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The Law of Gravitation, an example of Physical Law

It is odd, but on the infrequent occasions when I have been called upon in a formal place to play the bongo drums, the introducer never seems to find it necessary to mention that I also do theoretical physics. I believe that is probably because we respect the arts more than the sciences. The artists of the Renaissance said that man's main concern should be for man, and yet there are other things of interest in the world. Even the artists appreciate sunsets, and the ocean waves, and the march of the stars across the heavens. There is then some reason to talk of other things sometimes. As we look into these things we get an aesthetic pleasure from them directly on observation. There is also a rhythm and a pattern between the phenomena of nature which is not apparent to the eye, but only to the eye of analysis; and it is these rhythms and patterns which we call Physical Laws. What I want to discuss in this series of lectures is the general characteristic of these Physical Laws; that is another level, if you will, of higher generality over the laws themselves. Really what I am considering is nature as seen as a result of detailed analysis, but mainly I wish to speak about only the most overall general qualities of nature.

Now such a topic has a tendency to become too philosophical because it becomes so general, and a person talks in such generalities, that everybody can understand him. It is then considered to be some deep philosophy. I would like to be rather more special, and I would like to be understood in an honest way rather than in a vague way. So in this first lecture I am going to try to give, instead of only the generalities, an example of physical law, so that you have at least one example of the things about which I am speaking generally. In this way I can use this example again and again to give an instance, or to make a reality out of something which will otherwise be too abstract. I have chosen for my special example of physical law the theory of gravitation, the phenomena of gravity. Why I chose gravity I do not know. Actually it was one of the first great laws to be discovered and it has an interesting history. You may say, 'Yes, but then it is old hat, I would like to hear something about a more modern science'. More recent perhaps, but not more modern. Modern science is exactly in the same tradition as the discoveries of the Law of Gravitation. It is only more recent discoveries that we would be talking about. I do not feel at all bad about telling you about the Law of Gravitation because in describing its history and methods, the character of its discovery, its quality, I am being completely modern.

This law has been called 'the greatest generalization achieved by the human mind', and you can guess already from my introduction that I am interested not so much in the human mind as in the marvel of a nature which can obey such an elegant and simple law as this law of gravitation. Therefore our main concentration will not be on how clever we are to have found it all out, but on how clever nature is to pay attention to it.

The Law of Gravitation is that two bodies exert a force upon each other which varies inversely as the square of the distance between them, and varies directly as the product of their masses. Mathematically we can write that great law down in the formula:

$$F = G\frac{mm'}{r^2}$$

some kind of a constant multiplied by the product of the two masses, divided by the square of the distance. Now if I add the remark that a body reacts to a force by accelerating, or by changing its velocity every second to an extent inversely as its mass, or that it changes its velocity more if the mass is lower, inversely as the mass, then I have said everything about the Law of Gravitation that needs to be said. Everything else is a mathematical consequence of those two things. Now I know that you are not all mathematicians, and you cannot immediately see all of the consequences of these two remarks, so what I would like to do here is to tell you briefly of the story of the discovery, what some of the consequences are, what effect this discovery had on the history of science, what kind of mysteries such a law entails, something about the refinements made by Einstein, and possibly the relation to the other laws of physics.

The history of the thing, briefly, is this. The ancients first observed the way the planets seemed to move in the sky and concluded that they all, along with the earth, went around the sun. This discovery was later made independently by Copernicus, after people had forgotten that it had already been made. Now the next question that came up for study was: exactly how do they go around the sun, that is, with exactly what kind of motion? Do they go with the sun as the centre of a circle, or do they go in some other kind of curve? How fast do they move? And so on. This discovery took longer to make. The times after Copernicus were times in which there were great debates about whether the planets in fact went around the sun along with the earth, or whether the earth was at the centre of the universe and so on. Then a man named Tycho Brahe* evolved a way of answering the question. He thought that it might perhaps be a good idea to look very very carefully and to record exactly where the planets appear in the sky, and then the alternative theories might be distinguished from one another. This is the key of modern science and it was the beginning of the true understanding of Nature – this idea to look at the thing, to record the details, and to hope that in the information thus obtained might lie a clue to one or another theoretical interpretation. So Tycho, a rich man who owned an island near

*Tycho Brahe, 1546–1601, Danish astronomer.

Copenhagen, outfitted his island with great brass circles and special observing positions, and recorded night after night the position of the planets. It is only through such hard work that we can find out anything.

When all these data were collected they came into the hands of Kepler,* who then tried to analyse what kind of motion the planets made around the sun. And he did this by a method of trial and error. At one stage he thought he had it; he figured out that they went round the sun in circles with the sun off centre. Then Kepler noticed that one planet, I think it was Mars, was eight minutes of arc off, and he decided this was too big for Tycho Brahe to have made an error, and that this was not the right answer. So because of the precision of the experiments he was able to proceed to another trial and ultimately found out three things.

First, he found that the planets went in ellipses around the sun with the sun as a focus. An ellipse is a curve all artists know about because it is a foreshortened circle. Children also know because someone told them that if you put a ring on a piece of cord, anchored at each end, and then put a pencil in the ring, it will draw an ellipse (fig. 1).

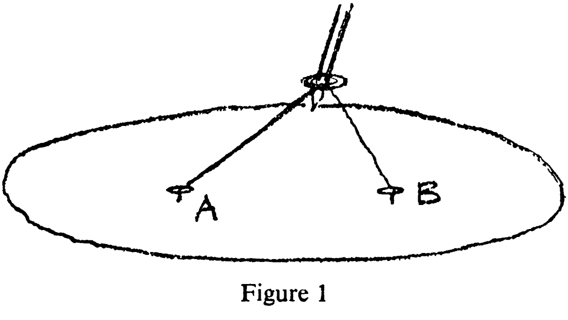

Figure 1

The two points A and B are the foci. The orbit of a planet around the sun is an ellipse with the sun at one focus. The

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*Johann Kepler, 1571–1630, German astronomer and mathematician, assistant to Brahe.

next question is: In going around the ellipse, how does the planet go? Does it go faster when it is near the sun? Does it go slower when it is farther from the sun? Kepler found the answer to this too (fig. 2).

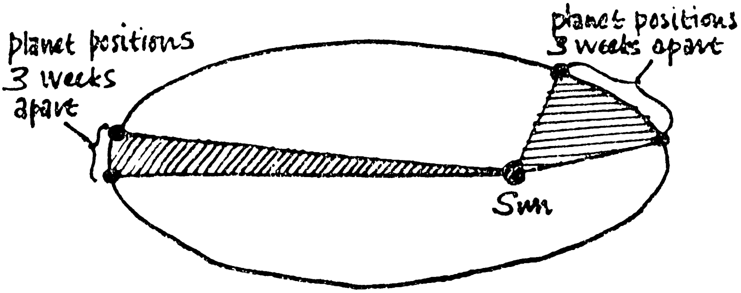

Figure 2

He found that, if you put down the position of a planet at two times, separated by some definite period, let us say three weeks – then in another place on its orbit two positions of the planet again separated by three weeks, and draw lines (technically called radius vectors) from the sun to the planet, then the area that is enclosed in the orbit of the planet and the two lines that are separated by the planet's position three weeks apart is the same, in any part of the orbit. So that the planet has to go faster when it is closer to the sun, and slower when it is farther away, in order to show precisely the same area.

Some several years later Kepler found a third rule, which was not concerned only with the motion of a single planet around the sun but related various planets to each other. It said that the time the planet took to go all around the sun was related to the size of the orbit, and that the times varied as the square root of the cube of the size of the orbit and for this the size of the orbit is the diameter across the biggest distance on the ellipse. Kepler then had these three laws which are summarized by saying that *the orbit forms an ellipse*, and that *equal areas are swept in equal times* and that the *time to go round varies as a three half power of the size*, that is, the square root of the cube of the size. These three laws of Kepler give a complete description of the motion of the planets around the sun.

The next question was – what makes planets go around the sun? At the time of Kepler some people answered this problem by saying that there were angels behind them beating their wings and pushing the planets around an orbit. As you will see, the answer is not very far from the truth. The only difference is that the angels sit in a different direction and their wings push inwards.

In the meantime, Galileo was investigating the laws of motion of ordinary objects at hand on the earth. In studying these laws, and doing a number of experiments to see how balls run down inclined planes, and how pendulums swing, and so on, Galileo discovered a great principle called the principle of inertia, which is this: that if an object has nothing acting on it and is going along at a certain velocity in a straight line it will go at the same velocity in exactly the same straight line for ever. Unbelievable as that may sound to anybody who has tried to make a ball roll for ever, if this idealization were correct, and there were no influences acting, such as the friction of the floor and so on, the ball would go at a uniform speed for ever.

The next point was made by Newton, who discussed the question: 'When it does not go in a straight line *then* what?' And he answered it this way: that a force is needed to change the velocity in any manner. For instance, if you are pushing a ball in the direction that it moves it will speed up. If you find that it changes direction, then the force must have been sideways. The force can be measured by the product of two effects. How much does the velocity change in a small interval of time? That's called the acceleration, and when it is multiplied by the coefficient called the mass of an object, or its inertia coefficient, then that together is the force. One can measure this. For instance, if one has a stone on the end of a string and swings it in a circle over the head, one finds one has to pull, the reason is that although the speed is not changing as it goes round in a circle, it is changing its direction; there must be a perpetually in-pulling force, and this is proportional to the mass. So that if we were to take two different objects, and swing first one and then the other at the same speed around the head, and measure the force in the second one, then that second force is bigger than the other force in proportion as the masses are different. This is a way of measuring the masses by what force is necessary to change the speed. Newton saw from this that, to take a simple example, if a planet is going in a circle around the sun, *no force is needed to make it go sideways, tangentially;* if there were no force at all then it would just keep coasting along. But actually the planet does not keep coasting along, it finds itself later not way out where it would go if there were no force at all, but farther down towards the sun.

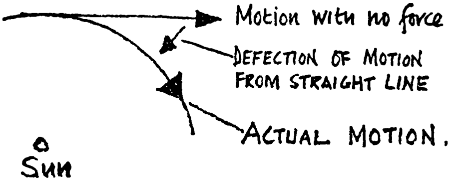

Figure 3

(fig. 3.) In other words, its velocity, its motion, has been deflected towards the sun. So that what the angels have to do is to beat their wings in towards the sun all the time.

But the motion to keep the planet going in a straight line has no known reason. The reason why things coast for ever has never been found out. The law of inertia has no known origin. Although the angels do not exist the continuation of the motion does, but in order to obtain the falling operation we do need a force. It became apparent that the origin of the force was towards the sun. As a matter of fact Newton was able to demonstrate that the statement that equal areas are swept in equal times was a direct consequence of the simple idea that all the changes in velocity are directed exactly towards the sun, even in the elliptical case, and in the next lecture I shall be able to show you how it works, in detail.

From this law Newton confirmed the idea that the force is towards the sun, and from knowing how the periods of the different planets vary with the distance away from the sun, it is possible to determine how that force must weaken at different distances. He was able to determine that the force must vary inversely as the square of the distance.

So far Newton has not said anything, because he has only stated two things which Kepler said in a different language. One is exactly equivalent to the statement that the force is towards the sun, and the other is exactly equivalent to the statement that the force is inversely as the square of the distance.

But people had seen in telescopes Jupiter's satellites going around Jupiter, and it looked like a little solar system, as if the satellites were attracted to Jupiter. The moon is attracted to the earth and goes round the earth and is attracted in the same way. It looks as though everything is attracted to everything else, and so the next statement was to generalize this and to say that every object attracts every object. If so, the earth must be pulling on the moon, just as the sun pulls on the planet. But it is known that the earth is pulling on things – because you are all sitting tightly on your seats in spite of your desire to float into the air. The pull for objects on the earth was well known in the phenomena of gravitation, and it was Newton's idea that maybe the gravitation that held the moon in orbit was the same gravitation that pulled the object towards the earth.

It is easy to figure out how far the moon falls in one second, because you know the size of the orbit, you know the moon takes a month to go around the earth, and if you figure out how far it goes in one second you can figure out how far the circle of the moon's orbit has fallen below the straight line that it would have been in if it did not go the way it does go. This distance is one twentieth of an inch.

The moon is sixty times as far away from the earth's centre as we are; we are 4,000 miles away from the centre, and the moon is 240,000 miles away from the centre, so if the law of inverse square is right, an object at the earth's surface should fall in one second by $\frac{1}{20}$ inch × 3,600 (the square of 60) because the force in getting out there to the moon, has been weakened by 60 × 60 by the inverse square law. $\frac{1}{20}$ inch × 3,600 is about 16 feet, and it was known already from Galileo's measurements that things fall in one second on the earth's surface by 16 feet. So this meant that Newton was on the right track, there was no going back now, because a new fact which was completely independent previously, the period of the moon's orbit and its distance from the earth, was connected to another fact, how long it takes something to fall in one second at the earth's surface. This was a dramatic test that everything is all right.

Further, Newton had a lot of other predictions. He was able to calculate what the shape of the orbit should be if the law were the inverse square, and he found, indeed, that it was an ellipse – so he got three for two as it were. In addition, a number of new phenomena had obvious explanations. One was the tides. The tides were due to the pull of the moon on the earth and its waters. This had sometimes been thought of before, with the difficulty that if it was the pull of the moon on the waters, making the water higher on the side where the moon was, then there would only be one tide a day under the moon (fig. 4), but actually we know

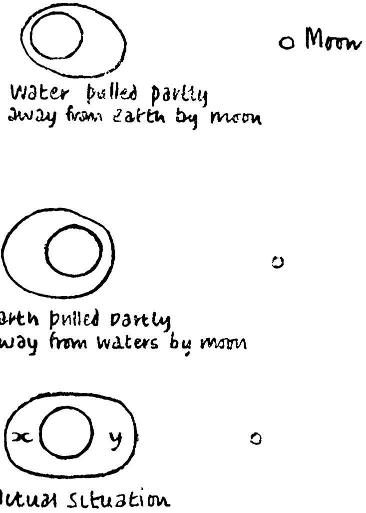

Figure 4

there are tides roughly every twelve hours, and that is two tides a day. There was also another school of thought that came to a different conclusion. Their theory was that it was the earth pulled by the moon away from the water. Newton was actually the first one to realize what was going on; that the force of the moon on the earth and on the water is the same at the same distance, and that the water at y is closer to the moon and the water at x is farther from the moon than the rigid earth. The water is pulled more towards the moon at y, and at x is less towards the moon than the earth, so there is a combination of those two pictures that makes a double tide. Actually the earth does the same trick as the moon, it goes around in a circle. The force of the moon on the earth is balanced, but by what? By the fact that just as the moon goes in a circle to balance the earth's force, the earth is also going in a circle. The centre of the circle is somewhere inside the earth. It is also going in a circle to balance the moon. The two of them go around a common centre so the forces are balanced for the earth, but the water at x is pulled less, and at y more by the moon and it bulges out at both sides. At any rate tides were then explained, and the fact that there were two a day. A lot of other things became clear: how the earth is round because everything gets pulled in, and how it is not round because it is spinning and the outside gets thrown out a little bit, and it balances; how the sun and moon are round, and so on.

As science developed and measurements were made more accurate, the tests of Newton's Law became more stringent, and the first careful tests involved the moons of Jupiter. By accurate observations of the way they went around over long periods of time one could check that everything was according to Newton, and it turned out to be not the case. The moons of Jupiter appeared to get sometimes eight minutes ahead of time and sometimes eight minutes behind time, where the time is the calculated value according to Newton's Laws. It was noticed that they were ahead of schedule when Jupiter was close to the earth and behind schedule when it was far away, a rather odd circumstance. Mr Roemer,* having confidence in the Law of Gravitation, came to the interesting conclusion that it takes light some time to travel from the moons of Jupiter to the earth, and what we are looking at when we see the moons is not how they are now but how they were the time ago it took the light to get here. When Jupiter is near us it takes less time for the light to come, and when Jupiter is farther from us it takes longer time, so Roemer had to correct the observations for the differences in time and by the fact that they were this much early or that much late. In this way he was able to determine the velocity of light. This was the first demonstration that light was not an instantaneously propagating material.

I bring this particular matter to your attention because it illustrates that when a law is right it can be used to find another one. If we have confidence in a law, then if something appears to be wrong it can suggest to us another phenomenon. If we had not known the Law of Gravitation we would have taken much longer to find the speed of light, because we would not have known what to expect of Jupiter's satellites. This process has developed into an avalanche of discoveries, each new discovery permits the tools for much more discovery, and this is the beginning of the avalanche which has gone on now for 400 years in a continuous process, and we are still avalanching along at high speed.

Another problem came up – the planets should not really go in ellipses, because according to Newton's Laws they are not only attracted by the sun but also they pull on each other a little – only a little, but that little is something, and will alter the motion a little bit. Jupiter, Saturn and Uranus were big planets that were known, and calculations were made about how slightly different from the perfect ellipses of Kepler the planets ought to be going by the pull of each on the others. And at the end of the calculations and observations it was noticed that Jupiter and Saturn went according

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*Olaus Roemer, 1644–1710, Danish astronomer.

to the calculations, but that Uranus was doing something funny. Another opportunity for Newton's Laws to be found wanting; but take courage! Two men, Adams and Leverrier,* who made these calculations independently and at almost exactly the same time, proposed that the motions of Uranus were due to an unseen planet, and they wrote letters to their respective observatories telling them – 'Turn your telescope and look there and you will find a planet'. 'How absurd,' said one of the observatories, 'some guy sitting with pieces of paper and pencils can tell us where to look to find some new planet.' The other observatory was more . . . well, the administration was different, and they found Neptune!

More recently, in the beginning of the twentieth century, it became apparent that the motion of the planet Mercury was not exactly right. This caused a lot of trouble and was not explained until it was shown by Einstein that Newton's Laws were slightly off and that they had to be modified.

The question is, how far does this law extend? Does it extend outside the solar system? And so I show on Plate 1 evidence that the Law of Gravitation is on a wider scale than just the solar system. Here is a series of three pictures of a so-called double star. There is a third star fortunately in the picture so that you can see they are really turning around and that nobody simply turned the frames of the pictures around, which is easy to do on astronomical pictures. The stars are actually going around, and you can see the orbit that they make on figure 5. It is evident that they are attracting each other and that they are going around in an ellipse according to the way expected. These are a succession of positions at various times going around clockwise. You will be happy except when you notice, if you have not noticed already, that the centre is not a focus of the ellipse but is quite a bit off. So something is the matter with the law? No, God has not presented us with this orbit face-on; it is tilted

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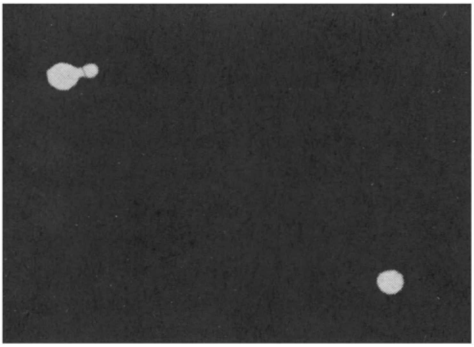
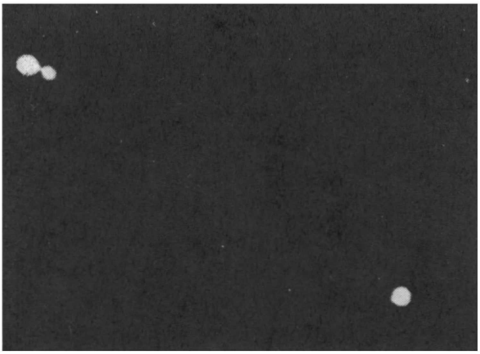
*John Couch Adams, 1819–92, mathematical astronomer. Urbain Leverrier, 1811–77, French astronomer.



21 July, 1908



September, 1915

10 July, 1920

Plate 1. Three photographs taken at different times of the same double star system.

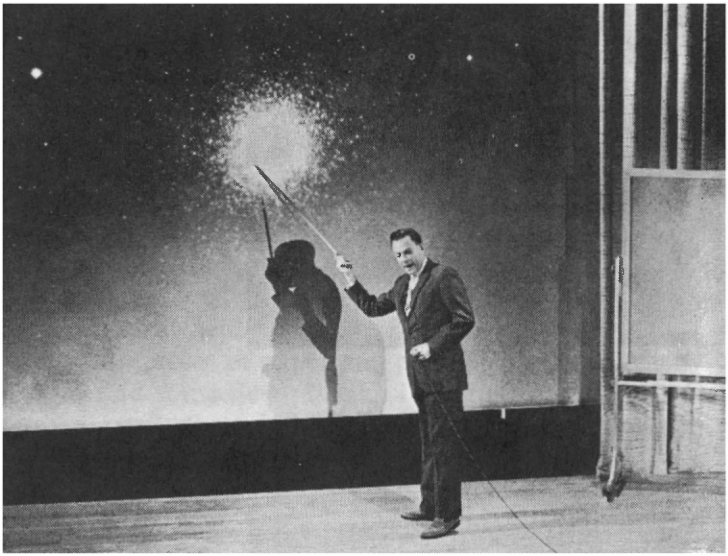


Plate 2. A globular star cluster

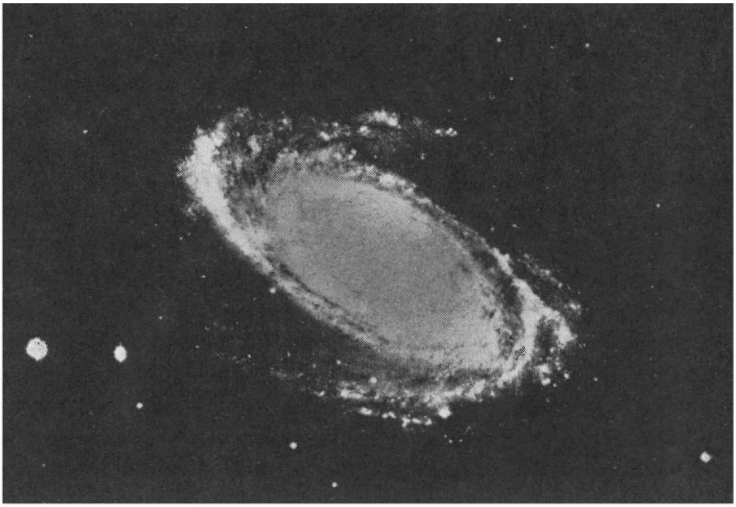


Plate 3. A spiral galaxy

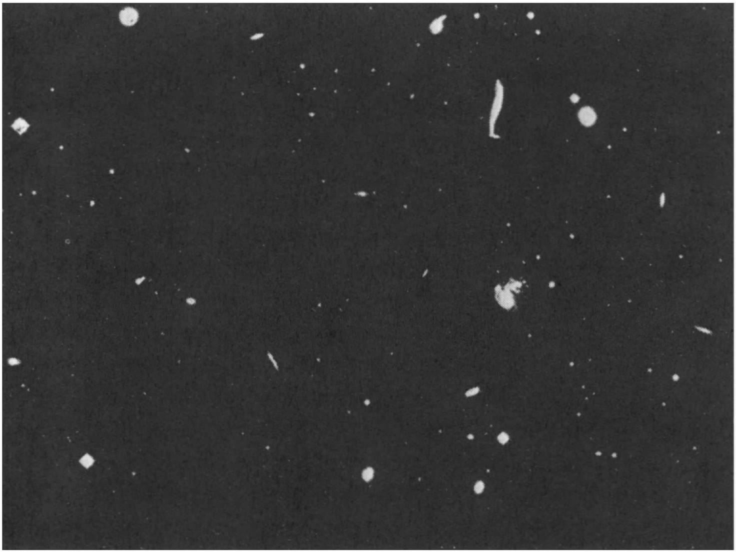
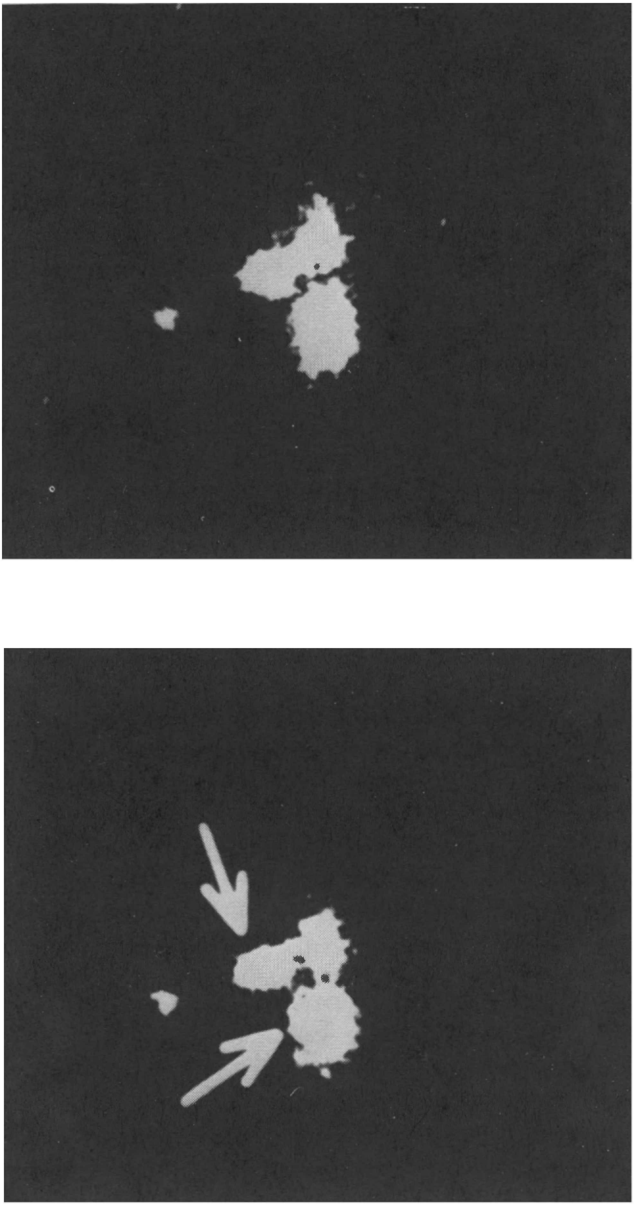


Plate 4. A cluster of galaxies

Plate 5. A gaseous nebula



1947

1954

Plate 6. Evidence of the creation of new stars

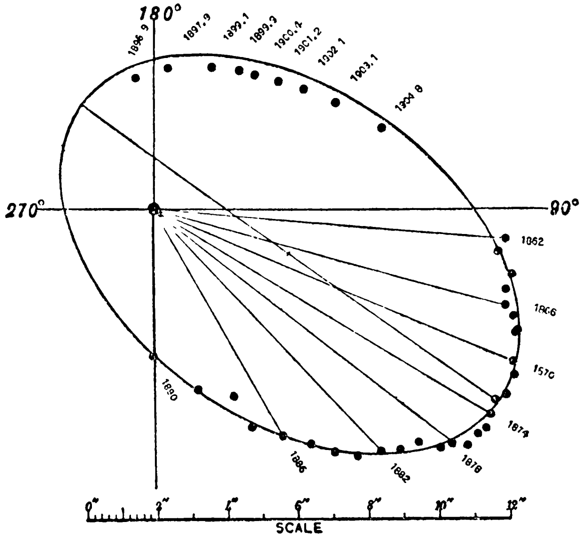


Figure 5

at a funny angle. If you take an ellipse and mark its focus and hold the paper at an odd angle and look at it in projection, you will find that the focus does not have to be at the focus of the projected image. It is because the orbit is tilted in space that it looks that way.

How about a bigger distance? This force is between two stars; does it go any farther than distances which are not more than two or three times the solar system's diameter? Here is something in plate 2 that is 100,000 times as big as the solar system in diameter; this is a tremendous number of stars. This large white spot is not a solid white spot; it appears like that because of the failure of the instruments to resolve it, but there are very very tiny spots just like other stars, well separated from each other, not hitting one another, each one falling through and back and forth in this great globular cluster. It is one of the most beautiful things in the sky; it is as beautiful as sea waves and sunsets. The distribution of this material is perfectly clear. The thing that holds this galaxy together is the gravitational attraction of the stars for each other. The distribution of the material and the sense of distance permits one to find out roughly what the law of force is between the stars . . . and, of course, it comes out that it is roughly the inverse square. Accuracy in these calculations and measurements is not anywhere near as careful as in the solar system.

Onward, gravity extends still farther. That cluster was just a little pin-point inside the big galaxy in plate 3, which shows a typical galaxy, and it is clear that again this thing is held together by some force, and the only candidate that is reasonable is gravitation. When we get to this size we have no way of checking the inverse square law, but there seems to be no doubt that in these great agglomerations of stars – these galaxies are 50,000 to 100,000 light years across, while the distance from the earth to the sun is only eight light minutes – gravity is extending even over these distances. In plate 4 is evidence that it extends even farther. This is what is called a cluster of galaxies; they are all in one lump and analogous to the cluster of stars, but this time what is clustered are those big babies shown in plate 3.

This is as far as about one tenth, maybe a hundredth, of the size of the Universe, as far as we have any direct evidence that gravitational forces extend. So the earth's gravitation has no edge, although you may read in the papers that something gets outside the field of gravitation. It becomes weaker and weaker inversely as the square of the distance, divided by four each time you get twice as far away, until it is lost in the confusion of the strong fields of other stars. Together with the stars in its neighbourhood it pulls the other stars to form the galaxy, and all together they pull on other galaxies and make a pattern, a cluster, of galaxies. So the earth's gravitational field never ends, but peters out very slowly in a precise and careful law, probably to the edges of the Universe.

The Law of Gravitation is different from many of the others. Clearly it is very important in the economy, in the machinery, of the Universe; there are many places where gravity has its practical applications as far as the Universe is concerned. But atypically the knowledge of the Laws of Gravitation has relatively few practical applications compared with the other laws of physics. This is one case where I have picked an atypical example. It is impossible, by the way, by picking one of anything to pick one that is not atypical in some sense. That is the wonder of the world. The only applications of the knowledge of the law that I can think of are in geophysical prospecting, in predicting the tides, and nowadays, more modernly, in working out the motions of the satellites and planet probes that we send up, and so on; and finally, also modernly, to calculate the predictions of the planets' positions, which have great utility for astrologists who publish their predictions in horoscopes in the magazines. It is a strange world we live in – that all the new advances in understanding are used only to continue the nonsense which has existed for 2,000 years.

I must mention the important places where gravitation does have some real effect in the behaviour of the Universe, and one of the interesting ones is in the formation of new stars. Plate 5 is a gaseous nebula inside our own galaxy; it is not a lot of stars; it is gas. The black specks are places where the gas has been compressed or attracted to itself. Perhaps it starts by some kind of shock waves, but the remainder of the phenomenon is that gravitation pulls the gas closer and closer together so that big mobs of gas and dust collect and form balls; and as they fall still farther, the heat generated by falling lights them up, and they become stars. And we have in plate 6 some evidence of the creation of new stars.

So this is how stars are born, when the gas collects together too much by gravitation. Sometimes when they explode the stars belch out dirt and gases, and the dirt and gases collect back again and make new stars – it sounds like perpetual motion.

I have already shown that gravitation extends to great distances, but Newton said that everything attracted everything else. Is it really true that two things attract each other?

Can we make a direct test and not just wait to see whether the planets attract each other? A direct test was made by Cavendish* on equipment which you see indicated in figure 6. The idea was to hang by a very very fine quartz fibre a rod with two balls, and then put two large lead balls in the positions indicated next to it on the side. Because of the attraction of the balls there would be a slight twist to the fibre, and the gravitational force between ordinary things is very very tiny indeed. It was possible to measure the force between the two balls. Cavendish called his experiment 'weighing the earth'. With pedantic and careful teaching today we would not let our students say that; we would have to say 'measuring the mass of the earth'. By a direct experiment Cavendish was able to measure the force, the two masses and the distance, and thus determine the gravitational constant, G. You say, 'Yes, but we have the same situation here. We know what the pull is and we know what the mass of the object pulled is, and we know how far away we are, but we do not know either the mass of the earth or

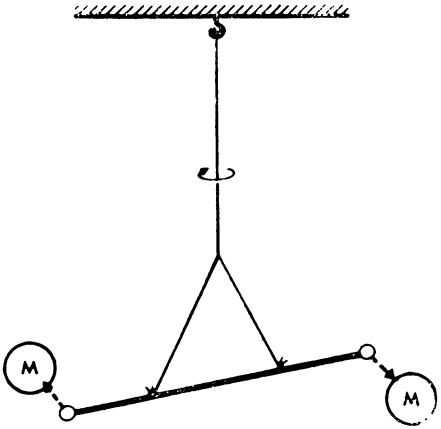


Figure 6

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*Henry Cavendish, 1731–1810, English physicist and chemist.

the constant, only the combination'. By measuring the constant, and knowing the facts about the pull of the earth, the mass of the earth could be determined.

Indirectly this experiment was the first determination of how heavy or massive is the ball on which we stand. It is an amazing achievement to find that out, and I think that is why Cavendish named his experiment 'weighing the earth', instead of 'determining the constant in the gravitational equation'. He, incidentally, was weighing the sun and everything else at the same time, because the pull of the sun is known in the same manner.

One other test of the law of gravity is very interesting, and that is the question whether the pull is exactly proportional to the mass. If the pull is exactly proportional to the mass, and the reaction to force, the motions induced by forces, changes in velocity, are inversely proportional to the mass. That means that two objects of different mass will change their velocity in the same manner in a gravitational field; or two different things in a vacuum, no matter what their mass, will fall the same way to the earth. That is Galileo's old experiment from the leaning tower of Pisa. It means, for example, that in a man-made satellite, an object inside will go round the earth in the same kind of orbit as one on the outside, and thus apparently float in the middle. The fact that the force is exactly proportional to the mass, and that the reactions are inversely proportional to the mass, has this very interesting consequence.

How accurate is it? It was measured in an experiment by a man named Eötvös* in 1909 and very much more recently and more accurately by Dicke,† and is known to one part in 10,000,000,000. The forces are exactly proportional to the mass. How is it possible to measure with that accuracy? Suppose you wanted to measure whether it is true for the pull of the sun. You know the sun is pulling us all, it pulls the earth too, but suppose you wanted to know whether the

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*Baron Roland von Eötvös, 1848–1919, Hungarian physicist.
†Robert Henry Dicke, American physicist.

pull is exactly proportional to the inertia. The experiment was first done with sandalwood; lead and copper have been used, and now it is done with polyethylene. The earth is going around the sun, so the things are thrown out by inertia and they are thrown out to the extent that the two objects have inertia. But they are attracted to the sun to the extent that they have mass, in the attraction law. So if they are attracted to the sun in a different proportion from that thrown out by inertia, one will be pulled towards the sun, and the other away from it, and so, hanging them on opposite ends of a rod on another Cavendish quartz fibre, the thing will twist towards the sun. It does not twist at this accuracy, so we know that the sun's attraction to the two objects is exactly proportional to the centrifugal effect, which is inertia; therefore, the force of attraction on an object is exactly proportional to its coefficient of inertia; in other words, its mass.

One thing is particularly interesting. The inverse square law appears again – in the electrical laws, for instance. Electricity also exerts forces inversely as the square of the distance, this time between charges, and one thinks perhaps that the inverse square of the distance has some deep significance. No one has ever succeeded in making electricity and gravity different aspects of the same thing. Today our theories of physics, the laws of physics, are a multitude of different parts and pieces that do not fit together very well. We do not have one structure from which all is deduced; we have several pieces that do not quite fit exactly yet. That is the reason why in these lectures instead of having the ability to tell you what *the law* of physics is, I have to talk about the things that are common to the various laws; we do not understand the connection between them. But what is very strange is that there are certain things which are the same in both. Now let us look again at the law of electricity.

The force goes inversely as the square of the distance, but the thing that is remarkable is the tremendous difference in the strength of the electrical and gravitational forces. People who want to make electricity and gravitation out of the same thing will find that electricity is so much more powerful than gravity, it is hard to believe they could both have the same origin. How can I say one thing is more powerful than another? It depends upon how much charge you have, and how much mass you have. You cannot talk about how strong gravity is by saying: ‘I take a lump of such a size’, because *you* chose the size. If we try to get something that Nature produces – her own pure number that has nothing to do with inches or years or anything to do with our own dimensions – we can do it this way. If we take a fundamental particle such as an electron – any different one will give a different number, but to give an idea say electrons – two electrons are two fundamental particles, and they repel each other inversely as the square of the distance due to electricity, and they attract each other inversely as the square of the distance due to gravitation.

Question: What is the ratio of the gravitational force to the electrical force? That is illustrated in figure 7. The ratio

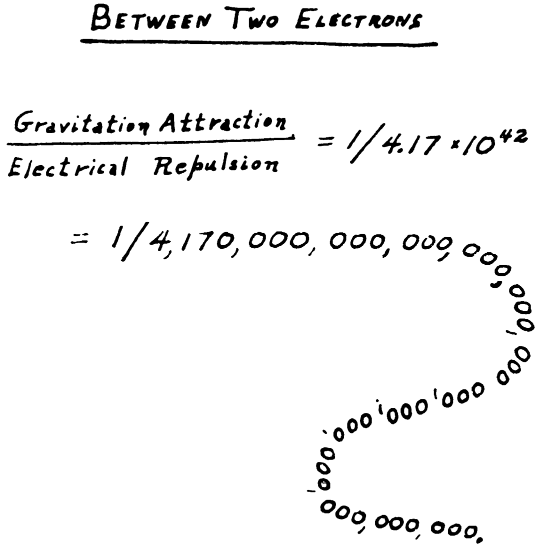

Figure 7

of the gravitational attraction to electrical repulsion is given by a number with 42 digits tailing off. Now therein lies a very deep mystery. Where could such a tremendous number come from? If you ever had a theory from which both of these things are to come, how could they come in such disproportion? What equation has a solution which has for two kinds of forces an attraction and repulsion with that fantastic ratio?

People have looked for such a large ratio in other places. They hope, for example, that there is another large number, and if you want a large number why not take the diameter of the Universe to the diameter of a proton – amazingly enough it also is a number with 42 digits. And so an interesting proposal is made that this ratio is the same as the ratio of the size of the Universe to the diameter of a proton. But the Universe is expanding with time and that means that the gravitational constant is changing with time, and although that is a possibility there is no evidence to indicate that it is a fact. There are several partial indications that the gravitational constant has not changed in that way. So this tremendous number remains a mystery.

To finish about the theory of gravitation, I must say two more things. One is that Einstein had to modify the Laws of Gravitation in accordance with his principles of relativity. The first of the principles was that 'x' cannot occur instantaneously, while Newton's theory said that the force was instantaneous. He had to modify Newton's laws. They have very small effects, these modifications. One of them is that all masses fall, light has energy and energy is equivalent to mass. So light falls and it means that light going near the sun is deflected; it is. Also the force of gravitation is slightly modified in Einstein's theory, so that the law has changed very very slightly, and it is just the right amount to account for the slight discrepancy that was found in the movement of Mercury.

Finally, in connection with the laws of physics on a small scale, we have found that the behaviour of matter on a small scale obeys laws very different from things on a large scale. So the question is, how does gravity look on a small scale? That is called the Quantum Theory of Gravity. There is no Quantum Theory of Gravity today. People have not succeeded completely in making a theory which is consistent with the uncertainty principles and the quantum mechanical principles.

You will say to me, 'Yes, you told us what happens, but what is gravity? Where does it come from? What is it? Do you mean to tell me that a planet looks at the sun, sees how far it is, calculates the inverse square of the distance and then decides to move in accordance with that law?' In other words, although I have stated the mathematical law, I have given no clue about the mechanism. I will discuss the possibility of doing this in the next lecture, 'The relation of mathematics to physics'.

In this lecture I would like to emphasize, just at the end, some characteristics that gravity has in common with the other laws that we mentioned as we passed along. First, it is mathematical in its expression; the others are that way too. Second, it is not exact; Einstein had to modify it, and we know it is not quite right yet, because we have still to put the quantum theory in. That is the same with all our other laws – they are not exact. There is always an edge of mystery, always a place where we have some fiddling around to do yet. This may or may not be a property of Nature, but it certainly is common to all the laws as we know them today. It may be only a lack of knowledge.

But the most impressive fact is that gravity is simple. It is simple to state the principles completely and not have left any vagueness for anybody to change the ideas of the law. It is simple, and therefore it is beautiful. It is simple in its pattern. I do not mean it is simple in its action – the motions of the various planets and the perturbations of one on the other can be quite complicated to work out, and to follow how all those stars in a globular cluster move is quite beyond our ability. It is complicated in its actions, but the basic pattern or the system beneath the whole thing is simple. This is common to all our laws; they all turn out to be simple things, although complex in their actual actions.

Finally comes the universality of the gravitational law, and the fact that it extends over such enormous distances that Newton, in his mind, worrying about the solar system, was able to predict what would happen in an experiment of Cavendish, where Cavendish's little model of the solar system, two balls attracting, has to be expanded ten million million times to become the solar system. Then ten million million times larger again we find galaxies attracting each other by exactly the same law. Nature uses only the longest threads to weave her patterns, so each small piece of her fabric reveals the organization of the entire tapestry.