of quality prevents a sharp estimate. At the transmitting end a person talks in a steady, monotonous manner, and it is arranged so that the person listening at the other end can switch instantly from the loaded line to the artificial cable standard, and *vice versâ*, and alter the length of the cable standard to secure an equal volume of transmission over the two. The comparison gave the equivalents of the two circuits for commercial service, which answered the questions which I had before me.

For a complete scientific investigation it is desirable to use a sinusoidal current and make qualitative measurements, and we have made some tests in this way. A great deal of experimental work has been done, both upon this cable with the T-14 and other loading coils with different separations between loads, with iron-cored loading coils and with terminal transformers, also with other cables and with loaded aerial lines several hundred miles in length, but the results are incomplete, and I am not prepared to attempt a discussion of them at present.

Any description of the experimental work must include a discussion of the actual performance of loading coils under periodic currents, which is an extended subject. For the mathematical work it has been assumed that the effective inductance and resistance of the coil is the same for all frequencies, which cannot be assumed in experimental work.

For the extended experimental work which has been done, which has been laborious and most painstaking, I am indebted to Mr. E. H. Colpitts, who has had charge of the experimental tests described above.

June 7, 1901.