

Electro-magnetically controlled acoustic metamaterials with adaptive properties

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A design of actively controlled metamaterial is proposed and discussed. The metamaterial consists of layers of electrically charged nano or micro particles exposed to external magnetic field. The particles are also attached to compliant layers in a way that the designed structure exhibits two resonances: mechanical spring-mass resonance and electro-magnetic cyclotron resonance. It is shown that if the cyclotron frequency is greater than the mechanical resonance frequency, the designed structure could be highly attenuative (40–60 dB) for vibration and sound waves in very broad frequency range even for wavelength much greater than the thickness of the metamaterial. The approach opens up wide range of opportunities for design of adaptively controlled acoustic metamaterials by controlling magnetic field and/or electrical charges.

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I. INTRODUCTION

Emerging metamaterials are envisioned as one of disruptive technologies that can potentially marginalize a number of existing dominant technologies. Metamaterial is a manmade material which gains its properties from its structure rather than directly from its composition, designed to exhibit unusual properties not available in nature. Electro-magnetic metamaterial with negative values for both permittivity and permeability results in structures with a negative refractive index allowing creation of microwave and optical cloaks, superlenses, super directional antennas, etc.,^{1–5} until recently considered just science fiction. While rapidly growing research efforts have been invested on electromagnetic metamaterials,⁶ the abnormal behaviors of elastodynamic/nonlinear/acoustic metamaterial (with negativity of mass density/stiffness) and the design/fabrication technologies just begun to be explored.^{7–9} Furthermore, metamaterial research and developmental efforts mostly focuses on passive approaches,¹⁰ i.e., the metamaterial properties are fixed by design and cannot not be manipulated ones the material is fabricated.

Recent efforts in developing acoustic metamaterials have been focused on constructing mechanical structures having so-called negative effective dynamic bulk modulus and negative effective dynamic mass density.^{11–14} Here the term “negative” is intrinsically bound with the term “effective dynamic” as a dynamic behavior of structured material with local resonances is treated as dynamic response of equivalent (effective) continuum medium. The resonances change the phase of the structural dynamic response to an

external stimulus, thus creating an appearance of negative inertial and/or compression properties of the equivalent continuum medium. Thus, common approach for constructing negative mass employs mass-spring oscillator build into another mass.^{15,16} A lattice of such “mass-in-mass” elements could be highly attenuative for acoustic/vibration waves when the effective dynamic mass density becomes negative. However, the negative mass effect takes place within a narrow fixed frequency band determined by the oscillator resonance frequency.

In the present paper we propose a new class of broad-band metamaterial actively controlled using electro-magnetic interaction. Active control of material properties using magnetic fields is not new. For example, active magnetic dampers for vibration suppression have been studied since the 1980s.¹⁷ These devices utilize magnets whose vibrations generate eddy currents in adjacent conductor effectively dissipating vibration energy. Another approach for active adaptively controlled magnetic dampers utilizes magnetoreological elastomers.¹⁸ In this application the elastic moduli of the elastomers are controlled by an applied magnetic field allowing adaptive tuning the resonance frequency of a vibration absorber. Recently, active magnetoreological fluids were proposed to be used for acoustic metamaterials as well.¹⁹

Our approach for active adaptive control is fundamentally different utilizing the interaction of applied magnetic field with electrically charged nano or micro particles constituting the metamaterial lattice. The particles are attached to compliant layers, therefore, the designed structure exhibits two resonances: mechanical spring-mass resonance and electro-magnetic cyclotron resonance. The last one is usually considered in accelerators, plasma, or condense matter physics, there very light atomic or subatomic electrically charged particles move in a circle due to Lorentz force

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induced by static magnetic field. The frequency of this circular motion (cyclotron frequency) is determined as

$$\omega_H = qH/m, \quad (1)$$

where m is the particle mass, q is its electrical charge, and H is the magnetic field amplitude. Charged nano or micro particles may experience the same cyclic motion. It is shown that if their cyclotron frequencies are greater than the mechanical resonance frequencies, the designed structure could be highly attenuative (40–60 dB) for vibration and sound waves in a very broad frequency range even for wavelengths much greater than the thickness of the metamaterial.

II. NEGATIVE EFFECTIVE MASS MODEL OF FORCED HARMONIC OSCILLATOR

Consider a charged nanoparticle of mass m connected to a base by a spring with a stiffness constant k placed into an external magnetic field \vec{H} and electric field \vec{E} . The particle is a subject to some external mechanical force \vec{F} . The general equation of the particle motion has the form²⁰

$$m\dot{\vec{v}} = -k\vec{r} + q\vec{E} + q\vec{v} \times \vec{H} + \vec{F}, \quad (2)$$

where q is the charge on the particle and \vec{v} is the velocity.

Equation (2) can provide very complicated three-dimensional trajectory of the particle depending on the values of the external parameters. To demonstrate the concept of the proposed approach we consider simplified model when a homogeneous magnetic field is applied only in the z direction $\vec{H} = (0, 0, H)$, the electric field is zero, $\vec{E} = 0$, and the mechanical force is applied in the x direction only, $\vec{F} = F(t)$, as shown in Fig. 1. For this configuration vector product $\vec{v} \times \vec{H} = \vec{i}\dot{y}H - \vec{j}\dot{x}H$ and, respectively, the particle trajectory is two-dimensional in the x, y plane.

Introducing new variables $\omega_0 = \sqrt{k/m}$ (the natural mass-spring frequency) and $\omega_H = qH/m$ (the cyclotron frequency) the equations of motion can be rewritten in the following form:

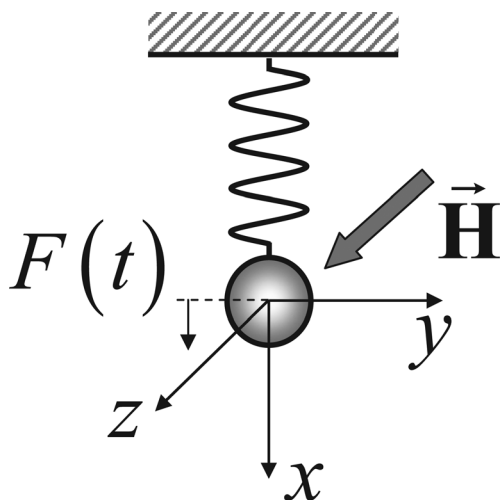


FIG. 1. Schematic of a charged particle which is a subject to a periodic external force in a uniform magnetic field.

$$\begin{cases} \ddot{x} = \omega_H \dot{y} - \omega_0^2 x - \gamma \dot{x} + \frac{F(t)}{m}, \\ \ddot{y} = -\omega_H \dot{x} - \gamma \dot{y}, \end{cases} \quad (3)$$

where γ is the damping coefficient.

Examples of the trajectory and dynamics of the charged particle governed by Eq. (3) is shown in Fig. 2. In the absence of the magnetic field, $H = 0$, the particle oscillates along the mechanical force direction (the x axis), Fig. 2(a), with the sinusoidal waveform shown in Fig. 2(b) by the solid line. There is no motion in the y direction. As the magnetic field is turned on such that $H > 0$, the trajectory becomes elliptical in the x, y plane with dominant motion along the y axis, as illustrated in Figs. 2(c) and 2(d). Figure 2(d) shows the particle oscillation vs time along the y axis by the dashed line, and along the x axis by the solid line, with the respective amplitude ratio about 100:1 for this example. That is, the particle oscillates primarily in the direction perpendicular to the direction of mechanical (acoustical/vibrational) force, and the oscillation in the x direction is considerably suppressed in the presence of the external magnetic field.

In order to provide some insight of the x -direction amplitude suppression effect, the Eq. (3) is solved for the steady-state spectral amplitude, $A_x(\omega)$, of the particle x -direction oscillation, assuming $F(t) = F \cos \omega t$, where F is the constant amplitude. The solution is

$$\begin{aligned} A_x(\omega) = \frac{F\sqrt{\gamma^2 + \omega^2}}{m} [(\gamma^2 + \omega^2)\omega^4 \\ - 2\omega^2\omega_0^2(\gamma^2 + \omega^2 - \omega_H^2) + \omega^2[(\gamma^2 + \omega^2)^2 \\ + 2(\gamma^2 - \omega^2)\omega_H^2 + \omega_H^4]]^{-1/2}. \end{aligned} \quad (4)$$

Figure 3 shows the particle spectral amplitude responses (with and without the external magnetic field) computed with the Eq. (4). In the absence of the magnetic field, the frequency response exhibits the mass-spring resonance with oscillation suppression above the resonance frequency ω_0 . With the magnetic field acting on the charged particle such that $\omega_H > \omega_0$, there is much more significant suppression effect in a very broad frequency range including frequencies below and within mass-spring resonance.

For an undamped oscillator, $\gamma = 0$, the Eq. (4) is reduced to

$$A_x(\omega, \gamma = 0) = \frac{F}{m} \frac{1}{|\omega_0^2 + \omega_H^2 - \omega^2|}. \quad (5)$$

Equation (5) evaluates the resonance frequency of the mass-spring electro-magnetic oscillator, which is equal to $\sqrt{\omega_H^2 + \omega_0^2}$. It also shows that the electro-magnetic interaction suppresses the vibration amplitude of the mechanical oscillator by a factor of

$$S = \frac{\omega_0^2 - \omega^2}{\omega_0^2 + \omega_H^2 - \omega^2}. \quad (6)$$

At the frequencies below ω_0 the amplitude suppression factor $S \cong \omega_0^2/\omega_H^2$, provided that $\omega_H \gg \omega_0$.

The effect of the amplitude suppression of the particle motion in the x direction can be interpreted by invoking the

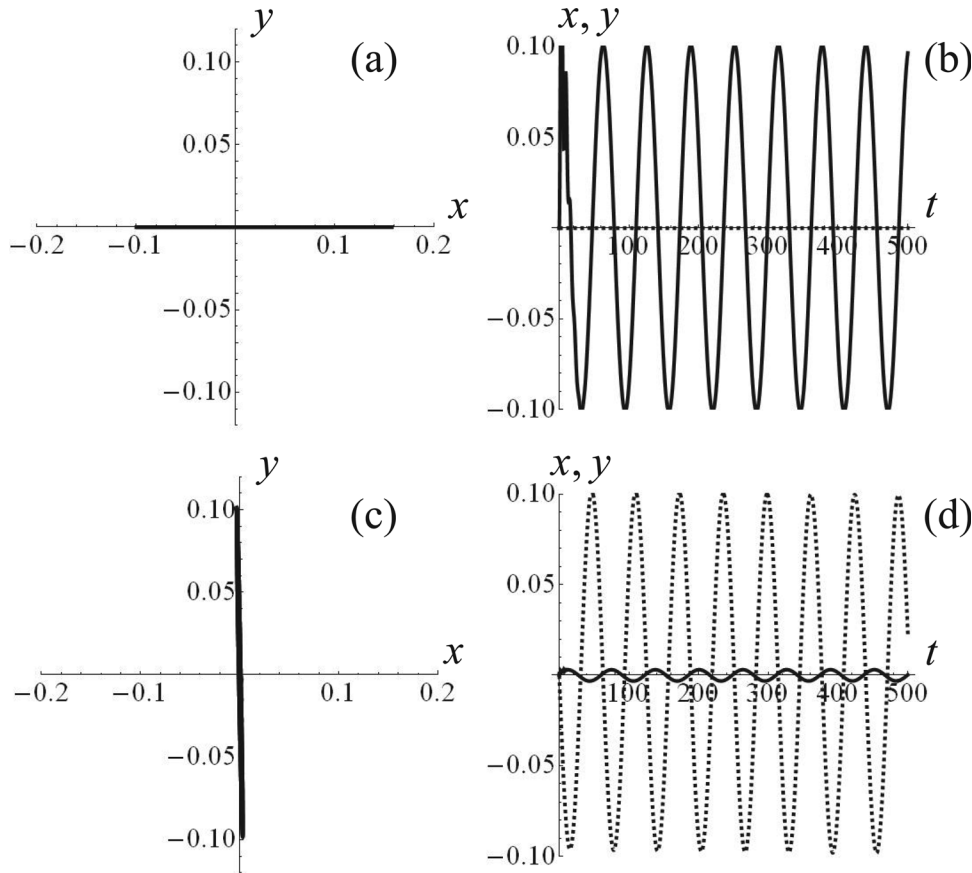


FIG. 2. Trajectory of a charged particle which is subject to a periodic external force in a uniform magnetic field. (a),(b)— $\omega_H/\omega_0 = 0$; (c),(d)— $\omega_H/\omega_0 = 10$; $\omega/\omega_0 = 0.1$, $F/k = 0.1$, and $\gamma/\omega_0 = 0.3$ [in (b) and (d) panels: x —solid line, y —dashed line].

negative effective mass paradigm, mentioned in the introductory section. To demonstrate this idea we consider further simplified model neglecting the damping terms in Eq. (3) and address the case with the external force in the x direction as $\vec{F} = F e^{-i\omega t}$. In this case, the general solution of the Eq. (3) has the form $x = A e^{-i\omega t}$ and $y = B e^{-i\omega t}$, where

$$A = \frac{F}{k - m_{\text{eff}}\omega^2}, \quad (7)$$

$$B = -i \frac{\omega_H}{\omega} \frac{F}{k - m_{\text{eff}}\omega^2}, \quad (8)$$

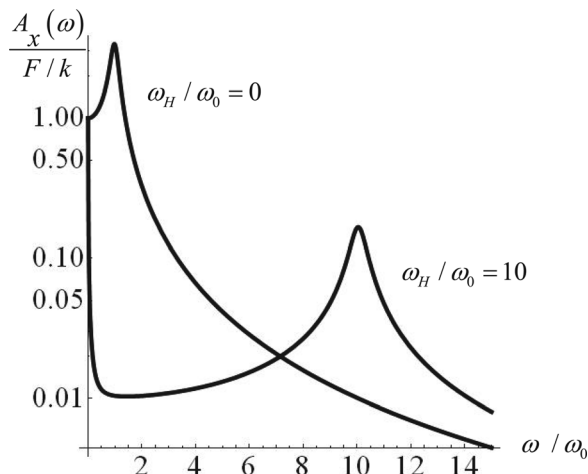


FIG. 3. Spectral response (in the x -direction) of a charged particle which is subject to a periodic external force in a uniform magnetic field; $\gamma/\omega_0 = 0.3$.

$$m_{\text{eff}} = m(1 - \omega_H^2/\omega^2). \quad (9)$$

The expressions for the coefficients A and B are written in the form which provides a standard solution of the forced one-dimensional harmonic oscillator problem²¹ when the effective mass, m_{eff} , is replaced by the oscillator mass m . Here the effective mass accounts for the interaction of the charged particle with the external magnetic field leading to the two-dimensional x - y trajectory of the mass. Note that the effective mass, m_{eff} , becomes negative in the wide range of frequencies $\omega < \omega_H$, as per Eq. (9). The “negative mass” translates into high attenuation of vibration or acoustic waves.

III. LINEAR CHAIN OF IDENTICAL EFFECTIVE MASSES

In this section we consider a one-dimensional lattice consisting of the effective masses connected by the linear springs as shown in Fig. 4. For simplicity we consider propagation of an elastic wave in an infinitely long lattice system with the goal to describe a steady-state harmonic motion of

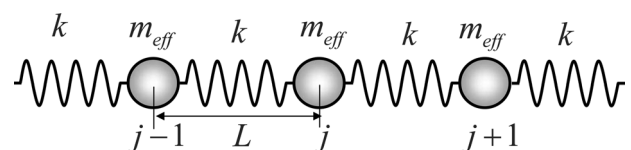


FIG. 4. Schematic of one-dimensional chain of the effective masses with the equilibrium distance L between them.

the masses and to investigate the attenuation mechanism of the wave propagating along the lattice.

Using the quasi elastic approximation, the equation of motion takes the following form:

$$m_{\text{eff}} \ddot{u}_j + k(2u_j - u_{j-1} - u_{j+1}) = 0, \quad (10)$$

where u_j is the displacement of the j -th mass.

Since the Eq. (10) does not change if we shift the whole system by the equilibrium distance between the masses, L , multiplied by an integer, the solution of this infinite set of equations can be written in the form

$$u_j = u(\eta) \exp\{i(\eta j L - \omega_\eta t)\}, \quad (11)$$

where η is the wavenumber.

Substituting Eq. (11) into Eq. (10) we find the characteristic equation

$$-m_{\text{eff}} \omega_\eta^2 + 2k(1 - \cos \eta L) = 0, \quad (12)$$

and obtain a dispersion relation for an elastic wave of wave-number η and the respective frequency ω_η

$$\omega_\eta = 2\sqrt{\frac{k}{m_{\text{eff}}}} \sin\left(\frac{\eta L}{2}\right). \quad (13)$$

Taking into account Eq. (7) with $\omega = \omega_\eta$ and expressing $k = m\omega_0^2$, Eq. (13) can be rewritten as follows:

$$\omega_\eta = \sqrt{4\omega_0^2 \sin^2\left(\frac{\eta L}{2}\right) + \omega_H^2}. \quad (14)$$

Figure 5 shows a general dispersion curve for the linear chain of the effective masses. As expected, there is only an acoustic branch in the spectrum of the chain, similar to the spectrum of a monoatomic linear chain.²² However, in our case there is a possibility to control the band gap width which is determined by the cyclotron frequency, ω_H . The frequency range of the acoustic wave which can propagate in the linear chain depends on both the cyclotron and the mechanical resonance frequencies. The low and upper boundaries of the propagating range are determined by ω_H and $\sqrt{4\omega_0^2 + \omega_H^2}$, respectively. Variation in frequency band structure is shown in Fig. 6 for several values of the ω_H/ω_0

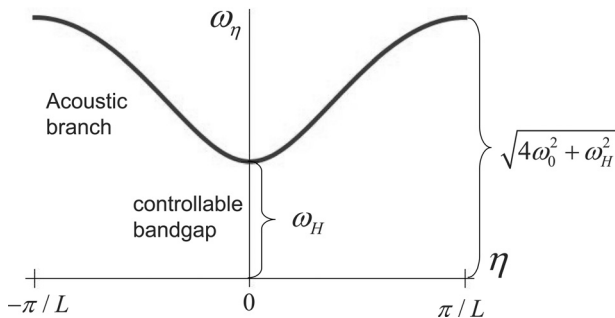


FIG. 5. Dispersion relation for one-dimensional chain composed of effective masses.

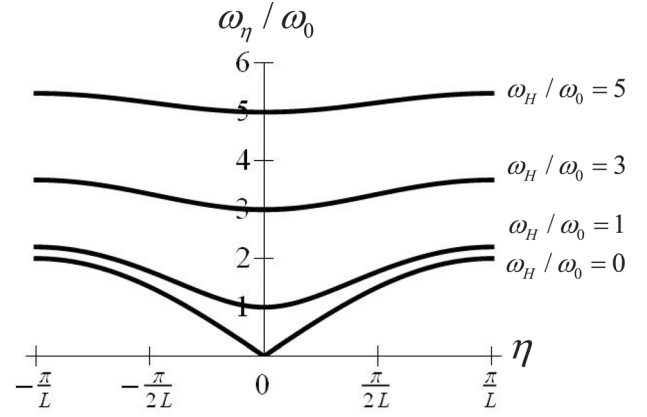


FIG. 6. Controllability of frequency band gap by external magnetic field in the linear chain of the effective masses.

ratio. The frequency range of the elastic waves propagating in the chain becomes narrower as the ω_H/ω_0 ratio is increasing.

The dispersion relationship, Eq. (14), points to appreciable attenuation of the wave propagating through the chain of the effective masses. This attenuation can be explained using the negative mass or the band gap paradigms. We focus on the long wavelength limit $\eta L \ll 1$ since the waves of the short wavelengths are attenuated in any case due to the mass-spring vibration isolation effect illustrated in Fig. 3.

In the long wavelength limit, $\eta L \ll 1$, Eq. (13) becomes

$$\omega_\eta = \sqrt{\frac{k}{m_{\text{eff}}}} \eta L. \quad (15)$$

Therefore the group velocity is

$$c = \frac{d\omega_\eta}{d\eta} = L\sqrt{\frac{k}{m_{\text{eff}}}}. \quad (16)$$

When $\omega_H^2 \gg \omega_\eta^2$ the effective mass becomes negative $m_{\text{eff}} \approx -m\omega_H^2/\omega_\eta^2$ and the wave vector, η , becomes purely imaginary

$$\eta = \frac{1}{L} \omega_\eta \sqrt{\frac{m_{\text{eff}}}{k}} = i \frac{\omega_H}{L\omega_0}. \quad (17)$$

That implies that there is no propagation of the elastic waves in the lattice.

The same conclusion can be reached using the band structure analysis. Thus, at $\eta L \ll 1$, Eq. (14) yields

$$\omega_\eta = \sqrt{\omega_0^2 \eta^2 L^2 + \omega_H^2}. \quad (18)$$

For $\omega_H^2 \gg \omega_0^2$, the dispersion relationship becomes $\omega_\eta \approx \omega_H$, that is, the frequency of the elastic wave is independent of the wave vector and the wave group velocity $c = d\omega_\eta/d\eta = 0$. Therefore there is no wave propagation.

IV. LINEAR CHAIN OF "MASS + EFFECTIVE MASS" UNIT CELLS

Now we consider a propagation of an acoustic wave in a linear chain of masses where each second mass is replaced by

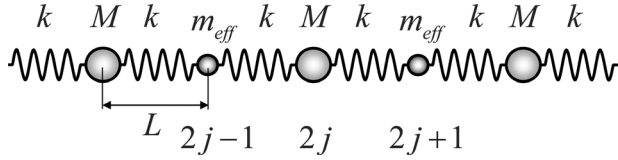


FIG. 7. Schematic of one-dimensional chain of the “mass + effective mass” unit cells.

an effective mass analyzed in the previous section. Figure 7 shows a fragment of the “mass + effective mass” chain.

The equations of motion of the linear chain in the quasi elastic approximation has the form

$$M\ddot{u}_{2j} + k(2u_{2j} - u_{2j-1} - u_{2j+1}) = 0, \quad (19)$$

$$m_{\text{eff}}\ddot{u}_{2j+1} + k(2u_{2j+1} - u_{2j} - u_{2j+2}) = 0, \quad (20)$$

where u_j is the displacement of the j -th mass.

Using the basic wave form solution

$$u_{2j,\eta} = u(\eta) \exp\{i(2j\eta L - \omega_\eta t)\}, \quad (21)$$

$$u_{2j+1,\eta} = v(\eta) \exp\{i[(2j+1)\eta L - \omega_\eta t]\}, \quad (22)$$

where L is the equilibrium distance between the masses, we obtain the characteristic equation

$$\begin{pmatrix} 2k - M\omega_\eta^2 & -2k \cos(\eta L) \\ -2k \cos(\eta L) & 2k - m_{\text{eff}}\omega_\eta^2 \end{pmatrix} \begin{pmatrix} u(\eta) \\ v(\eta) \end{pmatrix} = 0. \quad (23)$$

For a non-trivial solution the determinant of the matrix in Eq. (23) has to be equal to zero, yielding the following dispersion relation:

$$\omega_\eta^2 = k \left(\frac{1}{M} + \frac{1}{m_{\text{eff}}} \right) \pm \sqrt{k^2 \left(\frac{1}{M} + \frac{1}{m_{\text{eff}}} \right)^2 - \frac{4k^2}{Mm_{\text{eff}}} \sin^2(\eta L)}. \quad (24)$$

Taking into account that $m_{\text{eff}} = m(1 - \omega_H^2/\omega_\eta^2)$ and $k = m\omega_0^2$ we obtain

$$\omega_\eta^2 = \left(\omega_M^2 + \omega_0^2 + \frac{\omega_H^2}{2} \right) \pm \sqrt{\left(\omega_M^2 - \omega_0^2 - \frac{\omega_H^2}{2} \right)^2 + 4\omega_M^2\omega_0^2 \cos^2(\eta L)}, \quad (25)$$

where $\omega_M^2 = k/M$.

Figure 8 shows dispersion curves in the one-dimensional chain of the mass + effective mass unit cells for the case $M \gg m$ and $\omega_H > \omega_M$. Now there are two branches in the spectrum: optical and acoustic vibration branches, similar to the phonon vibrations in a crystal lattice.²³ The band gap between branches as well as band gap of acoustic branch is controllable by the external magnetic field amplitude.

Figure 9 shows variations in the frequency band structure of the chain for several values of the ω_H/ω_0 ratio. The

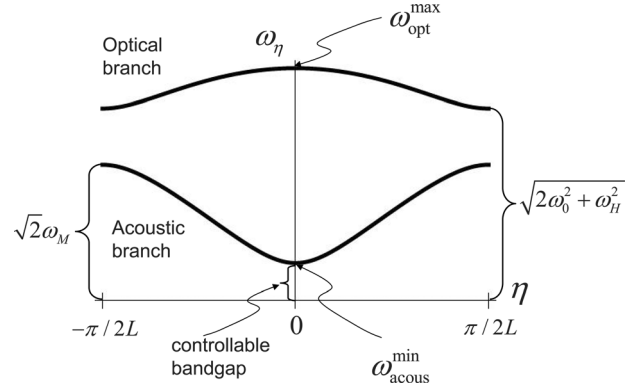


FIG. 8. Dispersion relation for one-dimensional chain composed of “mass + effective mass” unit cells.

wave frequency ω_η of the acoustic and the optical branches becomes independent of the wave vector when $\omega_H \gg \omega_0$. The acoustic branch becomes a horizontal line at $\omega_\eta = \sqrt{2}\omega_M$ while the optical branch becomes a horizontal line at $\omega_\eta = \omega_H$. The frequency range of the acoustic waves propagating in the chain is getting narrower as the ω_H/ω_0 ratio is increasing. For $\omega_H/\omega_0 \gg 1$ the wave propagation is almost completely suppressed.

V. CONTINUOUS MEDIA MODEL

The one-dimensional chain models discussed in the previous Sections are based on discrete elements but they can be adopted to introduce continuous elastic medium with effective mass density and elastic moduli. Here we consider a one-dimensional elastic medium with frequency-dependent effective mass density. The wave equation of the one-dimensional elastic medium can be written in the form¹⁵

$$E_{\text{eff}} \frac{\partial^2 u(x, t)}{\partial x^2} = \rho_{\text{eff}} \frac{\partial^2 u(x, t)}{\partial t^2}, \quad (26)$$

where $E_{\text{eff}} = Lk$ is the effective Young’s modulus which is determined by the stress-strain relation for a unit cell, and $\rho_{\text{eff}} = m_{\text{eff}}/L$ is the effective mass density.

Using the separation of variables method we obtain

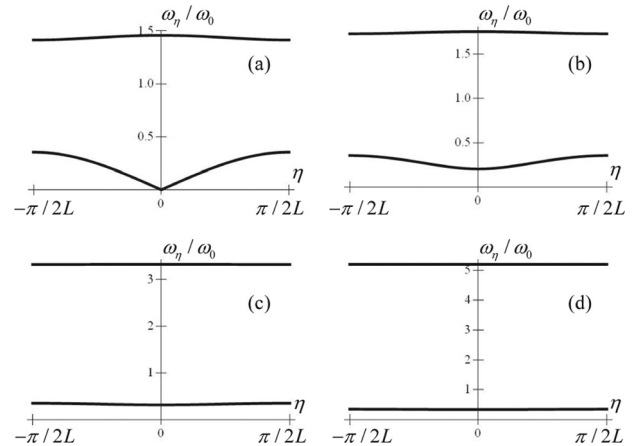


FIG. 9. Variations in the frequency band structure of the one-dimensional material representing by “mass + effective mass” chain; $m/M = 0.25$, (a) $\omega_H/\omega_0 = 0$, (b) $\omega_H/\omega_0 = 1$, (c) $\omega_H/\omega_0 = 3$, (d) $\omega_H/\omega_0 = 5$.

$$u(x, t) = A \exp \left\{ \pm i \left(\omega \sqrt{\frac{\rho_{\text{eff}}}{E_{\text{eff}}}} x \pm \omega t \right) \right\}$$

$$= A \exp \left\{ \pm i \left(\frac{\omega}{c} x \pm \omega t \right) \right\}, \quad (27)$$

where

$$c = \sqrt{\frac{E_{\text{eff}}}{\rho_{\text{eff}}}} = L \sqrt{\frac{k}{m_{\text{eff}}}} = \frac{L\omega_0}{\sqrt{1 - \omega_H^2/\omega^2}} \quad (28)$$

is the wave propagation velocity.

Equation (28) shows that the elastic wave propagation velocity is purely imaginary when the effective mass is negative, $\omega_H^2 \gg \omega^2$,

$$c \approx iL\omega_0 \frac{\omega}{\omega_H}, \quad (29)$$

and per the Eq. (27) the wave form is determined by

$$u(x, t) = A \exp \left\{ -\frac{\omega_H x}{\omega L} \pm i\omega t \right\}, \quad (30)$$

showing exponential attenuation of the elastic wave amplitude in a very broad range of frequencies determined by the cyclotron frequency ω_H .

VI. IMPLEMENTATION EXAMPLE

As an implementation example of the acoustic metamaterial, we consider a spherical electrically conductive particle coated with a dielectric material. The particle can be charged via attached to it energized electrode. The charge, q , carried by the particle is determined by its capacitance, C , and applied voltage, V ,

$$q = CV. \quad (31)$$

For a spherical conductor the capacitance is

$$C = 4\pi\epsilon_0\epsilon R, \quad (32)$$

where R is the radius of the sphere, ϵ is the dielectric constant (relative permittivity) of the sphere coating, and $\epsilon_0 = 8.85 \times 10^{-12}$ F/m is the vacuum permittivity. Now consider

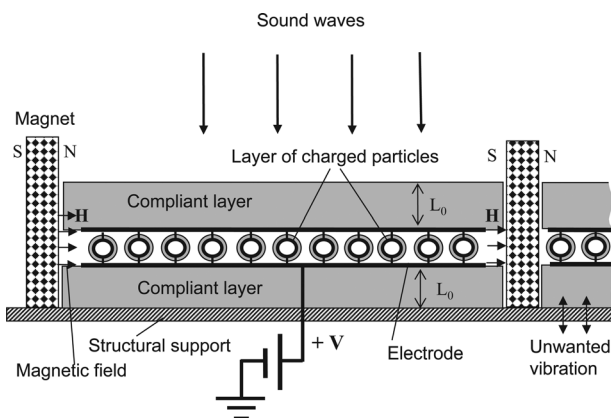


FIG. 10. Schematic diagram of the implementation example.

layer(s) of the particles attached to mechanically compliant layers as shown in Fig. 10. The particles are in magnetic field created by constant magnets. Such a structure could be used to reduce sound radiation from structure (due to structural support vibration) or protect the structure from incident sound waves.

The stiffness of the compliant layer is characterized by its Young's modulus, E , and it is subject of the Hook's law

$$\sigma = E\Delta\xi/L_0, \quad (33)$$

where σ is the normal stress (in the direction perpendicular the layer), $\Delta\xi$ is the layer's deformation, and L_0 is its thickness. From Eq. (33) it follows that the stiffness per unit area, K , is

$$K = E/L_0, \quad (34)$$

and the mechanical resonance frequency of the particle layer loaded with mass per unit area, M , is

$$\omega_0 = \sqrt{K/M} = \sqrt{\frac{E}{L_0 M}}. \quad (35)$$

Assume that the particles are uniformly distributed within the particle layer and spaced at the distance $2R$ from each other. Then $M = m/(4\pi R^2)$, where m is the particle mass. For solid sphere $m = \rho 4\pi R^3/3$. Here ρ is the density of the sphere and mass of the dielectric coating is neglected. Under these assumptions the mechanical and cyclotron resonance frequencies are

$$\omega_0 = \sqrt{\frac{3E}{L_0 \rho R^3}}, \quad (36)$$

$$\omega_H = \frac{3\epsilon_0 \epsilon V H}{\rho R^2}. \quad (37)$$

Note, that the sound or vibration suppression is taking place if

$$\omega_0 \ll \omega_H. \quad (38)$$

As an example consider an aluminum ($\rho = 2700$ kg/m³) sphere with the diameter $2R = 0.1$ μ m, coated with polyani-lyene/polyurethane blend,²⁴ $\epsilon = 1120$, charged with 400 V, and placed in magnetic field of 0.25 T produced by a typical permanent rare-earth magnet. The cyclotron frequency for this particle is $\omega_H/2\pi = 70.15$ kHz. In order to satisfy vibration suppression condition Eq. (38), we assume $\omega_0 = 0.1 \omega_H$ and from Eq. (36) obtain requirements for the compliant layer parameters

$$E/L_0 = \omega_0^2 \rho R^3/3 = 0.01 \omega_H^2 \rho R^3/3. \quad (39)$$

In our example $E/L_0 = 87331$ Pa/m. For $E = 225$ Pa (glass fiber), the thickness of the layer will be 2.6 mm as follows from Eq. (39). Using the metamaterial described above ($\omega_H = 10 \omega_0$) we see from Fig. 3 that the intensity of the acoustic waves below frequency $7 \omega_0$ are reduced by 40–60 dB

just within a single layer structure. This exhibits the possibility of constructing a multi-layer metamaterial based on this design. The multi-layered structure relaxes the condition Eq. (38) allowing for the decrease of the external magnetic field and particle charges (which can be more favorable in practice) without loss of attenuation efficiency.

VII. CONCLUSION

This paper is an initial step toward development and implementation of the proposed concept. There are still many interesting but unanswered questions. For example, two- or three-dimensional lattice and its response to external stimulus (vibration or acoustic waves) with arbitrary angular dependence, structural response to time/space dependent magnetic field or electric charges, electrical interaction between charged particles, and many others.

These questions emphasize that the proposed approach with combined electro-magnetic cyclotron and mechanical resonances offers many opportunities for innovative design of acoustic metamaterials. As discussed, it could be utilized as a superior sound and vibration suppressor in a very broad frequency range. Controlling applied voltage and magnetic field may enable even more advanced capabilities such as re-programmable acoustic characteristics of the material, super-lensing, and cloaking.

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