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Improvement of vector fitting by using a new method for selection of starting poles

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ABSTRACT

Vector fitting (VF) is known as the most popular method for frequency-dependent modelling. Using VF, frequency-domain data of a network can be converted into an equivalent circuit model. Some initial poles need to be selected before VF can be started. The starting poles affect the accuracy and convergence speed of the vector fitting method. In this paper, a new procedure is proposed to select starting poles. The proposed procedure selects starting poles by partitioning the frequency response, and then ranks the starting poles so that the most dominant poles can be used when low order approximation is desired. Case study results show that the proposed procedure improves the accuracy and the speed of VF significantly.

1. Introduction

The study of power quality often requires the detailed modelling of complex networks. However, detailed representations may lead to an excessive computation burden. A common practice to reduce the computational burden is to divide a system into a study zone and an external system encompassing the rest of the system [1]. The external system is commonly represented by a Frequency-Dependent Network Equivalent (FDNE) circuit [2,3]. Using equivalent circuit models in a general simulation environment is straightforward, enabling fast simulation in both time and frequency domain [2,4]. The equivalent network should be developed based on the frequency domain data of the system. The frequency domain data is the tabulated data including impedances and voltage harmonics for different frequencies. This data can be generated by using measurements or power quality simulation tools.

Vector fitting (VF) with its several formulations [3–10] is the most commonly used method for frequency-dependent modelling. Although new methods based on evolutionary computation such as Refs. [11–13] have been recently proposed, VF is still the most popular method due to its simplicity and robustness. VF gets the given frequency-response data and finds a rational function approximation with guaranteed stability. The identified rational function can then be converted to an equivalent electric circuit model using the algorithm proposed in Ref. [4].

To start the vector fitting method, some initial poles should be selected. The initial poles strongly affect the accuracy and convergence speed of the vector fitting method. The most commonly used procedure for the selection of starting poles in the one proposed in Ref. [3]. This procedure suggests that the starting poles should be complex conjugate with the imaginary parts linearly or logarithmically distributed over the frequency range of interest. By using this method, however, the starting poles might be very far from the actual poles. This will result in two problems:

- A high order approximation with a large number of iterations may be required to achieve an acceptable accuracy.
- In practical applications, one often wants to use a low order approximation for a high order function. In this case, the method might not be able to find the most dominant poles, resulting in a larger approximation error.

The above problems can be significantly magnified when the frequency response contains noise. Experience with the VF algorithm has shown that the existence of noise can significantly impair convergence, leading to possibly inaccurate models due to the presence of noise-induced spurious poles [16]. As discussed in Ref. [17], this problem is also related to the pole relocation of VF which may fail to reach the optimum poles. During the pole relocation, the poles may be relocated in small steps and the convergence may even stall. Noise-induced spurious poles may be identified by VF instead of actual dominant poles, resulting in inaccurate models.

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This paper aims to alleviate the above problems by proposing a new procedure for starting pole selection. The proposed procedure selects starting poles by partitioning the frequency response, and then ranks the starting poles so that the most dominant poles can be used when low order approximation is desired. The detailed concept and algorithms of the proposed procedure are presented in this paper. The effectiveness of the procedure is also confirmed using different case studies.

2. Vector fitting method

The aim of vector fitting is to find a rational function that approximates the frequency-domain data of the system. VF is an iterative technique based on pole-zero relocation technique. This method is reviewed in this section.

The frequency response $G(s)$ can be represented using rational functions. For an Nth order system, the rational function can be written as:

$$G(s) \approx \sum_{n=1}^{N} \frac{c_n}{s - p_n} + d + se \tag{1}$$

where the residues cn and poles pn are either real quantities or come in complex conjugate pairs, while e and d are real, and the problem is to estimate all coefficients in Eq. (1). Clearly, the optimization problem is non-linear in the poles p_i. To solve this problem, Gustavsen and Semlyen [3] propose to identify the parameters indirectly. For this purpose, two other transfer functions $\sigma(s)$ and $H(s)$ are defined as follows:

$$\sigma(s) = \sum_{n=1}^{N} \frac{\widehat{c}_n}{s - \widehat{p}_n} + 1 \tag{2}$$

$$H(s) = \sigma(s)G(s) \approx \sum_{n=1}^{N} \frac{c_n}{s - \widehat{p}_n} + d + se \tag{3}$$

$H(s)$ has the same structure as $G(s)$ and $\sigma(s)$ has a unit gain in high frequencies. As seen in Eqs. (2) and (3), $G(s)$ and $\sigma(s)$ have identical poles. These poles are supposed to be known at the beginning of each iteration.

2.1. Identification of parameters in $\sigma(s)$ and $H(s)$

By substituting Eq. (2) in Eq. (3), we will have:

$$\left(\sum_{n=1}^{N} \frac{\widehat{c}_n}{s - \widehat{p}_n} + 1 \right) G(s) \approx \sum_{n=1}^{N} \frac{c_n}{s - \widehat{p}_n} + d + se \tag{4}$$

Eq. (4) can be re-written as follows:

$$\left(\sum_{n=1}^{N} \frac{c_n}{s - \widehat{p}_n} + d + se \right) - \left(\sum_{n=1}^{N} \frac{\widehat{c}_n}{s - \widehat{p}_n} \right) G(s) \approx G(s) \tag{5}$$

In matrix forms, it can be rewritten as:

$$\begin{bmatrix} \frac{1}{s_1 - \widehat{p}_1} & \cdots & \frac{1}{s_1 - \widehat{p}_N} & \frac{-G(s_1)}{s_1 - \widehat{p}_1} & \cdots & \frac{-G(s_1)}{s_1 - \widehat{p}_N} & 1 & s_1 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \frac{1}{s_k - \widehat{p}_1} & \cdots & \frac{1}{s_k - \widehat{p}_N} & \frac{-G(s_k)}{s_1 - \widehat{p}_1} & \cdots & \frac{-G(s_k)}{s_1 - \widehat{p}_N} & 1 & s_k \end{bmatrix} \begin{bmatrix} c_1 \\ \vdots \\ c_N \\ \widehat{c}_1 \\ \vdots \\ \widehat{c}_N \\ d \\ e \end{bmatrix} = \begin{bmatrix} G(s_1) \\ \vdots \\ \vdots \\ G(s_k) \end{bmatrix} \tag{6}$$

where k is the number of data points in the frequency-domain data.

As explained before, $\widehat{p}_i$, the starting poles, are known. Therefore, Eq. (6) is a linear equation with respect to $\widehat{c}_n$, c_n, d and e. Thus, the least square method can be employed to obtain these parameters. By identification of these parameters, $\sigma(s)$ and $H(s)$ are determined.

2.2. Identification of poles in $G(s)$

Transfer functions $H(s)$ and $\sigma(s)$ can be written as:

$$H(s) = \sigma(s)G(s) = \frac{\prod_{n=1}^{N+1}(s - z_n)}{\prod_{n=1}^{N}(s - \widehat{p}_n)} \tag{7}$$

$$\sigma(s) = \frac{\prod_{n=1}^{N}(s - \widehat{z}_n)}{\prod_{n=1}^{N}(s - \widehat{p}_n)} \tag{8}$$

Using Eqs. (7) and (8), $G(s)$ can be written as follows:

$$G(s) = \frac{\sigma(s)G(s)}{\sigma(s)} = \frac{\prod_{n=1}^{N+1}(s - z_n)}{\prod_{n=1}^{N}(s - \widehat{z}_n)} \tag{9}$$

Eq. (9) indicates that poles of $G(s)$ are identical with the zeros of the estimated $\sigma(s)$. It should be noted that the starting poles are cancelled out because the same poles are used to estimate $H(s)$ and $\sigma(s)$. Since $\sigma(s)$ has been identified in the previous step, the poles of $G(s)$ which are equal to the poles of $G(s)$ are easily found.

2.3. Stability of the model

If during the calculation of poles of $G(s)$, some unstable poles are found, they should be made stable in order to ensure $G(s)$ stability. At this point, unstable poles, poles with a positive real part, should be modified to be stable poles with negative real parts. This can be easily done by changing the sign of the real part of the unstable poles [14]. This procedure ensures the stability of the approximated function.

2.4. Identification of residues of G(s)

As a result of the previous steps, the poles of the system are identified. The next step is to identify the residues and the constant terms. These parameters can be calculated from

$$G(s) = \sum_{n=1}^{N} \frac{c_n}{s - p_n} + d + se \tag{10}$$

In this equation, the poles *pn* have already been found. Only *cn*, *d*, and e have been remained unknown. In matrix form, Eq. (10) can be rewritten as [15]:

$$\begin{bmatrix} \frac{1}{s_1 - p_1} & \cdots & \frac{1}{s_1 - p_N} & 1 & s_1 \\ \vdots & \ddots & \vdots & \vdots & \vdots \\ \frac{1}{s_k - p_1} & \cdots & \frac{1}{s_k - p_N} & 1 & s_k \end{bmatrix} \begin{bmatrix} c_1 \\ \vdots \\ c_N \\ d \\ e \end{bmatrix} = \begin{bmatrix} G(s_1) \\ \vdots \\ G(s_k) \end{bmatrix} \tag{11}$$

Therefore, by using the above equation, all the unknown residues *cn* and constant terms *d* and *e* are obtained.

According to the above steps, the vector fitting algorithm can be represented as the flowchart shown in Fig. 1.

3. Problem definition

As explained in the previous section, some starting poles should be selected in order to start the vector fitting method. The starting poles strongly affect the accuracy and convergence speed of the vector fitting method. In Ref. [3], a procedure for the selection of starting poles has been recommended. The proposed procedure suggests that the starting poles should be complex conjugate with the imaginary parts covering the frequency range. Also the real parts of the starting poles should be small enough in order to avoid the ill-conditioned least square problem to be solved. As a result, each pair of the starting poles can be found according to the following equation.

$$p_n = -\alpha + j\beta, \quad p_{n+1} = -\alpha - j\beta \tag{12}$$

where $\alpha = (\beta/100)$ and β is determined to be linearly or logarithmically distributed over the frequency range of interest.

By using this kind of starting poles selection, the starting poles might be very far from the actual poles. This might result in two problems:

- A high order approximation with a large number of iterations may be required to achieve an acceptable accuracy.
- In practical applications, one often wants to use a low order approximation for a high order function. In this case, the method might not be able to find the most dominant poles, resulting in a large approximation error.

As an example, let us consider a simple transfer function as shown in Eq. (13).

$$(s) = \frac{1}{s+5} + \frac{30 + j40}{s - (-50 + j500)} + \frac{30 - j40}{s - (-50 - j500)} + \frac{10 + j20}{s - (-50 + j800)} + \frac{10 - j20}{s - (-50 - j800)} + \frac{1}{2} \tag{13}$$

This transfer function has one real pole (−5) and two complex conjugate poles (−50 ± *j*500, −50 ± *j*800). The frequency response of the transfer function is shown in Fig. 2. As seen in Fig. 2, the complex conjugate poles which result in the two resonance peaks are the dominant poles.

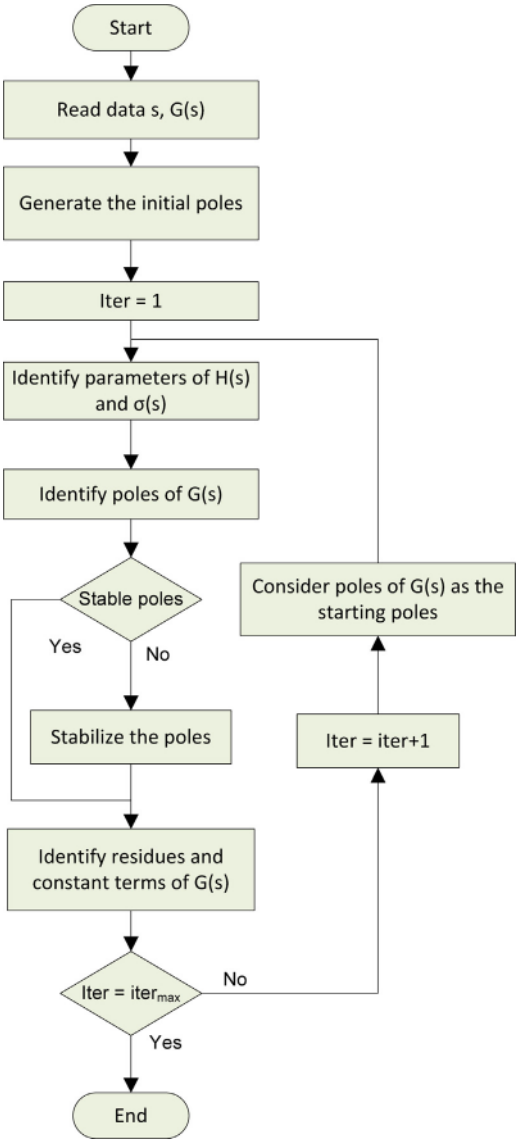


Fig. 1. Flowchart of the vector fitting algorithm.

If the vector fitting method with the typical procedure of starting poles selection is used to approximate the function with a 4th order approximation, Fig. 3 is obtained. Note that FRVF in this figure stands for fast relaxed vector fitting which is an improved version

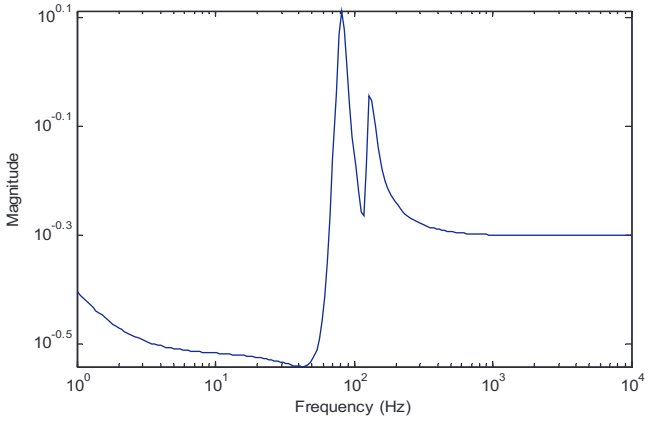


Fig. 2. The frequency response of the transfer function.

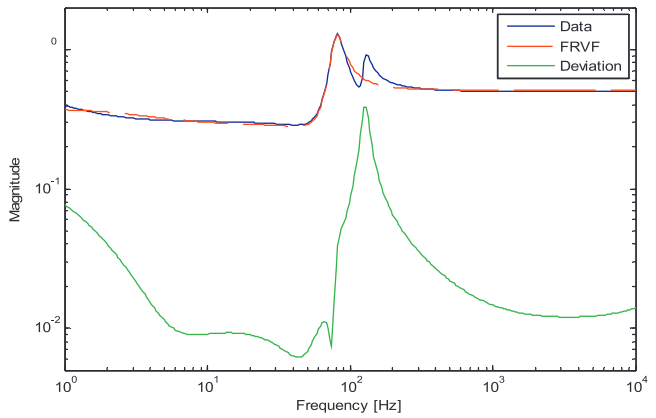


Fig. 3. Results of the vector fitting method using the typical method of starting poles selection.

of VF introduced in Ref. [5]. As seen in this figure, the method has failed to capture the characteristics of the transfer function at one of the resonance peaks, resulting in high approximation errors around that peak. This has occurred because the method has failed to identify one pair of the dominant poles (see Table 1).

In order to overcome these problems, a new method is proposed for selection of starting poles.

4. Proposed starting poles selection method

The starting poles should be close to the actual complex conjugate poles as much as possible. Consider the rational function approximation as follows:

$$f(j\omega) = \frac{c_k^{'} + jc_k^{''}}{j\omega - (-a_k^{'} + ja_k^{''})} + \frac{c_k^{'} - jc_k^{''}}{j\omega - (-a_k^{'} - ja_k^{''})} + \sum_{i=1}^{M} \frac{c_i}{j\omega - (-a_i)} + \sum_{\substack{i=1 \\ i \neq k}}^{\frac{1}{2}(N-M)} \left(\frac{c_i^{'} + jc_i^{''}}{j\omega - (-a_i^{'} + ja_i^{''})} + \frac{c_i^{'} - jc_i^{''}}{j\omega - (-a_i^{'} - ja_i^{''})} \right) + d + sh \tag{14}$$

where N is the total number of poles, M is the number of real poles, and $(N-M)/2$ is the number of pairs of complex conjugate poles.

In the above equation, the kth pair of complex poles has been extracted from the summation. Assume that the frequency is not too low and the imaginary parts of complex poles are not too close to each other. In this situation, the kth pair of complex poles more likely results in a resonance peak in the frequency response when the frequency is close to $a_k^{''}(\omega \approx a_k^{''})$ because the denominator of $(c_k^{'} + jc_k^{''})/(j\omega - (-a_k^{'} + ja_k^{''}))$ will be much smaller than the other denominators in Eq. (14). This gives us the idea to select the starting poles as follows:

- Find the frequencies ($freq_{\max}$) at which the frequency response has local maximums.

Table 1
Actual and estimated poles of the transfer function.

Actual poles	Identified poles by vector fitting method
$-50+j500$	$-52.49+j496.9$
$-50-j500$	$-52.49-j496.9$
$-50+j800$	-16.375
$-50-j800$	-720.25
-5	

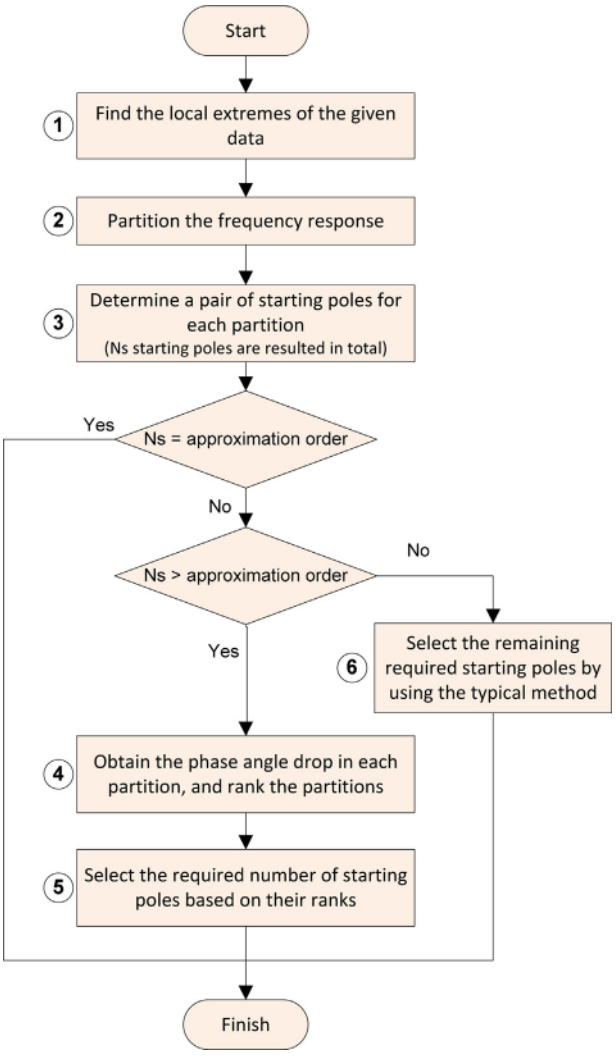


Fig. 4. The procedure for selection of starting poles.

- Set the imaginary part of the starting pole as $\beta = 2\pi \times freq_{\max}$.
- Set the starting poles as $p_i = -\alpha_i \pm j\beta_i$, where $\alpha_i = (\beta_i/100)$.

Assume that using the above procedure, Ns starting poles are determined. Then we might have one of the three following cases:

Case 1: The desired approximation order is equal to Ns. In this case, the obtained starting poles are used and VF is performed.
Case 2: The desired approximation order is larger than Ns. In this case, some more starting poles are required to perform VF. For this purpose, it is suggested that the remaining required starting poles are selected using the typical method as explained before.
Case 3: The desired approximation order is smaller than Ns. In this case, the obtained starting poles need to be ranked so that according to the desired approximation order, the most dominant poles can be selected and used. For this purpose, a procedure for ranking the starting poles is proposed in the following.

One approach is to rank the starting poles according to the magnitude of the frequency response at the associated local maximum. However, according to Eq. (14), the magnitude of each local maximum is affected not only by the associated pole but also by the other poles. In order to decrease the effect of the other poles, the variation of the phase angle of the frequency response can be used for ranking.

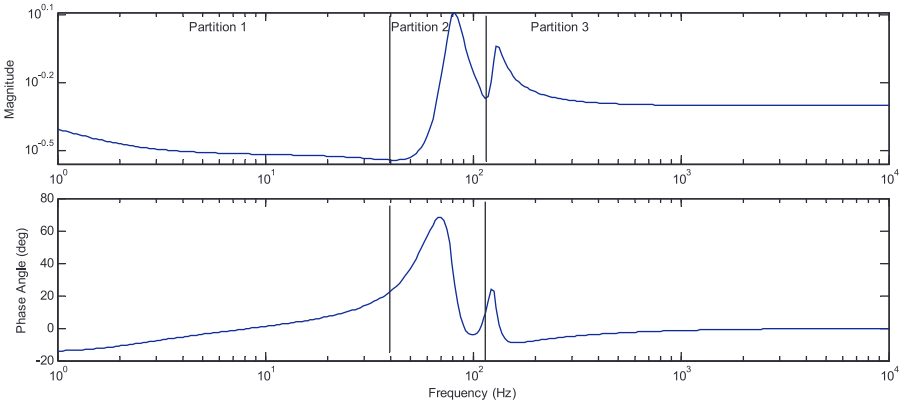


Fig. 5. Partitioning of the frequency response.

According to Eq. (14), when the frequency (ω) is reaching a_k'' and passing it, a large drop happens in the phase angle of the term associated with the kth pole i.e. $(c_k' + jc_k'')/(j\omega - (-a_k' + ja_k''))$ while the phase angle of the terms associated with the other poles do not change a lot. This drop makes the phase angle of the frequency response decrease significantly if the associated pole is a dominant pole. In other words, the more dominant the pole is, the larger the drop that the phase angle of the frequency response experiences is. Therefore, the amount of the phase angle drop around each pole can be used to rank the poles.

According to what was explained above, a procedure as shown in Fig. 4 is proposed for the selection of starting poles.

Step 1: local maximum and minimum points of the given data are found, and the frequencies at which these local extremes occur are stored.
Step 2: The frequency response is portioned so that each partition has one local maximum and two local minimums. Only the first and the last partitions may have only one local maximum and one local minimum.
Step 3: A pair of complex conjugate poles is determined for each partition based on the frequency ($freq_{max}$) at which the local maximum occurs; $p = -\alpha \pm j\beta$ where $\alpha = (\beta/100)$ and $\beta = 2\pi \times freq_{max}$.
Step 4: The phase angle drop in each partition is obtained, and the partitions are ranked. For each partition, the larger the drop is, the better the rank is.
Step 5: According to the desired order of approximation and the ranking obtained at *Step 4*, the required numbers of starting poles are selected from the poles obtained in *Step 3*.
Step 6: The remaining required starting poles are selected using the typical method ($p_i = -\alpha_i \pm j\beta_i$ where $\alpha = (\beta/100)$ and β is determined to be logarithmically distributed over the frequency range of interest.).

5. Case study results

The effect of the explained procedure on the vector fitting method is analyzed for two case studies.

5.1. Case 1 – the transfer function in Eq. (13)

The frequency response is partitioned as shown in Fig. 5. Also, the phase angle which is used for partition ranking is shown in this figure. The ranking of the partitions and the calculated starting poles associated for each partition is shown in Table 2.

If a 4th order approximation is desired, 4 poles are needed. Therefore, according to Table 2, the poles associated with partitions 2 and 3 should be used as the starting poles. If vector fitting with this starting poles selection is used, Fig. 6 is obtained. Comparison of Fig. 6 and Fig. 3 reveals that the proposed procedure improves the result of the vector fitting significantly. Also, according to Table 3, the explained procedure has a significant effect of on the accuracy and convergence of the vector fitting. Note that, relative RMS error (rRMSE) shown in Eq. (15) has been used for the accuracy comparison.

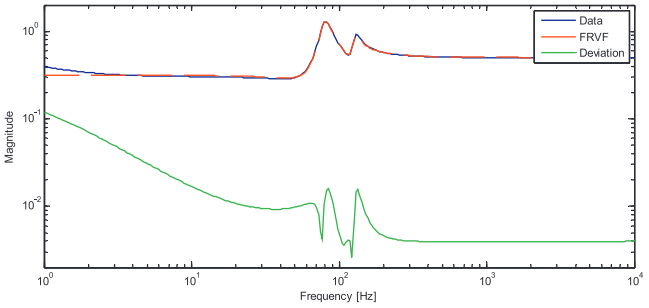


Fig. 6. Results with frequency partitioning.

$$\text{rRMSE} = \sqrt{\frac{1}{N}\sum_{k=1}^{N}\left(\frac{f_k - \hat{f}_k}{f_k}\right)^2} \tag{15}$$

where N is the number of the data points, f_k is the magnitude of the kth data point, and $\hat{f}_k$ is the estimated magnitude of the kth data point.

Table 2
Partition ranking and estimated starting poles of each partition.

Rank	Partition no.	Frequency of max point (Hz)	Estimated starting poles
1	2	81.85	$-5.1428 \pm j514.28$
2	3	129.90	$-8.1619 \pm j816.19$
3	1	1	$-6.283 \times 10^{-2} \pm j6.283$

Table 3
Effect of the proposed procedure on the accuracy and convergence of the vector fitting.

	Without the proposed procedure	With the proposed procedure
rRMSE	5.683e−2	4.415e−2
No. of iterations	11	5

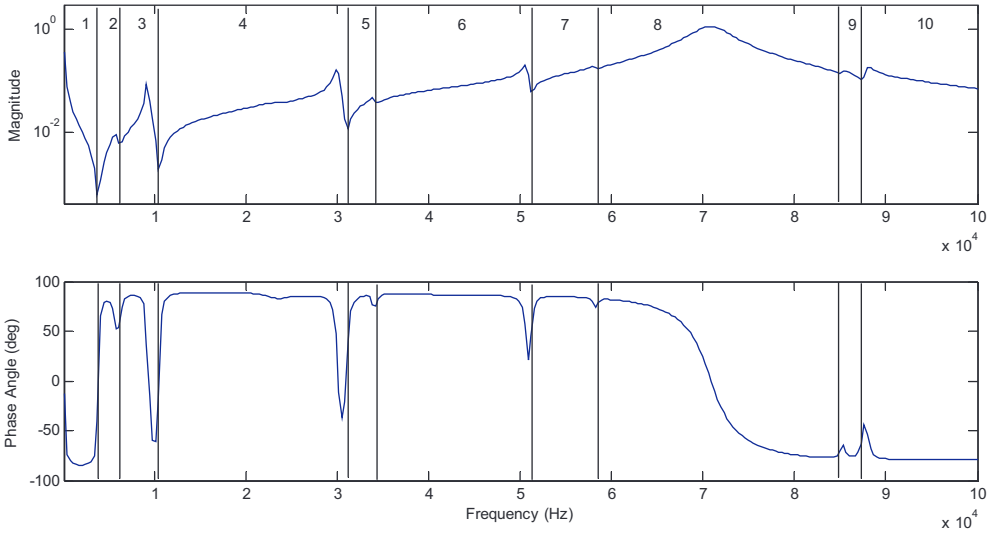
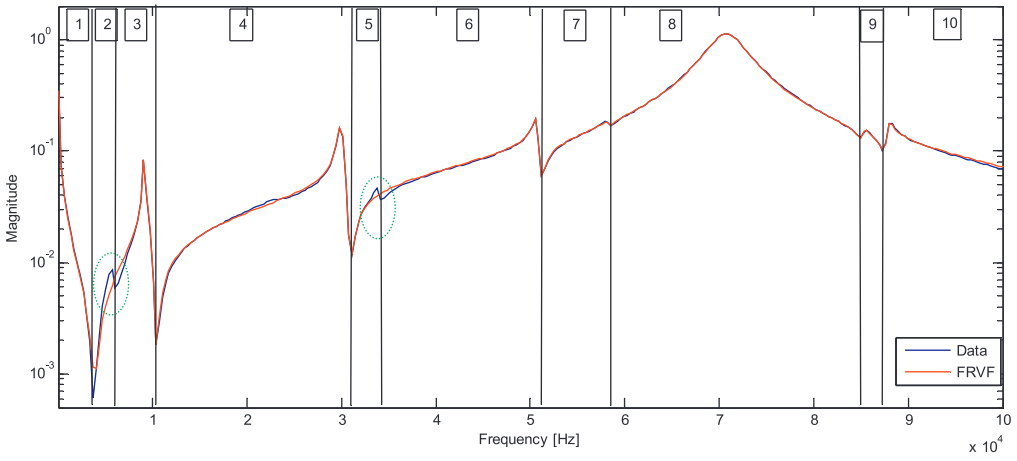


Fig. 7. Partitioning of the frequency response.

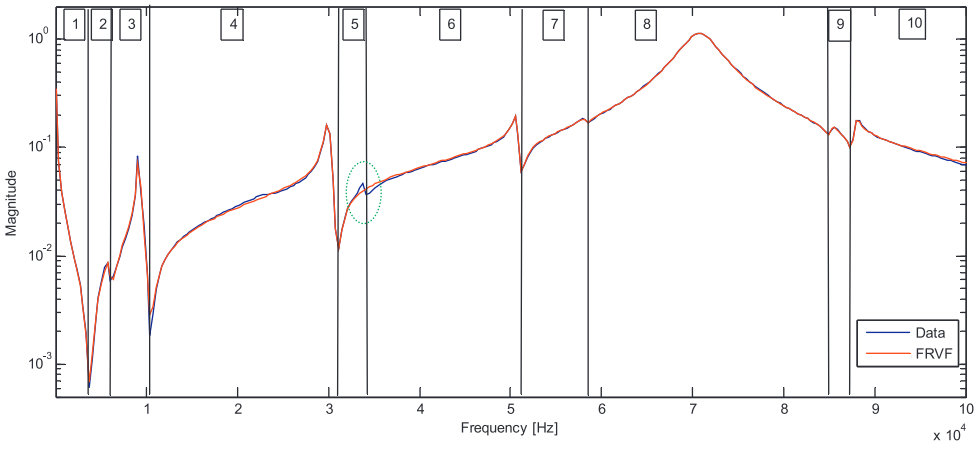
5.2. Case 2 – a distribution system

The calculated equivalent admittance of a distribution system provided in Ref. [9] is used for this case study. The frequency response is partitioned as shown in Fig. 7. The ranking of the partitions is shown in Table 4.

The results of the vector fitting method with and without using the proposed procedure for the 18th approximation order are



A: without the proposed procedure



B: with the proposed procedure

Fig. 8. Results for the 18th order with and without the proposed procedure.

Table 4
Partition ranking.

Rank	Partition no.	Frequency of max point (Hz)
1	8	1.0000e1
2	3	5.6950e3
3	4	9.0390e3
4	1	2.9770e4
5	6	3.3790e4
6	10	5.0510e4
7	2	5.7860e4
8	7	7.0910e4
9	9	8.5620e4
10	5	8.7960e4

Table 5
The effect of the proposed procedure on the accuracy and convergence of VF (18th order).

	Without the proposed procedure	With the proposed procedure
rRMSE	7.49e−2	4.77e−2
No. of iterations	11	4

Table 6
The effect of the proposed procedure on the accuracy and convergence of VF (16th order).

	Without the proposed procedure	With the proposed procedure
rRMSE	7.55e−2	5.12e−2
No. of iterations	7	6

Table 7
The effect of the proposed procedure on the accuracy and convergence of VF (the 20th order).

	Without the proposed procedure	With the proposed procedure
rRMSE	4.23e−2	4.23e−2
No. of iterations	12	4

shown in Fig. 8. As seen in this figure, without using the proposed procedure, the local maximums and minimums of partitions 2 and 5 are not captured. However, when the proposed procedure is used, as expected according to Table 4, only the local maximum and minimum of partition 5 are missed. As a result, by using the proposed procedure, the accuracy of the vector fitting is improved significantly. Table 5 shows the effect of the procedure on the rRMSE and convergence speed. As seen in this table, the proposed procedure decreases the error of the vector fitting method considerably. Also, it makes the method converge much faster.

Similarly, Table 6 presents the comparisons results for the 16th order approximation. It can be seen that the proposed procedure improves the accuracy of VF.

If the order of approximation is high enough i.e. if the order of approximation is at least equal to 20 in this system, similar results are obtained with and without the proposed procedure. However, results are achieved much faster if the proposed procedure is used. For example, Table 7 shows the results for the 20th approximation order. As seen in this table, by using the proposed procedure, the number of iterations is one third of when the procedure is not used.

6. Conclusion

A new procedure to select starting poles for vector fitting (VF) was proposed in this paper. The proposed procedure selects starting poles by partitioning the frequency response and ranks the starting poles so that the most dominant poles can be used when low order approximation is desired. The procedure makes two improvements in VF.

If the order of approximation is high enough to capture all resonance peaks of the given frequency-response, the proposed procedure makes the vector fitting converge much faster.

If the order of approximation is not high enough to capture all resonance peaks of the given data, the proposed procedure makes the vector fitting identify the most dominant poles of the system, resulting in a high accuracy. This is a significant improvement especially in practical applications when we work on measured data and an equivalent model with an enough high order is not practical. In this situation, the proposed method would lead to more accurate equivalents.

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