ON THE AGING OF STEEL GUITAR STRINGS

by

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INTRODUCTION

It is well-known by musicians who play steel string guitars that their strings are very susceptible to aging. The sound of a quality guitar can be totally lost within a few hours of playing because of string aging. This problem is such that professional musicians frequently change their strings during an intermission.

There exists a great deal of folklore concerning a cure for aging strings. It is a generally accepted fact that strings age more rapidly for some people than others -- an effect that is presumed to be attributable to the propensity of musicians to perspire. Many a thrift-minded guitar player has tried the technique of boiling his week-old strings in vinegar in an attempt to rejuvenate them. The relatively short life of the guitar string and the frequent necessity to replace strings (a 10-minute, mindless ordeal) inspired me to investigate this perplexing problem of aging.

THEORY

A guitar string may be modeled as a rigid string (bar) under tension [1,2]. The linearized differential equation describing the string and the physical model are shown in Fig. 1. The first term in this equation represents the wire's inherent rigidity. This term is small but contributes significantly to one's perception of the string's sound because it introduces inharmonicity. Since the ear is very sensitive to this inharmonicity, even the slightest degree of stiffness is important.

The second and third terms in the equation are the standard wave equation terms which make up classical string theory. The loss is introduced into the equation by the fourth term. This term represents the rate of change of the string's curvature and is a term which may be related to internal loss mechanisms. The quantitative validity of this term has not yet been confirmed; however, it is qualitatively in agreement with all of our present measured results. The right hand side of the equation is the source term, which for simplicity has been chosen to be a point source at $z=z_0$.

The model is specified by a two-dimensional vector differential equation. A vector is required because we assume that the degeneracy of the x and y motion has been removed. The physical mechanism which removes the degeneracy is presently unknown, so in our model we arbitrarily assume that the boundary conditions are responsible.

It has been suggested by Prof. Lothar Cremer[1] that nonlinearities are actually responsible for the removal of the degeneracy, but I have been unable to confirm this suggested theory. The double mode structure is important perceptually because of beating between neighboring modes. The resulting time dependence of the partials is clearly important and greatly complicates the analysis of real string impulse responses. The resulting homogeneous equation is

$$\epsilon\psi'''' - \psi'' - R\dot{\psi}'' + \frac{1}{c^2}\ddot{\psi} = 0 \qquad (1)$$

where the primes indicate differentiation with respect to z and the dots with respect to time. c is the velocity of transverse waves on the string, R is the loss coefficient and ϵ, which is small, is the stiffness coefficient. Equation (1) may be analyzed by the following argument. Assume we were able to force a standing-wavelike pattern on the string of the form

$$\psi_{ks}(x,t) = e^{st}cos(kx) \qquad (2)$$

Under these conditions, what (complex) frequency $s=\sigma+i\omega$ would be required to maintain this motion? Substituting (2) into (1) we may solve for the unknown complex frequencies s as a function of k.

$$\epsilon k^4 + k^2 + Rsk^2 + s^2/c^2 = 0 \qquad (3)$$

Solving for s, we find that

$$s^{\pm} = \frac{-Rc^2k^2}{2} \pm ick\sqrt{1+k^2[\epsilon - \frac{R^2C^2}{4}]} \qquad (4)$$

For simple pinned boundary conditions k may be found to be

$$k = n\pi/L; \quad n = \pm 1, \pm 2, \pm 3, \ldots \qquad (5)$$

corresponding to the vibrating modes of the string. Using (5) in (4) we may determine the complex resonant frequency of each mode

$$s_n^{\pm} = \frac{1}{2} - Rc^2(\frac{n\pi}{L})^2 \qquad (6)$$

$$\pm ic(\frac{n\pi}{L})\sqrt{1+(\frac{n\pi}{L})^2[\epsilon - \frac{R^2c^2}{4}]}$$

Since

$$s_n^{\pm} = \sigma_n \pm i\omega_n \qquad (7)$$

we may identify σ_n and ω_n

$$\sigma_n = -Rc^2\pi^2n^2/2L^2 \qquad (8)$$

$$\omega_n = \frac{n\pi c}{L}\sqrt{1+Bn^2} \qquad (9)$$

where we have defined

$$B = \frac{\pi^2}{L^2}[\epsilon - \frac{-R^2c^2}{4}] \qquad (10)$$

If we further define

$$\omega_0 = \pi c/L \qquad (11)$$

then (8, 9, 10) may be rewritten as

$$\sigma_n = \frac{-R\omega_0^2}{2}n^2 \quad (sec^{-1}) \qquad (12)$$

$$\omega_n = \omega_0 n\sqrt{1+Bn^2} \quad (rad/sec) \qquad (13)$$

$$B = \omega_0^2[\frac{\epsilon}{c^2} - \frac{R^2}{4}] \qquad (14)$$

Equation (12) defines the damping coefficient for each mode, ω_n is the frequency, and B is commonly called the coefficient of inharmonicity [3]. In Fig. 2 we plot (12, 13). When B is zero the modes (partials) are perfectly harmonic.

Note that B is zero for the lossless case when ϵ is zero. For the typical case (13) may be accurately approximated by expanding the square root since B is very small compared to 1.

$$\omega_n = \omega_0 n[1+\frac{Bn^2}{2}] \quad (rad/sec) \qquad (15)$$

Note that the frequency in Hertz is defined as

$$f_n = \omega_n/2\pi \quad (Hz) \qquad (16)$$

The 45 degree line of Fig. 2 is the case of the harmonic string. For a stiff string, the partials are increasingly sharpened. The damping constant also increases quadratically. If we define the reverberation time T_n as the time required for a mode to decay 60 dB, then

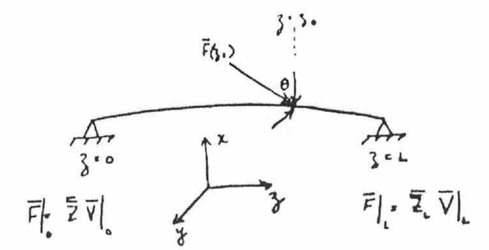
$$T_n = -6.91/\sigma_n \quad (sec) \qquad (17)$$

PHYSICAL MEASUREMENTS

The greatest problem in making the physical measurements was obtaining old guitar strings which were intact. D. Thompson of The Martin Guitar Company was kind enough to donate several dozen new base E strings. New sets of strings were given to guitar virtuosos in trade for their old, worn guitar strings.

The measurement jig consisted of a wooden electrical guitar body. Electrical pickup was made by placing a one-inch diameter X cut quartz disk under the metal bridge. A plastic pick served as the point source. The string was plucked as close to the bridge as possible while the spectrum was monitored on a Spectral Dynamics SD-

1. personal communication



BASIC EQ:

$$\underbrace{-\epsilon \frac{\partial^4}{\partial z^4}\begin{bmatrix}X(z)\\ Y(z)\end{bmatrix}}_{\text{STIFFNESS TERM}} + \frac{\partial^2}{\partial z^2}\begin{bmatrix}X(z)\\ Y(z)\end{bmatrix} + \frac{\omega^2}{c^2}\begin{bmatrix}X(z)\\ Y(z)\end{bmatrix} \underbrace{+ iR\omega \frac{\partial^2}{\partial z^2}\begin{bmatrix}X(z)\\ Y(z)\end{bmatrix}}_{\text{LOSS TERM}} = -\begin{bmatrix}A\cos\theta\\ A\sin\theta\end{bmatrix}\delta(z-z_0)$$

B.C. at $z=0$ and $z=L$:

$$\frac{\partial}{\partial z}\begin{bmatrix}X(z)\\ Y(z)\end{bmatrix} = +\frac{i\omega}{T}\begin{bmatrix}Z_{11} & Z_{12}\\ Z_{21} & Z_{22}\end{bmatrix}\begin{bmatrix}X(z)\\ Y(z)\end{bmatrix}, \; z=0,L; \quad \frac{\partial^2}{\partial z^2}\begin{bmatrix}X\\ Y\end{bmatrix} = 0, \; z=0,L$$

FIG. 1 MATHEMATICAL MODEL

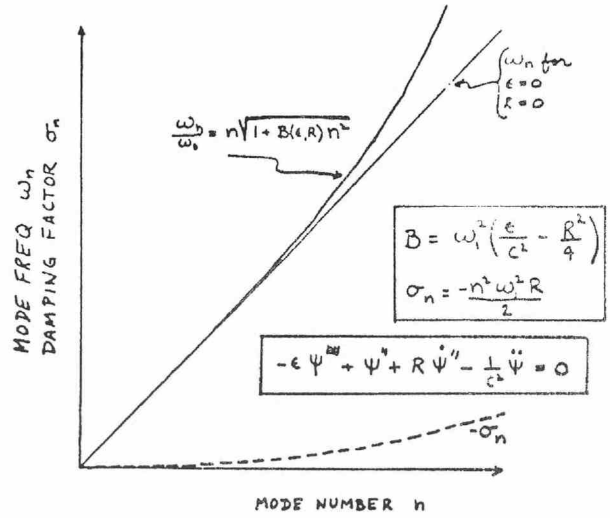


FIG. 2

301 real time spectrum analyzer. An attempt was made to create a spectrum which was as uniform as possible. When the plucking technique was mastered, the signal was digitized with a 12 bit A/D converter and the time waveform was stored on digital magnetic tape. The final responses were chosen by listening to the digitized samples via a 12 bit D/A converter. One new and one old sample were chosen as being characteristic of the sound quality we wished to study.

The first 50 ms of the waveforms for the new and old string are shown in Figs. 3a,b. A great deal of information may be obtained from the time waveforms. As the initial pulse propagates on the wire the high frequencies separate out. This effect is due to the stiffness of the wire which causes dispersion. Because of the stiffness, the high frequencies propagate with a greater velocity. When the pulse reaches the end of the wire it is reflected back. Thus we see a series of pulses with a high frequency precursor separating out from the main part of the pulse. It is easy to understand why the modes become sharper as the frequency increases. The increased velocity of propagation at higher frequencies implies a shorter time period for the round trip of the pulse at those frequencies. Thus the high frequency partials are shifted up in frequency relative to the frequency predicted by a constant sound velocity.

We wish to demonstrate that ϵ does not change as the string ages. We may do this if we measure the mode frequencies of the new and old string and plot them as a function of the mode number. The string time response was Fourier transformed by the use of a discrete Fourier transform (DFT). From a visual inspection, the mode frequencies were determined and are plotted in the upper right hand corners of each panel of Fig. 4a,b. In the insert we have plotted

$$f_n/nf_0 - 1 \tag{18}$$

which from (15) is equal to

$$\tfrac{1}{2}Bn^2 \tag{19}$$

In this manner both f_0 and B have been determined.

It is easily seen from the figures that B is the same for both the new and old strings since the curvature of the plots is the same. Thus it appears that the stiffness does not change as the string ages.

In Fig. 4a,b the double mode structure is also apparent above 1 kHz. The double structure complicates the analysis greatly when two modes lie very close to each other and are not resolved by the DFT. Under this condition, the modes beat together and cause problems in measuring the decay constant σ_n. Even when a single mode was apparently resolved there were problems measuring σ_n. When each of the modes was inverse-transformed, a very irregular decay curve was common. One possible cause of this phenomenon is the slight change in the mode frequency of each of the two modes as the response decays. Because of the irregular decay, we have not yet found a reliable method of measuring σ_n.

However, one qualitative conclusion is possible. The main difference observed in comparing Fig. 4a and 4b is the loss of the high frequency modes in the old string spectrum 4b. These modes, which are inharmonic, are initially excited but decay away so rapidly they are not perceived. In the new string the loss factor R is smaller and therefore the decay time is long enough that the inharmonicity is audible.

To confirm these conclusions we synthetically generated string impulse responses on the computer so that the perceptual effect of different R values could be determined.

SIMULATIONS

To simulate the sound of a guitar string we evaluated the sum

$$h(t) = \sum_{n=1}^{N} e^{-\sigma_n t}\cos(\omega_n t) \tag{20}$$

where σ_n and ω_n are given by (12) and (13). It was necessary to modify (12) slightly by adding a small constant. Without this modification the lower modes would have excessive reverberation time. The modification is equivalent to an extra term in (1) porportional to ψ. Although the naturalness of the synthetic strings left something to be desired, the change in quality which was perceived with increased R was distinctly similar to the difference between a new and an old string. In Fig. 3c we show an example of a synthetic impulse response. The dispersion we described earlier is clearly present.

SUMMARY

We have discussed in some detail the theory of a plucked stiff steel string under tension. The stiffness is important perceptually because it introduces inharmonicity. In a new string the loss is such that the inharmonicity is easily perceived. As the string ages the loss increases and damps out the higher harmonics making the inharmonicity imperceptible. If one could stabilize the loss coefficient R of a string he might be able to greatly improve its aging properties.

REFERENCES

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[2] Rayleigh, J.W.S., *The Theory of Sound*, Vol I, (1894), pp 296-297, Dover, N.Y.

[3] Fletcher,H., "Normal Vibration Frequencies of a Stiff Piano String , " JASA , (1964) , 36 , 1 , pp 203-9 [see also Errata , 36 , 6 , 1214 , June 1964]

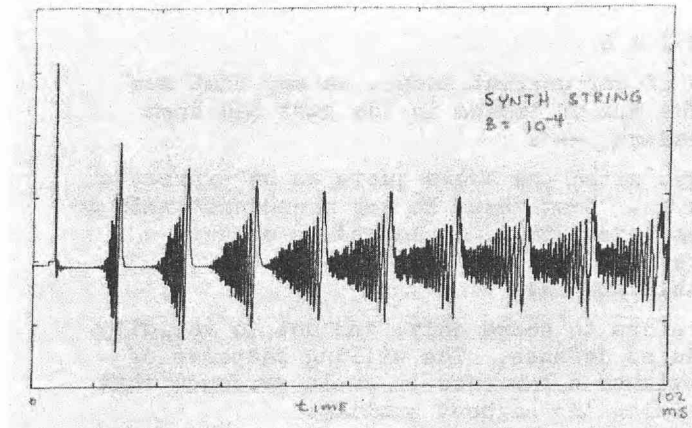


FIG. 3C

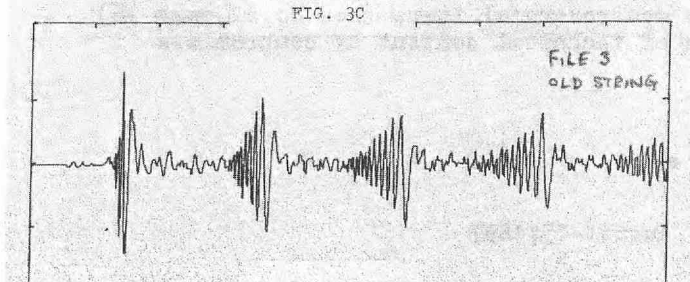


FIG. 3B

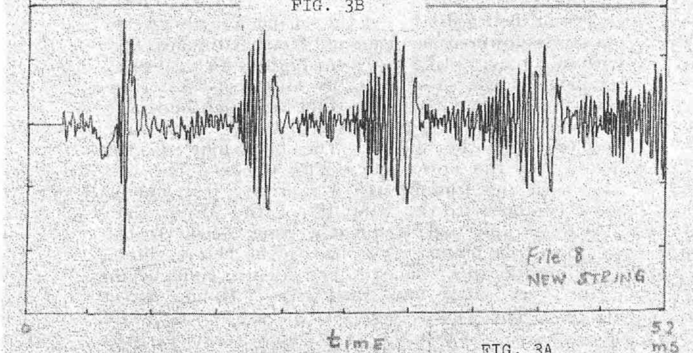
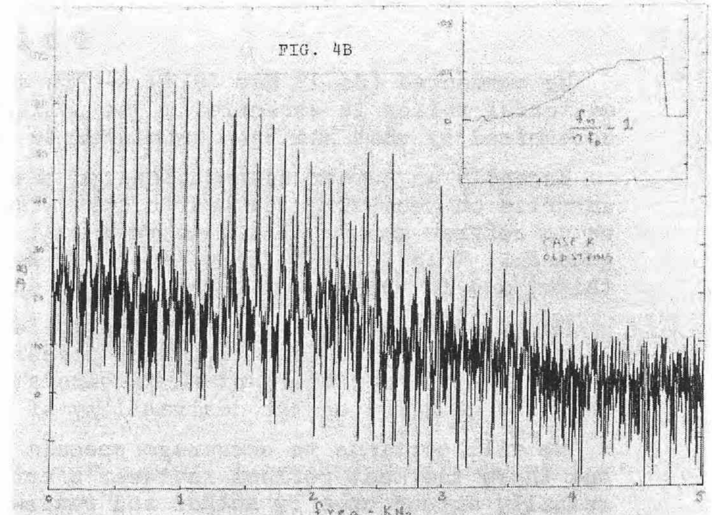


FIG. 3A

FIG. 4B



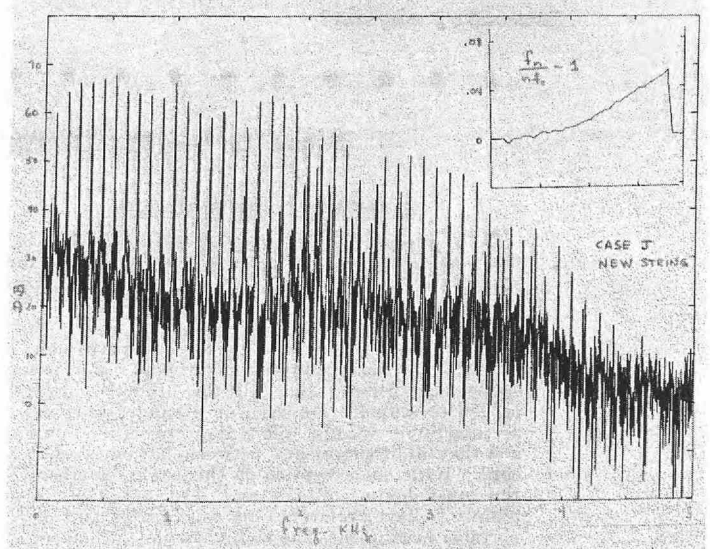


FIG. 4A

Found in the VIOLIN TIMES

of October 15, 1895

Here is a good story of Joachim, which has the advantage of being absolutely authentic. During one of his visits to London some years ago, the great virtuoso had occasion to enter a barber's shop for a shave. The barber's acquaintance with illustrious musicians was limited, and Joachim preserved his *incognito*.

"Hair cut, sir?" demanded the obsequious assistant, eyeing Joachim's flowing locks with an air of proprietorship.

Joachim intimated his perfect satisfaction with the existing length of his hair; but the barber was not to be so easily baffled.

"Trifle long at the back, sir," he suggested diplomatically.

Joachim explained that that was the way he liked it, and the barber was silent for a little.

"Rather thin on top, sir," was his next remark, whereby he sought to convey his own firm conviction that to sacrifice thickness to length in the matter of hair was altogether a poor policy; but Joachim only glared at the barber and tossed his lion mane. And the barber went on shaving, but in a moody, discontented kind of way. Hope springs eternal in the human breast, and the barber's breast was no exception.

"Just trim the edges for you, sir? Half-an-inch all round, sir?"

Joachim remained obdurate, and the barber's stock of patience and ingenuity deserted him at the same time. He vented his indignation in the most scathing expression of contempt that suggested itself to his tonsorial mind.

"Well, of course, if you want to look like a German musician," he remarked, "it's no good talking."

of January 15, 1894

Scene: Doorstep of a house. Landlady just coming out when an itinerant fiddler accosts her. "Patronise the wandering minstrel, kind lady?" Kind Lady: "Certainly not, one scraper at the door is quite enough."

Landlady (to lodger): "Beg pardon, sir. Did I understand as you were a doctor of music?"—Lodger: "I am, ma'am. Why?"—Landlady: "Well, sir, my Billy 'ave just bin and broke his concertina, and I thawt as 'ow I should be glad to put a hodd job in yer way."

Here's a yarn, not new, but good, and attributed to the composer Cherubini. One day a young fellow called on him to have his voice tried. Cherubini heard him give a song or two, and then the youth asked, "What branch of the profession do you advise me to go in for?" "Auctioneer," promptly replied the maestro; and then the interview ended.

A thief broke into a large mansion early the other morning, and found himself in a music-room. Hearing footsteps approaching he hid behind a screen. From 7 to 8 o'clock the eldest daughter had a lesson on the piano. From 8 to 9 o clock the second daughter had a singing lesson. From 9 to 10 o'clock the eldest son had a violin lesson. From 10 to 11 o'clock the other son had a lesson on the flute. At 11 o'clock all the brothers and sisters assembled, and studied an ear-splitting piece for piano, violin, flute, and voice. The thief staggered out from behind the screen at 11.30, and falling at their feet, cried—"For mercy's sake, have me arrested or give me a rest!"