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Short communication

## On the method of Hunt's parameter calibration

Noori Kim <sup>a,\*</sup>, Jont B. Allen <sup>b</sup><sup>a</sup> School of Mechanical and Aerospace Engineering, Nanyang Technological University, 50 Nanyang Ave, Singapore 639798, Singapore<sup>b</sup> Department of Electrical and Computer Engineering, University of Illinois at Urbana-Champaign, 1206 W. Green Street, Urbana, IL 61801, USA

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## ABSTRACT

This note comments on the observations of Bernier et al. (2016) regarding errors in Appendix A of Kim and Allen (2013). We acknowledge that the equations in the Appendix are in error, but wish to point out that these equations were not actually used for our analysis. We appreciate their effort in pointing out the errors, and offering corrected equations.

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## 1. Introduction

In Kim and Allen (2013) the Hunt parameters of the BAR receiver (Hunt, 1954) were calculated based on the electrical impedance measurements, by varying acoustic loads and then were then directly compared to model simulation. As indicated by Bernier et al. (2016), there are mathematical errors in appendix A of Kim and Allen (2013).

While the equations of Appendix A are indeed incorrect, they were not the equations that were actually used in our analysis. Appendix was added as an afterthought during the preparation of the manuscript, and obviously, the final result was not properly proof-read. The Appendix was intended as a quick guide to demonstrate that the equations could be solved, but clearly it is not helpful to provide solutions that are wrong. If we had used the equations of Appendix A in our analysis, we would not have been able to match the data with experiment. If we had done so, we would have quickly found these errors.

Since the publication of (Kim and Allen, 2013) we have discovered that these Equations and their solution have been previously discussed by (Ramo et al., 1965, p. 543). Below we compare their solution to that of (Bernier et al., 2016).

## 2. Summary of Ramo et al. (1965) vs. Bernier et al. (2016)

## 2.1. Equations from Ramo et al. (1965)

The equations from Ramo et al. (1965) are

\* Corresponding author. Current address: Department of Electrical and Computer Engineering, DigiPen Institute of Technology, 510 Dover Road, #03-01 SIT@SP Building, Singapore 139660, Singapore.

E-mail address: [noorimail@gmail.com](mailto:noorimail@gmail.com) (N. Kim).

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$$Z_{ip} = Z_{11} - \frac{Z_{21}Z_{12}}{Z_{22} + Z_{lp}}, \quad p = 1, 2, 3,$$

where  $Z_{ip}$  and  $Z_{lp}$  are the three input and output impedances, respectively. For the anti-reciprocal case  $Z_{12} = -Z_{21}$ . One must also be careful with the definition of the sign of the output voltage  $V_2$  to obey both the definition of the impedance and that of the Transmission matrix.

Ramo et al. (1965, p. 543) provide the formulas for the Hunt parameters

$$Z_{11} = \frac{(Z_{i1} - Z_{i3})(Z_{i1}Z_{L1} - Z_{i2}Z_{L2}) - (Z_{i1} - Z_{i2})(Z_{i1}Z_{L1} - Z_{i3}Z_{L3})}{(Z_{i1} - Z_{i3})(Z_{L1} - Z_{L2}) - (Z_{i1} - Z_{i2})(Z_{L1} - Z_{L3})}$$

$$Z_{22} = \frac{(Z_{i1}Z_{L1} - Z_{i2}Z_{L2}) - Z_{11}(Z_{L1} - Z_{L2})}{Z_{i2} - Z_{i1}}$$

$$Z_{12}^2 = (Z_{11} - Z_{ip})(Z_{22} - Z_{lp}), \quad p = 1, 2, 3$$

## 2.2. Equations from Kim and Allen (2013)

Three measured electrical impedances which include three unknown Hunt parameters ( $Z_e$ ,  $Z_a$  and  $T_a$ ) are

$$Z_{in|q} = \frac{E}{I} = Z_e + \frac{T_a^2}{Z_a + Z_{l|q}}, \quad q = A, B, C \quad (1)$$

From these equations we wish to solve for the Hunt parameters  $Z_a$ ,  $T_a$ , and  $Z_e$ , via the following procedure:

- a. Subtract two electrical impedance measuring data to eliminate  $Z_e$ ,

$$Z_{in|C} - Z_{in|A} = \frac{T_a^2}{Z_a + Z_{L|C}} - \frac{T_a^2}{Z_a + Z_{L|A}}. \quad (2)$$

- b. Take the ratio of various different terms defined in step a, thereby removing  $T_a^2$ :

$$\frac{(Z_{in|B} - Z_{in|C})}{(Z_{in|C} - Z_{in|A})} = \frac{(Z_{L|A} + Z_a)}{(Z_{L|B} + Z_a)} \frac{(Z_{L|B} - Z_{L|C})}{(Z_{L|C} - Z_{L|A})}.$$

From this we may solve for the first unknown parameter  $Z_a$ ,

$$Z_a = \frac{X \cdot Z_{L|A} - Z_{L|B}}{1 - X}, \quad (3)$$

$$\text{where } X = \frac{(Z_{L|B} - Z_{L|C})}{(Z_{L|C} - Z_{L|A})} \frac{(Z_{in|C} - Z_{in|A})}{(Z_{in|B} - Z_{in|C})}.$$

- c. Next we find  $T_a^2$  by substituting  $Z_a$  into Eq. (2)

$$T_a^2 = \frac{(Z_{in|C} - Z_{in|A})(Z_a + Z_{L|C})(Z_a + Z_{L|A})}{Z_{L|A} - Z_{L|C}}. \quad (4)$$

The sign of  $T_a$  cannot be decided by electrical input impedance measurements alone as only  $T_a^2$  term participates in Eq. (2).

- d. Finally  $Z_e$  is given by Eq. (1)

$$Z_e = Z_{in|A} - \left( \frac{T_a^2}{Z_{L|A} + Z_a} \right). \quad (5)$$

Specific solutions for  $Z_e$ ,  $Z_a$ , and  $T_a^2$  may vary based on the dependence of the terms. For example, if one uses the expression for  $T_a^2$  in the Eq. for  $Z_e$ , the relationship will appear quite different, resulting in the equations of Ramo et al. (1965):

$$Z_e = \frac{(Z_{in|A} - Z_{in|C})(Z_{in|A}Z_{L|A} - Z_{in|B}Z_{L|B}) - (Z_{in|A} - Z_{in|B})(Z_{in|A}Z_{L|A} - Z_{in|C}Z_{L|C})}{(Z_{in|A} - Z_{in|C})(Z_{L|A} - Z_{L|B}) - (Z_{in|A} - Z_{in|B})(Z_{L|A} - Z_{L|C})}, \quad (6)$$

$$Z_a = \frac{(Z_{in|A}Z_{L|A} - Z_{in|B}Z_{L|B}) - Z_e(Z_{L|A} - Z_{L|B})}{Z_{in|B} - Z_{in|A}}, \quad (7)$$

$$T_a^2 = (Z_e - Z_{in|A,B,C})(Z_a + Z_{L|A,B,C}). \quad (8)$$

This is the same as the solution by Ramo et al. (1965) if we let  $T_a = Z_{21} = -Z_{12}$ ,  $Z_e = Z_{11}$  and  $Z_a = Z_{22}$ .

### 3. Conclusions

There are many ways to express the solution, depending on the choice of independent variables. We do agree that the equations in Appendix A of Kim and Allen (2013) are incorrect, as observed by Bernier et al. (2016), and we thank them for point out these errors.

The point by Bernier et al. (2016) about the sign ( $\pm$ ) on  $T_a$  is really not relevant in our view, since it is a matter of definition, and for the anti-reciprocal transducer, both signs are present, since  $T_{ea} = -T_{ae}$ . Based on Eq. (1), the sign of  $T_a$  cannot be determined by electrical input impedance measurements alone as only  $T_a^2$  term contributes (Ramo et al., 1965, p. 543).

To our knowledge, the first attempt to derive impedance matrix parameters for a two-port system from input measurements was introduced by Ramo et al. (1965, pp. 541-545). Their original work applies for microwave systems with two wave guide terminals, but the method is valid for any types of two-port networks. Finally we would like to mention the work of Weece and Allen (2010), which also used this set of equations.

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