CHAPTER

Introduction and terminology

1

CHAPTER OUTLINE

PART I: INTRODUCTION

1.1 A LITTLE HISTORY

Acoustics has entered a new age—the era of precision engineering. One hundred years ago acoustics was an art. The primary measuring instruments used by engineers in the field were their ears. The only controlled noise sources available were whistles, gongs, sirens and gun shots. Microphones consisted of either a diaphragm connected to a mechanical scratcher that recorded the shape of the wave on the smoked surface of a rotating drum or a flame whose height varied with the sound pressure. About that time the great names of Rayleigh, Stokes, Thomson, Lamb, Helmholtz, König, Tyndall, Kundt, and others appeared on important published papers. Their contributions to the physics of sound were followed by the publication of Lord Rayleigh's two-volume treatise *Theory of Sound* in 1877/1878 (revised in 1894/1896). In the late 19th century, Alexander Graham Bell invented the magnetic microphone and with it the telephone. Thomas Edison created the carbon microphone, which was the transmitter used in standard telephone handsets for almost 100 years. The next big advance was Edison's phonograph, which made it possible for the human voice and other sounds to be preserved for posterity.

In a series of papers published between 1900 and 1915, W. C. Sabine advanced architectural acoustics to the status of a science. He measured the duration of reverberation in rooms using organ pipes as the source of sound and a chronograph for the precision measurement of time. He showed that

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reverberation could be predicted for auditoria from knowledge of room volume, audience size, and the characteristics of the sound-reflecting surfaces—sidewalls and ceiling.

Even though the contributions of these earlier workers were substantial, the greatest acceleration of research in acoustics followed the invention of the triode vacuum tube (1907) and the advent of radio broadcasting (1920). When vacuum tube amplifiers and loudspeakers became available, loud sounds of any desired frequency could be produced. With the invention of moving coil and condenser microphones the intensity of very faint sounds could be measured. Above all, it became feasible to build acoustical measuring instruments that were compact, rugged, and insensitive to air drafts, temperature and humidity.

The progress of communication acoustics was hastened through the efforts of the Bell Telephone Laboratories (1920*ff*), which were devoted to perfection of the telephone system in the United States. During the First World War the biggest advances were in underwater sound. In the next two decades (1936*ff*) architectural acoustics strode forward through research at Harvard, the Massachusetts Institute of Technology, the University of California at Los Angeles, and several research centers in England and Europe, especially Germany. Sound decay in rectangular rooms was explained in detail, the impedance method of specifying acoustical materials was investigated, and the computation of sound attenuation in ducts was put on a precise basis. The advantages of skewed walls and use of acoustical materials in patches rather than on entire walls were demonstrated. Functional absorbers were introduced to the field, and a wider variety of acoustical materials came on the market. In particular, Morse, Stenzel, Bouwkamp, and Spence contributed to the mathematical theory of sound radiation and diffraction.

The science of psychoacoustics was rapidly developing. At the Bell Telephone Laboratories, under the leadership of Harvey Fletcher, the concepts of loudness and masking were quantified, and many of the factors governing successful speech communication were determined (1920–1940). Acoustics, through the medium of ultrasonics, entered the fields of medicine and chemistry. For example, ultrasonic diathermy was being tried, and acoustically accelerated chemical reactions were reported.

Then came the Second World War with its demand for the successful detection of submerged submarines and for highly reliable speech communication in noisy environments such as in armored vehicles and high-flying non-pressurized aircraft. Government financing of improvements in these areas led to the formation of great laboratories in England, Germany, and France, and in the United States at Columbia University, Harvard, and the University of California. During this period research in acoustics reached proportions undreamed of a few years before and has continued unabated since.

In the last fifty years, the greatest revolution has undoubtedly been the vast increase in computing power accompanied by a rapid rate of miniaturization which has lead to a previously unimaginable plethora of hand portable products, including cell phones, palmtop computers and measuring devices. Size has presented new challenges for the acoustical designer as the pressure to reduce dimensions is ever increasing. Contrary to popular expectations, electroacoustic transducers do not obey Moore's law, [1] so it cannot be assumed that a reduction in size can be achieved without sacrificing performance, although new materials such as polysilicon membranes for microphones and neodymium magnets for loudspeakers have helped preserve performance to some extent. Reducing the size of loudspeakers usually compromises their maximum sound power output, particularly at low frequencies, and in the case of microphones, the signal-to-noise ratio in their output deteriorates. Therefore, the ability of the acoustical engineer to optimise the design of transducers and electronics has never been more important.

In addition to the changes in products in which electro-acoustic transducers are employed, computers have revolutionized the way in which the transducers themselves are modeled. [2] The first wave of tools came in the 1960s and early 1970s for simulating electrical circuits. Acoustical engineers were quick to adapt these for modelling loudspeakers and microphones using lumped mechanical and acoustical circuit elements analogous to electrical ones, as given in *Acoustics*. However, simulation by this method was largely a virtual form of trial and error experimentation, albeit much faster than actual prototyping, until Thiele and Small applied filter theory to the transfer function so that the designer could choose a target frequency response shape for a loudspeaker and engineer the electro-mechano-acoustic system accordingly. Finite element modeling (FEM) and boundary element modeling (BEM) both followed, quickly. Unlike lumped element simulation, the range was no longer limited to that where the acoustical wavelength is much greater than the largest dimension of the device. With the wide availability of modern tools for acoustical simulation, it is perhaps tempting to neglect more traditional analytical methods, which are a focus of this text, especially in Chaps. 12 and 13. However, analytical (mathematical) methods can offer some distinct benefits:

- According to Richard Hamming, [3] "The purpose of computation is insight not just numbers." By examining the mathematical relationships, we can gain a better understanding of the physical mechanisms than when the calculations are all "hidden" in a computer. This helps us to create improved systems, especially when we can manipulate the equations to arrive at formulas that enable us to design everything correctly first time such as those given in Chap 7 for loudspeaker systems. By contrast, a simulation tool can only simulate the design we load into it. It cannot tell us directly how to design it, although it may be possible to tweak parameters randomly in a Monte-Carlo optimization or "evolve" a design using a Darwinian genetic algorithm. Even then, a global optimum is not necessarily guaranteed.
- Although exact analytical formulas are generally limited to simple rectangular, cylindrical or spherical geometries, many electroacoustic transducers have, or can be approximated by, these simple geometries. (Note this restriction does not apply to lumped elements which can have almost any shape.) It may sometimes take time and effort to derive a formula, but once it is done it can be used to generate as many plots as you like simply by varying the parameters. Furthermore, the right formula will give the fastest possible computation with the least amount of memory space. If a picture paints a thousand words, it could be said that an equation paints a thousand pictures. In the words of Albert Einstein, "Politics is for the moment, but an equation is for eternity."
- Analytical solutions often yield simple asymptotic expressions for very low/high frequencies or the far field, which can form the basis for elements in circuit simulation programs.
- It is often useful to have an analytical benchmark against which to check FEM/BEM simulation results. This can tell us much about the required element size and what kind of meshing geometry to use. Of course, having two ways of solving a problem gives us increased confidence in both methods.
- Analytical formulas are universal and not restricted to a particular simulation tool: They can be written into a wide choice of programming languages.

Another area in which computers have contributed is that of symbolic computation. For example, if we did not know that the integral of $\cos x$ was $\sin x$, we would have to integrate $\cos x$ numerically, which is a relatively slow and error-prone process compared with evaluating $\sin x$

directly. Modern mathematical tools are capable of solving much more complicated integrals than this symbolically, which has led to new analytical solutions in sound radiation. For example, the previous formulas for radiation from a circular disk in free space were too lengthy to include in the original *Acoustics* but a more compact solution, with or without a circular baffle, is given in the new Chapter 13.

Not only have computers led to the advances mentioned above, but they have fallen dramatically in price, so much so, that many devices such as cellphones, hearing aids, and sound level meters now contain a digital signal processor (DSP) as well as electro-acoustical transducers. This enables an acoustical designer to design a complete system including DSP equalization. Although DSP algorithms are beyond the scope of this book, Chapter 14 has been written with the intention of aiding this part of the design process. It describes state-variable circuit-simulation theory, which can be used to obtain a transfer function of the electro-acoustical system. The inverse transfer function can then be used as a basis for DSP equalization. However, any form of equalization should come with a health warning, since it cannot be used to compensate for a poor acoustical design. On the other hand, a DSP can be used in real time to monitor changes in the electro-mechano-acoustical parameters and to adjust the drive levels accordingly in order to extract the maximum possible performance, whilst avoiding burn-out.

In 1962, Sessler and West invented a new kind of capacitor microphone, which contained a permanently stored charge on a metalized membrane as well as a pre-amplifier, which has become known as the foil electret microphone. This device has been followed by Micro-Electro-Mechanical Systems (MEMS), now incorporated into microphones and vibration pickups (accelerometers) and gyroscopes, which have dimensions in the order of microns. One embodiment of MEMS is widely used in hearing aids and cellphones, where the trend is to incorporate more microphones for noise cancellation and beam forming. It consists of a freely vibrating diaphragm made from polysilicon which is spaced from a perforated backplate that is coated with vapour-deposited silicon nitride. When the device is moved, there is a change in capacitance in the order of femtofarads. The combination of low cost, small size, reliability and near studio quality has made the "crackly" carbon microphone obsolete. Hence, the electret and MEMS models are also described in this text. Other new additions include call loudspeakers for cellphones and an improved tube model for very small diameters.

Today, acoustics is no longer a tool of the telephone industry, a few enlightened architects, and the military. It is a concern in the daily life of nearly every person. International movements legislate and provide quiet housing. Labor and office personnel demand safe and comfortable acoustic environments in which to work. Architects in rapidly increasing numbers are hiring the services of acoustical engineers as a routine part of the design of buildings. Manufacturers are using acoustic instrumentation on their production lines. In addition there has been great progress in the abatement of noise from jet-engine propelled aircraft—which efforts were instigated by the Port of New York Authority and its consultant Bolt Beranek and Newman in the late 1950s, and have been carried on by succeeding developments in engine design. Acoustics is coming into its own in the living room, where high-fidelity reproduction of music has found a wide audience. Overall, we witness the rapid evolution of our understanding of electroacoustics, architectural acoustics, structural acoustics, underwater sound, physiological and psychological acoustics, musical acoustics, and ultrasonics.

It is difficult to predict the future with any certainty, although nano-technologies look as though they will play a steadily increasing role. One can truly say that although over one hundred years have passed since the publication of Rayleigh's *Theory of Sound*, there is still plenty to explore.

This book covers first the basic aspects of acoustics: wave propagation in the air, the theory of mechanical and acoustical circuits, the radiation of sound into free space, and the properties of acoustic components. Then follow chapters dealing with microphones, loudspeakers, enclosures for loudspeakers, and horns. The basic concepts of sound in enclosures are treated next, and methods for solving problems related to the radiation and scattering of sound are given. The final chapter describes a computer method for analyzing circuits. Throughout the text we shall speak to *you*—the student of this modern and exciting field.

1.2 WHAT IS SOUND?

In reading the material that follows, your goal should be to form and to keep in mind a picture of what transpires when the diaphragm on a loudspeaker, or any surface for that matter, is vibrating in contact with the air.

A sound is said to exist if a disturbance propagated through an elastic material causes an alteration in pressure or a displacement of the particles of the material which can be detected by a person or by an instrument. Because this text deals primarily with devices for handling speech and music, the only types of elastic material with which we shall concern ourselves are gases (more particularly, air). Because the physical properties of gases are relatively easy to express, it is not difficult to describe the way that sound travels in such media.

Imagine that we could cut a tiny cubic "box" out of air and hold it in our hands as we would a block of wood. What physical properties would it exhibit? First, it would have weight and, hence, mass. In fact, a cubic meter of air has a mass of a little over one kilogram. If a force is applied to this box it will accelerate according to Newton's second law, which states that force equals mass times acceleration.

If we exert forces that compress two opposing sides of the little cube, the four other sides will bulge outward. The incremental pressure produced in the gas by this force will be the same throughout this small volume. This happens because pressure in a gas is a scalar, i.e., a nondirectional quantity.

Now imagine that the little box of air is held tightly between your hands. Move one hand toward the other so as to distort the cube into a parallelepiped. You find that no opposition to the distortion of the box is made by the air outside the two distorted sides. This indicates that air does not support a shearing force. [4]

Further, if we constrain five sides of the cube and push on the sixth one, we find that the gas is elastic; i.e., a force is required to compress the gas. The magnitude of the force is in direct proportion to the displacement of this unconstrained side of the container. A simple experiment proves this. Close off the hose of a bicycle tire pump so that the air is confined in the cylinder. Push down on the plunger. You will find that the confined air behaves like a simple spring.

What is air? The air that surrounds us consist of tiny molecules which are about 0.33 nm in diameter, but are 3.3 nm apart, so they only occupy 0.1% of the space! Even so, at room temperature, a cubic meter weighs 1.18 kg. Although the mean free path between collisions is 60 nm, air molecules travel at an average speed of 500 m/s at room temperature so that each one experiences 8.3×10^9 collisions per second! It is the force with which the molecules bombard the boundary of a confined space which explains the elastic (and viscous) properties of air

The spring constant of the gas varies, however, with the speed of the compression. A displacement of the gas particles occurs when it is compressed. An incremental change in the volume of our box will cause an incremental increase in the pressure that is directly proportional to the displacement. If the compression takes place slowly this relation obeys the formula:

$$\Delta P = -K\,\Delta V\text{—slow process}$$

where K is a constant.
If, on the other hand, the incremental change in volume takes place rapidly, and if the gas is air, oxygen, hydrogen, or nitrogen, the incremental pressure that results is equal to $1.4K$ times the incremental change in volume:

$$\Delta P = -\gamma K\,\Delta V\text{—fast process, diatomic gas,}$$

where γ is the ratio of specific heats for a gas and is equal to 1.4 for air and other diatomic gases. Note that a positive increment (increase) in pressure produces a negative increment (decrease) in volume. Processes that take place at intermediate rates are more difficult to describe, even approximately, and fortunately need not be considered here.

What is the reason for the difference between these two occurrences? For slow variations in volume the compressions are *isothermal*, which means that they take place at constant temperature throughout the volume. There is time for the heat generated in the gas during the compression to flow out through the walls of the container. Hence, the temperature of the gas remains constant. For rapid alternations in the volume, however, the temperature rises when the gas is compressed and falls when the gas is expanded. If the alternations are rapid enough, there is not enough time during a cycle of compression and expansion for the heat to flow away. Such rapid alternations in the compression of the gas are said to be *adiabatic*.

In either isothermal or adiabatic processes, the pressure in a gas is due to collisions of the gas molecules with the container walls. You will recall that pressure is force per unit area, or, from Newton, time rate of change of momentum per unit area. Let us investigate the mechanism of this momentum change in a confined gas. The direction of motion of the molecules changes when they strike a wall, so that the resulting change in momentum appears as pressure in the gas. The *rate* at which the change of momentum occurs, and so the magnitude of the pressure change, depends on two quantities. It increases either if the number of collisions per second between the gas particles and the walls increases, or if the amount of momentum transferred per collision becomes greater, or both.

During an isothermal compression of a gas, an increase of pressure results because a given number of molecules are forced into a smaller volume and they necessarily collide with the container walls more frequently.

During an adiabatic compression of a gas, the pressure increase partly results from an increase in the number of wall collisions as described above, and additionally from the greater momentum transfer per collision. Both of these increases are due to the temperature change which accompanies the adiabatic compression. From kinetic theory we know that the velocity of gas molecules varies as the square root of the absolute temperature of the gas. As contrasted with the isothermal, in the adiabatic process the molecules get hotter, they move faster, collide with the container walls more frequently, and, having greater momentum themselves, transfer more momentum to the walls during each individual collision. For a given volume change ΔV, the rate of momentum change, and therefore the pressure increase, is seen to be greater in the adiabatic process. Hence, the gas is stiffer—it takes more force to expand or compress it. We shall see later in the text that sound waves are adiabatic alternations.

1.3 PROPAGATION OF SOUND THROUGH GAS

The propagation of sound through a gas can be fully predicted and described if we take into account the factors just discussed, viz., the mass and stiffness of the gas, and its conformance with basic physical laws. Such a mathematical description will be given in detail in later chapters. We are now concerned with a qualitative picture of sound propagation.

If we put a sinusoidally vibrating wall in a gas (see Fig. 1.1*a*), it will accelerate adjacent air particles and compress that part of the gas nearest to it as it moves forward from rest. This initial compression is shown in Fig. 1.1*b* as a crowding of dots in front of the wall. The dots represent air particles. These closely crowded air particles have, in addition to their random velocities, a forward momentum gained from the wall. They collide with their neighbors to the right and, during the collision, transfer forward momentum to these particles, which were at rest. These particles in turn move closer to their neighbors, with which they collide, and so on. Progressively more and more remote parts of the medium will be set into motion. In this way, through successive collisions, the force built up by the original compression may be transferred to distant parts of the gas.

When the wall reverses its motion, a rarefaction occurs immediately in front of it (see Fig. 1.1*c* and Fig. 1.1*d*). This rarefaction causes particles to be accelerated backward, and the above process is now repeated in the reverse direction, and so on, through successive cycles of the source.

It is important to an understanding of sound propagation that you keep in mind the relative variations in pressure, particle displacement, and particle velocity. Note that, at any one instant, the maximum particle displacement and the maximum pressure do not occur at the same point in the wave. To see this, consider Fig. 1.1*c*. The maximum pressure occurs where the particles are most tightly packed, i.e., at $D_2 = 1.7$ m. But at D_2 the particles have not yet moved from their original rest position, as we can see by comparison with Fig. 1.1*a*. At D_2, then, the pressure is a maximum, and the particle displacement is zero. At this instant, the particles next to the wall are also at their zero-displacement position, for the wall has just returned to its zero position. Although the particles at both D_2 and d_0 have zero displacement, their environments are quite different. We found the pressure at D_2 to be a maximum, but the air particles around d_0 are far apart, and so the pressure there is a minimum. Halfway between d_0 and D_2 the pressure is found to be at the ambient value (zero incremental pressure), and the displacement of the particles at a maximum. At a point in the wave where pressure is a maximum, the particle displacement is zero. Where particle displacement is a maximum, the incremental pressure is zero. Pressure and particle displacement are then 90° out of phase with each other.

At any given point on the wave the pressure and particle displacement are varying sinusoidally in time with the same frequency as the source. If the pressure is varying as $\cos 2\pi ft$, the particle displacement, 90° out of phase, must be varying as $\sin 2\pi ft$. The velocity of the particles, however, is the time derivative of displacement and must be varying as $\cos 2\pi ft$. At any one point on the wave, then, pressure and particle velocity are in phase.

We have determined the relative phases of the particle displacement, velocity, and pressure at a point in the wave. Now we ask, "What phase relationship exists between values of, say, particle

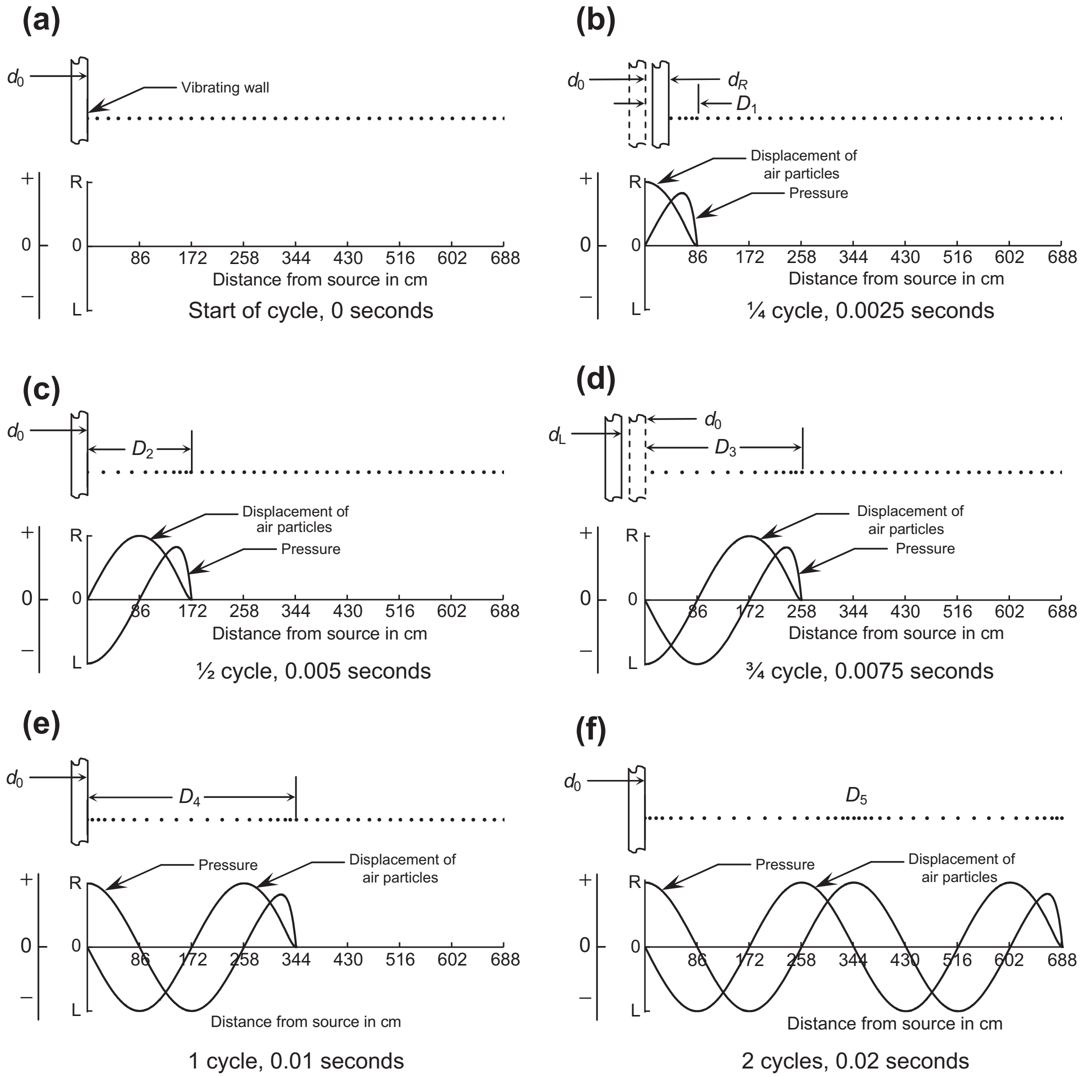

FIG. 1.1 Pressure and displacement in a plane sound wave produced by a sinusoidally vibrating wall.

D_1 = one-fourth wavelength; D_2 = one-half wavelength; D_3 = three-fourths wavelengths; D_4 = one wavelength; D_5 = two wavelengths. *R* means displacement of the air particles to the right, *L* means displacement to the left and *O* means no displacement. Crowded dots mean positive excess pressure and spread dots mean negative excess pressure. The frequency of vibration of the piston is 100 Hz.

displacement measured at two different points on the wave?" If the action originating from the wall were transmitted instantaneously throughout the medium, all particles would be moving in phase with the source and with each other. This is not the case, for the speed of propagation of sound is finite, and at points increasingly distant from the source there is an increasing delay in the arrival of the signal. Each particle in the medium is moved backward and forward with the same frequency as the wall, but not at the same time. This means that two points separated a finite distance from each other along the wave in general will not be moving in phase with each other. Any two points that are vibrating in exact phase will, in this example of a plane wave, be separated by an integral number of wavelengths. For example, in Fig. 1.1*f* the 344.8- and 689.6-cm points are separated by exactly one wavelength. A disturbance at the 689.6-cm point occurs at about 0.01 s after it occurs at the 344.8-cm point. At room temperature, 22°C, this corresponds to a speed of propagation of 344.8 m/s. Mathematically stated, a wavelength is equal to the speed of propagation divided by the frequency of vibration.

$$\lambda = \frac{c}{f} \tag{1.1}$$

where λ is the wavelength in meters, c is the speed of propagation of the sound wave in m/s, and f is the frequency in hertz (or cycles/s).

What is a plane wave? In Fig. 1.1 it is assumed that the wall is infinite in size so that everywhere in front of it the air particles are moving as shown by the line at the center of the wall. Such a sound wave is called plane because it is behaving the same at every plane parallel to the surface of the oscillating wall.

Sound waves in air are longitudinal; i.e., the direction of the vibratory motion of air particles is the same as the direction in which the wave is traveling. This can be seen from Fig. 1.1. Light, heat, or radio waves in free space are transverse; i.e., the vibrations of the electric and magnetic fields are perpendicular to the direction in which the wave advances. By contrast, waves on the surface of water are circular. The vibratory motion of the water molecules is in a small circle or ellipse, but the wave travels horizontally.

1.4 MEASURABLE ASPECTS OF SOUND

Consider first what measurements might be made on the medium before a sound wave or a disturbance is initiated in it. The gas particles (molecules) are, on the average, at rest. They do have random motion, but there is no net movement of the gas in any direction. Hence, we say that the *particle displacement* is zero. It follows that the *particle velocity* is zero. Also, when there is no disturbance in the medium, the *pressure* throughout is constant and is equal to the *ambient pressure*, so that the *incremental pressure* is zero. A value for the ambient pressure may be determined from the readings of a barometer. The *density*, another measurable quantity in the medium, is defined as usual as the mass per unit volume. It equals the *ambient density* when there is no disturbance in the medium.

When a sound wave is propagated in the medium, several measurable changes occur. The particles are accelerated and as a result are displaced from their rest positions. The particle velocity at any point is not zero except at certain instants during an alternation. The pressure at any point varies above and below the ambient pressure. Also, the temperature at a point fluctuates above and below its ambient value. The *incremental* variation of pressure is called the *sound pressure* or the *excess pressure*. An incremental pressure variation, in turn, causes a change in the density called the *incremental density*. An increase in sound pressure at a point causes an increase in the density of the medium at that point.

The speed with which an acoustical disturbance propagates outward through the medium is different for different gases. For any given gas, the speed of propagation is proportional to the square root of the absolute temperature of the gas [see Eq. (1.8)]. As is the case for all types of wave motion, the speed of propagation is given by Eq. (1.1).

PART II: TERMINOLOGY

You now have a general picture of the nature of a sound wave. To proceed further in acoustics, you must learn the particular "lingo," or accepted terminology. Many common words such as pressure, intensity, and level are used in a special manner. Become well acquainted with the special meanings of these words at the beginning as they will be in constant use throughout the text. The list of definitions below is not exhaustive, and some additional terminology will be presented as needed in later chapters.[5] If possible, your instructor should have you make measurements of sounds with a sound-level meter and a sound analyzer so that the terminology becomes intimately associated with physical phenomena.

1.5 GENERAL

Acoustic. The word "acoustic," an adjective, means intimately associated with sound waves or with the individual media, phenomena, apparatus, quantities, or units discussed in the science of sound waves. Examples: "Through the acoustic medium came an acoustic radiation so intense as to produce acoustic trauma. The acoustic filter has an output acoustic impedance of 10-acoustic ohms." Other examples are acoustic horn, transducer, energy, wave, admittance, refraction, mass, component, propagation.

Acoustical. The word "acoustical," an adjective, means associated in a general way with the science of sound or with the broader classes of media, phenomena, apparatus, quantities, or units discussed in the science of sound. Example: "Acoustical media exhibit acoustical phenomena whose well-defined acoustical quantities can be measured, with the aid of acoustical apparatus, in terms of an acceptable system of acoustical units." Other examples are acoustical engineer, school, glossary, theorem, circuit diagram.

Imaginary unit. The symbol $j = \sqrt{-1}$ is the imaginary unit. In the case of a complex quantity $z = x + jy$, we denote the real part by $\Re(z) = x$, where $\Re$ is a capital R in the Fraktur typeface, and the imaginary part by $\Im(z) = y$, where $\Im$ is a capital I in the Fraktur typeface. It is worth noting that $\sqrt{-1}$ can be positive or negative, as with the square root of any number, and many texts use the negative square root i, where $i = -j$. Mathematica also uses the positive square root, but denotes it by a double-struck $\mathbb{i}$ instead of j. A complex quantity $z = x + jy$ can also be represented in terms of magnitude $|z|$ and phase angle θ by

$$z = |z|e^{j\theta} = |z|(\cos\theta + j\sin\theta),$$

where the magnitude (aka modulus or absolute value) is given by

$$|z| = \sqrt{x^2 + y^2}$$

and the phase angle is given by

$$\theta = \arctan(y/x)$$

Harmonically varying quantity. In this text, a harmonically varying quantity will be denoted by a tilde. Hence, if ψ represents a generic quantity, we denote that it is harmonically varying by writing it as $\tilde{\psi}$. In the analyses that follow, the tilde will help us to distinguish time-dependent (or signal) variables from system parameters or constants. Although the wave is periodic, it may be of arbitrary shape. The simplest case is a sinusoidal wave, which means

$$\tilde{\psi} = \psi e^{j\omega t}, \tag{1.2}$$

where $\omega = 2\pi f$ is the angular frequency in rad/s, and f is the frequency in Hz (hertz, formerly cycles/second). Note that if ψ is real, it represents the peak amplitude of $\tilde{\psi}$. However, it may be complex, containing phase information such as that relating to the position in space, in which case the peak amplitude is the magnitude of ψ or $|\psi|$. Since most of the problems dealt with in this text relate to steady-state linear systems driven by sinusoidal sources, this simple shorthand saves us from having to carry the time dependency term, $e^{j\omega t}$, through the calculations. If we are deriving an expression for one harmonically varying quantity in terms of another, the exponent will cancel in the final transfer function.

Instantaneous value. In the steady state, the instantaneous value $\psi(t)$ is defined by

$$\psi(t) = \Re(\tilde{\psi}) = \Re(\psi\, e^{j\omega t}). \tag{1.3}$$

This is the actual value that would be observed at any instant in time t. However, the real part of a complex quantity should only be taken at the end of deriving an expression so that the correct phase relationships are maintained throughout. It is often the case that the product of two imaginary quantities yields a real one.

Root-mean-square value. The root-mean-square or rms value of a time-varying quantity is that which delivers the same amount of power on average as a constant quantity of the same value. For example, a direct electrical current passing through an electrical resistance produces the same amount of heat as an alternating current having the same rms value. The rms value ψ_{rms} is given by

$$\psi_{\mathrm{rms}} = \sqrt{\frac{1}{T}\int_0^T (\psi(t))^2 dt}. \tag{1.4}$$

If the quantity $\psi\,(t)$ is a periodic function of time, then $T = 1/f$ is the period of repetition. It turns out that in the case of the simple sinusoidal function described by Eq. (1.3):

$$\psi_{\mathrm{rms}} = \frac{|\tilde{\psi}|}{\sqrt{2}}, \tag{1.5}$$

where $|\tilde{\psi}|$ denotes the magnitude of $\tilde{\psi}$. However, the periodic function can have any arbitrary shape, which is expandable by a Fourier series of harmonics. The rms value is then given by the Euclidian norm (or root of the sum of the squares) of the peak amplitudes of the harmonics divided by $\sqrt{2}$. In the case of nonperiodic quantities, the interval T should be long enough to make the value obtained essentially independent of small changes in the length of the interval.

1.6 STANDARD INTERNATIONAL (SI) UNITS

The SI system of units is used throughout this book. Conversion tables for other systems are given in Appendix III. Some of the fundamental units are listed in Table 1.1. However, this list is by no means exhaustive. Other units used in electroacoustics can be derived from them and will be introduced as and when needed.

Also, for very large or very small quantities, it is useful to use the prefixes shown in Table 1.2.

1.7 PRESSURE AND DENSITY

The standard unit of pressure in the SI system is the pascal (Pa), where 1 Pa = 1 N/m^2.

***Static pressure* (P_0).** The static pressure at a point in the medium is the pressure that would exist at that point with no sound waves present. At normal barometric pressure, P_0 equals approximately

Table 1.1 List of SI units

Quantity	Unit	Symbol
Length	meter	m
Mass	kilogram	kg
Time	second	s
Temperature	kelvin	K
Magnetic flux density	tesla	T
Force	newton	N
Power	watt	W

Table 1.2 Prefixes for large or small quantities

Multiples	***Name***	deca	hecto	kilo	mega	giga	tera	peta	exa	zetta	yotta
	Symbol	da	h	k	M	G	T	P	E	Z	Y
	Factor	10^1	10^2	10^3	10^6	10^9	10^{12}	10^{15}	10^{18}	10^{21}	10^{24}
Subdivisions	***Name***	deci	centi	milli	micro	nano	pico	femto	atto	zepto	yocto
	Symbol	d	c	m	μ	n	p	f	a	z	y
	Factor	10^{-1}	10^{-2}	10^{-3}	10^{-6}	10^{-9}	10^{-12}	10^{-15}	10^{-18}	10^{-21}	10^{-24}

10^5 Pa. Standard atmospheric pressure is usually taken to be 0.760 m Hg at 0°C. This is a pressure of 101 325 Pa. In this text when solving problems we shall assume $P_0 = 10^5$ Pa.

***Microbar* (μbar).** Although not an SI unit, a microbar is a unit of pressure often used in acoustics. One microbar is equal to 0.1 Pa.

***Instantaneous sound pressure* [$p(t)$].** The instantaneous sound pressure at a point is the incremental change from the static pressure at a given instant caused by the presence of a sound wave. The unit is the pascal (Pa).

***Effective sound pressure* (p_{rms}).** The effective sound pressure at a point is the root-mean-square (rms) value of the instantaneous sound pressure. The unit is the pascal (Pa).

***Density of air* (ρ_0).** The ambient density of air is given by the formula

$$\rho_0 = \frac{P_0}{287T} \text{ kg/m}^3, \tag{1.6}$$

where T is the absolute temperature and P_0 is the static pressure. At a normal room temperature of $T = 295$°K (22°C or 71.6°F), and for a static pressure $P_0 = 10^5$ Pa, the ambient density is

$$\rho_0 = 1.18 \text{ kg/m}^3.$$

This value of ρ_0 will be used in solving problems unless otherwise stated. Note that here the temperature in °C is obtained by subtracting 273 from the one in °K.

1.8 SPEED AND VELOCITY

***Speed of sound* (c).** The speed of sound in air is given approximately by the formula

$$c = 331.4 + 0.607\theta \text{ m/s} \tag{1.7}$$

where θ is the ambient temperature in °C. For temperatures above 30°C or below −30°C, the velocity of sound must be determined from the exact formula:

$$c = 331.4\sqrt{\frac{T}{273}} = 331.4\sqrt{1 + \frac{\theta}{273}} \text{ m/s}, \tag{1.8}$$

where T is the ambient temperature in °K. At a normal room temperature of $\theta = 22$°C (= 71.6°F),

$$c = 344.8 \text{ m/s}, \text{ or } 1131.2 \text{ ft/sec}.$$

These values of c will be used in solving problems unless otherwise stated.

***Instantaneous particle velocity* (*particle velocity*) [$u(t)$].** The instantaneous particle velocity at a point is the velocity, due to the sound wave only, of a given infinitesimal part of the medium at a given instant. It is measured over and above any motion of the medium as a whole. The unit is m/s.

***Effective particle velocity* (u_{rms}).** The effective particle velocity at a point is the root mean square of the instantaneous particle velocity (see *Effective Sound Pressure* for details). The unit is m/s.

***Instantaneous volume velocity* [$U(t)$].** The instantaneous volume velocity, due to the sound wave only, is the rate of flow of the medium perpendicularly through a specified area S. That is, $U(t) = Su(t)$, where $u(t)$ is the instantaneous particle velocity. The unit is m^3/s.

1.9 IMPEDANCE

***Acoustic impedance* (Z_A). (*American standard acoustic impedance*).** The acoustic impedance at a given surface is defined as the complex ratio [6] of effective sound pressure averaged over the surface to effective volume velocity through it. The surface may be either a hypothetical surface in an acoustic medium or the moving surface of a mechanical device. The unit is N·s/m^5, or rayls/m^2. [7]

$$Z_A = \frac{\tilde{p}}{\tilde{U}} \ \mathrm{N \cdot s/m^5 (rayls/m^2)}. \tag{1.9}$$

***Specific acoustic impedance* (Z_s).** The specific acoustic impedance is the complex ratio of the effective sound pressure at a point of an acoustic medium or mechanical device to the effective particle velocity at that point. The unit is N·s/m^3, or rayls. [8] That is,

$$Z_S = \frac{\tilde{p}}{\tilde{u}} \ \mathrm{N \cdot s/m^3 (rayls)}. \tag{1.10}$$

***Mechanical impedance* (Z_M).** The mechanical impedance is the complex ratio of the effective force acting on a specified area of an acoustic medium or mechanical device to the resulting effective linear velocity through or of that area, respectively. The unit is N·s/m, or rayls·m^2. That is,

$$Z_M = \frac{\tilde{f}}{\tilde{u}} \ \mathrm{N \cdot s/m (rayls \cdot m^2)}. \tag{1.11}$$

***Characteristic impedance* ($\rho_0 c$).** The characteristic impedance is the ratio of the effective sound pressure at a given point to the effective particle velocity at that point in a free, plane, progressive sound wave. It is equal to the product of the density of the medium times the speed of sound in the medium ($\rho_0 c$). It is analogous to the characteristic impedance of an infinitely long, dissipationless electric transmission line. The unit is N·s/m^3 or rayls.

In the solution of problems in this book we shall assume for air that

$$\rho_0 c = 407 \text{ rayls}$$

which is valid for a temperature of 22°C (71.6°F) and a static pressure of 10^5 Pa.

1.10 INTENSITY, ENERGY DENSITY, AND LEVELS

***Sound intensity* (I).** The sound intensity measured in a specified direction at a point is the average rate at which sound energy is transmitted through a unit area perpendicular to the specified direction at the point considered. The unit is W/m^2. In a plane or spherical free-progressive sound wave the intensity *in the direction of propagation* is

$$I = \frac{p_{rms}^2}{\rho_0 c} = \frac{\left|\tilde{p}\right|^2}{2\rho_0 c} \ \mathrm{W/m^2}. \tag{1.12}$$

***Sound energy density* (D).** The sound energy density is the sound energy in a given infinitesimal part of the gas divided by the volume of that part of the gas. The unit is W·s/m^3. In many acoustic environments, such as in a plane wave, the sound energy density at a point is

$$D = \frac{p_{rms}^2}{\rho_0 c^2} = \frac{p_{rms}^2}{\gamma P_0} \text{ W}\cdot\text{s/m}^3. \tag{1.13}$$

where γ is the ratio of specific heats for a gas and is equal to 1.4 for air and other diatomic gases. The quantity γ is dimensionless.

Electric power level, or acoustic intensity level. The electric power level, or the acoustic intensity level, is a quantity expressing the ratio of two electrical powers or of two sound intensities in logarithmic form. The unit is the decibel (dB). Definitions are:

$$\text{Electric power level} = 10 \log_{10}\frac{W_1}{W_2} \text{ dB} \tag{1.14}$$

$$\text{Acoustic intensity level} = 10 \log_{10}\frac{I_1}{I_2} \text{ dB} \tag{1.15}$$

where W_1 and W_2 are two electrical powers and I_1 and I_2 are two sound intensities.

Extending this thought further, we see from Eq. (1.14) that

$$\text{Electric power level} = 10 \log_{10}\frac{e_{1rms}^2}{R_1}\frac{R_2}{e_{2rms}^2} \text{ dB}$$

$$= 20 \log_{10}\frac{e_{1rms}}{e_{2rms}} + 10 \log_{10}\frac{R_2}{R_1} \text{ dB} \tag{1.16}$$

where e_{1rms} is the voltage across the resistance R_1 in which a power W_1 is being dissipated and e_{2rms} is the voltage across the resistance R_2 in which a power W_2 is being dissipated. Similarly,

$$\text{Acoustic intensity level} = 20 \log_{10}\frac{p_{1rms}}{p_{2rms}} + 10 \log_{10}\frac{R_{S2}}{R_{S1}} \text{ dB} \tag{1.17}$$

where p_{1rms} is the pressure at a point where the specific acoustic resistance (i.e., the real part of the specific acoustic impedance) is R_{S1} and p_{2rms} is the pressure at a point where the specific acoustic resistance is R_{S2}. We note that

$$10 \log_{10}(W_1/W_2) = 20 \log_{10}(E_1/E_2)$$

only if $R_1 = R_2$ and that

$$10 \log_{10}(I_1/I_2) = 20 \log_{10}(p_{1rms}/p_{2rms})$$

only if $R_{S2} = R_{S1}$.

The word "level" implies a position relative to another position, for example, "the water level is 2 meters above its normal level." This means that any quantity in acoustics designated as "level" is the magnitude of a quantity, expressed in logarithmic units, which is so many units above the magnitude of another quantity, also expressed in logarithmic units. Hence, 10 log A is not a level, but

$$10 \log \text{A} - 10 \log \text{B} = 10 \log(\text{A/B})$$

is a level for A measured above a level for B. In electronics and acoustics, the level differences are measured in decibels. The quantity B is usually referred to as the "reference quantity."

Levels involving voltage and pressure alone are sometimes spoken of with no regard to the equalities of the electric resistances or specific acoustic resistances. This practice leads to serious confusion. It is emphasized that the manner in which the terms are used should be clearly stated always by the user in order to avoid confusion.

***Sound pressure level* (SPL).** The sound pressure level of a sound, in decibels, is 20 times the logarithm to the base 10 of the ratio of the measured effective sound pressure of this sound to a reference effective sound pressure. That is,

$$\mathrm{SPL} = 20 \log_{10} \frac{p_{rms}}{p_{ref}} \ \mathrm{dB}. \tag{1.18}$$

In the United States p_{ref} is either

$p_{ref} = 20\ \mu\mathrm{Pa}$ rms (0.0002 μbar rms) or
$p_{ref} = 0.1\ \mathrm{Pa}$ rms (1 μbar rms)

The reference pressure listed first above is in general use for measurements dealing with hearing and for sound-level and noise measurements in air (e.g., in rooms and outdoors) and sometimes in liquids. The second reference pressure has gained widespread use for calibrations of transducers and some types of sound-level measurements in liquids. The two reference levels are almost exactly 74 dB apart. The reference pressure must always be stated explicitly. Thus, in the case of Eq. (1.18), using the first reference pressure, the result of a measurement would be expressed, "The sound pressure level is X decibels *re* (0.0002 μbar)."

***Intensity level* (IL).** The intensity level of a sound, in decibels, is 10 times the logarithm to the base 10 of the ratio of the intensity of this sound to a reference intensity. That is,

$$\mathrm{IL} = 10 \log_{10} \frac{I}{I_{ref}} \ \mathrm{dB} \tag{1.19}$$

In the United States the reference intensity is usually taken to be $10^{-12}\ \mathrm{W/m^2}$. This reference at standard atmospheric conditions in a plane or spherical progressive wave was originally selected as corresponding approximately to the reference pressure (0.0002 μbar).

The exact relation between intensity level and sound pressure level in a plane or spherical progressive wave may be found by substituting Eq. (1.12) for intensity in Eq. (1.19):

$$\mathrm{IL} = \mathrm{SPL} + 10 \log_{10} \frac{p_{ref}^2}{\rho_0 c I_{ref}} \ \mathrm{dB}. \tag{1.20}$$

Substituting $p_{ref} = 20\ \mu\mathrm{Pa}$ rms and $I_{ref} = 10^{-12}\ \mathrm{W/m^2}$ yields

$$\mathrm{IL} = \mathrm{SPL} + 10 \log_{10} \frac{400}{\rho_0 c} \ \mathrm{dB}. \tag{1.21}$$

It is apparent that the intensity level IL will equal the sound pressure level SPL only if $\rho_0 c = 400$ rayls. For certain combinations of temperature and static pressure this will be true, although for $T = 22°\mathrm{C}$ and $P_0 = 10^5$ Pa, $\rho_0 c = 407$ rayls. For this common case then, the intensity level is smaller than the sound pressure level by about 0.1 dB. The reference quantity must always be stated explicitly.

Acoustic power level **(PWL).** The acoustic power level of a sound source, in decibels, is 10 times the logarithm to the base 10 of the ratio of the acoustic power radiated by the source to a reference acoustic power. That is,

$$\mathrm{PWL} = 10\log_{10}\frac{W}{W_{ref}}\ \mathrm{dB}. \tag{1.22}$$

In most countries, W_{ref} is 1 pW (i.e., 10^{-12} W). This means that a source radiating 1 acoustic watt has a power level of 120 dB.

If the temperature is 20°C (67°F) and the pressure is 101 325 Pa (0.76 m Hg), the sound pressure level in a duct with an area of 1 m^2 cross section, or at a distance of 0.282 m from the center of a "point" source (at this distance, the spherical surface has an area of 1 m^2), is, from Eqs. (1.12) and (1.18),

$$\begin{aligned}\mathrm{SPL}_{1\mathrm{m}^2} &= 10\log_{10}\frac{I\rho_0 c}{p_{ref}^2} = 10\log_{10}\frac{W\rho_0 c}{Sp_{ref}^2}\\ &= 10\log_{10}\left(W\times 412.5\times\frac{1}{(2\times 10^{-5})^2}\right)\\ &= 10\log_{10}\frac{W}{10^{-12}} + 0.1,\end{aligned}$$

where W is acoustic power in W, $\rho_0 c$ is characteristic impedance = 412.5 rayls, $S = 1\ \mathrm{m}^2$ of area, and p_{ref} is rms reference sound pressure = 20 μPa rms.

In words, the sound pressure level equals the acoustic power level plus 0.1 dB under the special conditions that the power passes uniformly through an area of 1 m^2, the temperature is 20°C (67°F), and the barometric pressure is 0.76 m (30 in.) Hg.

Sound level. The sound level at a point in a sound field is the reading in decibels (dB) of a sound-level meter constructed and operated in accordance with the latest edition of "American National Standard Specification for Sound Level Meters." [9]

The meter reading (in decibels, dB) corresponds to a value of the sound pressure integrated over the audible frequency range with a specified frequency weighting and integration time. The standard sound level meter has three frequency weightings, A, B, and C, as shown in Fig. 1.2. The C scale treats all frequencies within the operating range approximately equally. The B scale is seldom used. The A scale discriminates against frequencies below 800 Hz. When reporting measurements, if the C scale has been used the result is usually given in dB. If the A scale has been used the result must be given in dBA.

Band power level **(PWL_n).** The band power level for a specified frequency band is the acoustic power level for the acoustic power contained within the band. The width of the band and the reference power must be specified. The unit is the decibel. The letter n is the designation number for the band being considered.

Band pressure level **(BPL_n).** The band pressure level of a sound for a specified frequency band is the effective sound pressure level for the sound energy contained within the band. The width of the band and the reference pressure must be specified. The unit is the decibel. The letter n is the designation number for the band being considered.

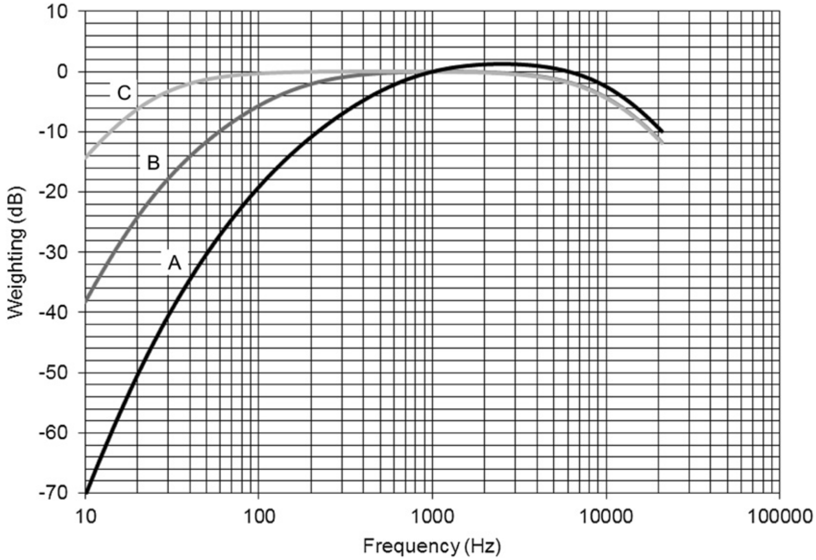

FIG. 1.2 Weighting curves for sound level measurements.

The A, B, and C curves can be regarded as very rough approximations to the contours of equal loudness [10] at 40, 70, and 100 phons respectively in order to compensate for the reduced sensitivity of the ear to very low and very high frequencies.

Power spectrum level. The power spectrum level of a sound at a specified frequency is the power level for the acoustic power contained in a band one cycle per second wide, centered at this specified frequency. The reference power must be specified. The unit is the decibel (see also the discussion under Pressure Spectrum Level).

Pressure spectrum level. The pressure spectrum level of a sound at a specified frequency is the effective sound pressure level for the sound energy contained within a band one cycle per second wide, centered at this specified frequency. The reference pressure must be explicitly stated. The unit is the decibel.

Discussion. The concept of pressure spectrum level ordinarily has significance only for sound having a continuous distribution of energy within the frequency range under consideration.

The level of a band of uniform noise with a continuous spectrum exceeds the spectrum level by

$$C_n = 10 \log_{10}(f_b - f_a) \text{ dB}, \tag{1.23}$$

where f_b and f_a are the upper and lower frequencies of the band, respectively.

The level of a uniform noise with a continuous spectrum in a band of width $f_b - f_a$ Hz is therefore related to the spectrum level by the formula

$$L_n = C_n + S_n, \tag{1.24}$$

where L_n is sound pressure level in dB of the noise in the band of width $f_b - f_a$, for C_n see Eq. (1.23), S_n is spectrum level of the noise, and n is designation number for the band being considered.

Notes

[1] Moore's law was originated by Gordon E. Moore, a co-founder of Intel, in 1965 and states that the number of transistors on an integrated circuit for minimum component cost doubles every 24 months.

[2] For those who have wondered how those impedance and directivity plots were calculated for Acoustics in 1952, MIT hired a room of over 50 women, each were given a motorized mechanical computer of that era, and they did the computations for professors. They were efficient, accurate and got large jobs done in short times by working long hours.

[3] Hamming RW. Numerical Methods for Scientists and Engineers. 2nd ed. Dover Publications; 1987. Preface.

[4] This is only approximately true, as the air does have viscosity, but the shearing forces are very small compared with those in solids.

[5] A good manual of terminology is American National Standard Acoustical Terminology, ANSI S1.1-1994 (R2004), published by the American National Standards Institute, New York, N.Y. http://webstore.ansi.org/

[6] Complex ratio has the same meaning as the complex ratio of voltage and current in electric-circuit theory.

[7] This notation is taken from Table 12.1 of American National Standard Acoustical Terminology, ANSI S1.1-1994 (R2004).

[8] Named in honor of Lord Rayleigh.

[9] American National Standard Specification for Sound Level Meters, ANSI S1.4−1983 (R2006)/ANSI S1.4A−1985 (R2006), American National Standards Institute, New York, N.Y.

[10] See ISO 226. Acoustics: Normal equal-loudness-level contours. available from, http://www.iso.org; 2003.