Causal Black body Radiation is quantized in time and frequency

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Abstract

Possibly, for the first time, we describe the inner relationship of black body radiation, due to the causality. Radiation from black holes and Josephsons manifests the discrete nature of black body radiation as they are at very low temperatures. Light and gravitational waves emitted by the Sun or any black body are found to be in agreement with these findings.

1 Introduction

A blackbody absorbs nd reemits all incident radiation at thermal equilibrium. Planck created a precise description of the atomic processes that occur in the walls of a hollow having a small hole that presents a model of the black surface, laying the groundwork for contemporary Quantum Physics [1, 2]. Wien determined the maximum radiation a surface in thermal equilibrium produces as a function of temperature [3]. Due to equilibrium condition, no surface emits more energy than a blackbody. Planck spectral density formula is given by [1]

$$S(\nu) = \frac{\nu^3}{e^{h\nu/kT} - 1}\,, \tag{1}$$

where $[J/k]k$ the Boltzman constat, $h[Js]$ the Planck constant, $T[K]$ the temperature and $\nu[Hz]$ the frequency of the emitted radiation. The wavelength $\lambda_{max.}$ at which Planck's distribution reaches a maximum is given by the Wien law as

$$\lambda_{max.} = \frac{2.898}{T} \times 10^{-3}[m]\,. \tag{2}$$

The Planck formula resulted in the Stefan-Boltzmann equation, which relates the intensity of emitted radiation to the black body temperature as $I = \sigma T^4[W/m^2]$, where σ is Stefan's constant. The energy density of radiation diverges at high frequencies, causing what is known as the *ultraviolet catastrophe*.

Radiation exists even at low temperature, examples include black holes and the Josephson junction [4]. A Josephson junction consists of two superconductors sandwiched with a thin insulator. Unlike, the black body at high temperatures which exhibits a continuous spectrum, these emissions typically are discrete.

The power radiated by an electric dipole calculated by Larmor, is

$$P_L = \frac{2}{3\pi\varepsilon_0 c^3}\, e^2 a^2[W/m^2]\,, \tag{3}$$

where a is the electron acceleration, $\varepsilon_o 0$ is thevacuum permittivity and c is the speed of light. One also can attribute the black body's power emission.

Quantum field theory predicts that an accelerating observer in a vacuum sees quanta not seen by an inertial observer, derived by Unruh [5]

$$kT = \frac{\hbar a}{2\pi c}\,, \qquad\qquad \hbar = \frac{h}{2\pi}\,. \tag{4}$$

In a gravitational theory, a similar effect is suggested for radiation emitted by a black hole [6]. He asserts that black holes emit as black bodies. The temperature of the thermal radiation emitted by a black hole is given by

$$kT = \frac{\hbar g}{2\pi c}\,, \qquad\qquad g = \frac{c^4}{4GM}\,, \tag{5}$$

where g is the surface acceleration of the black hole and M is its mass.

If the Planck formula is made causal, by expressing the frequency in Eq.(1) as the Laplace frequency

$$s = \sigma + j2\pi\nu \tag{6}$$

one obtains an infinity of poles, forcing the power spectrum to be discrete. The average of this yields Planck's continuous spectrum. The power spectrum is real (not complex analytic), thus $S(\nu)$ has one pole at $\nu = 0$. This formula is known to match the experimental data of the smoothed (non-analytic) black-body power spectrum [2].

Allen [7, 8] found that there is a characteristic frequency associated with this spectrum making a connection with the Bohr radius. Bear in mind that the emission from a hydrogen atom is a line emission. This seems consistent with the quantum nature of the radiation emitted by hydrogen and a black body. To see this, we apply Eq.(6) in Eq.(1) to obtain the eigen frequencies s_nn

$$\exp(-\frac{\hbar s_n}{kT}j) = 1\,, \tag{7}$$

which is satisfied by

$$\frac{\hbar\nu_n}{kT} = 2\pi n\,, \qquad\qquad n = 0, \pm 1, \pm 2, \cdots\,, \tag{8}$$

giving the frequency eigen modes. This determines the temperature T_n. Another pole is at $s = 0$.

Therefore, all radiations are line emission. Additionally, Allen found that there is a delay in the emission of the radiation [7, 8]. In this case, there is no ultraviolet catastrophe in the blackbody power distribution. The characteristic time delay is given by

$$\tau_0 = \frac{\hbar}{kT_c}\,, \tag{9}$$

where $T_c[K]$ the temperature. Thus, at $T_c \sim 1[K]$, one has $\tau_0 \sim 10^{-11} sec$. A typical wavelength associated with τ_0 at $T \sim 1[K]$ is that $\lambda_0 \sim \frac{\pi}{20}[\mathring{A}]$ that is very closed to the Bohr radius. The causality requirement for a blackbody emission results in the quantum nature of the radiation. The delay time for the Sun with $T \sim 10^3[K]$ is $\tau \sim 10^{-14} sec$. As $\tau_0 \to 0$, the emission becomes continuous without significant delay.

If we now assume that the emission of a black hole is connected with some electron acceleration inside an atom, then Eq.(4) suggests an acceleration

$$a_0 = \frac{2\pi c}{\tau_o}\,, \tag{10}$$

that amounts to $a_0 \sim 10^{20} m/s^2$, for $T_c \sim 1K$. A typical system reflecting this low-temperature range is the Josephson junction which would have a delay time $\tau_o \sim 10^{-11} sec$.

However, a solar black hole has a radiating temperature of $1nK$. According to Eq.(9), it corresponds to $\tau_o \sim 10^{-2} sec$. Since the radiation by a black hole is a blackbody, the

emission must have a discrete spectrum. It is understood that the black hole radiation occurs in the vicinity of a black hole where particle-antiparticle pairs are created [6]. One of the pairs (with negative energy) is attracted to the black hole and the other (with positive energy) is ejected. Using Eqs.(9) and (5), the characteristic frequency for a black hole will be

$$\nu_o = \frac{c^3}{8\pi GM}\,, \qquad \nu_o = \frac{c}{4\pi R_s}\,, \qquad R_s = \frac{2GM}{c^2}\,, \tag{11}$$

where R_s is the Schwarzschild radius of a black hole. The emission from a black hole can be considered as stochastic at a temperature of $\sim 1[nK]$ assuming a typical solar black hole. Owing to Eq.(9), the delay time in its emission will be $\tau_0 \sim 10^2 sec$. Incidentally, the gravitational waves recorded by LIGO reveal a frequency of this order of magnitude [9].

According to the forgoing analysis, the black hole radiates discretely, separated by a mean time lap of τ_o. This could be connected with the time to create a pair of particles. Applying Eq.(9) in Eq.(3) yields

$$P_0 = \frac{8\pi e^2 \nu_o^2}{3\varepsilon_0 c}\,, \tag{12}$$

that may account for the quantum thermal power associated with accelerating electrons inside matter. A quantum power is recently derived that is given by [10]

$$P_A = \frac{\hbar}{2c^2}\, a^2\,. \tag{13}$$

Based on Eqs.(13) and (4), the radiation power of Hawking radiation becomes

$$P_U = 2\pi^2\hbar \left(\frac{kT}{\hbar}\right)^2\,, \qquad P_U = 2\pi^2\hbar\,\nu_o^2. \tag{14}$$

Katayama *et al.* connecting Hawking radiation from the black holes to the Josephson tunneling current [11]. They have proposed a quantum circuit that acts as a black hole laser, providing a lab-bench black hole equivalent. The power of a black hole in 1+1 dimensions is given by Katayama *et al.*

$$P_1 = \frac{\pi\hbar}{12}\left(\frac{kT}{\hbar}\right)^2\,, \tag{15}$$

which agrees with the quantum power (Eq.(13)). The working frequency in Josephson junction is $\sim 100GHz$ that makes its power $P_U \sim 10^{-11}[W]$. Under application of a voltage (V), the current in the Josephson junction oscillates with a frequency given by $\nu_J = 2eV/h$ [4]. Thus according to our foregoing analysis, the Josephson junction radiates like a black body having a significant quantization signature.

2 Concluding remarks

Possibly, for the first time, we describe the inner relationship of black body radiation imposed by causality. Radiation from black holes and Josephson junction manifests the discrete nature of black body radiation as they are at very low temperatures. The visible light emitted by the Sun a reflects a discrete pattern and that by a black hole too which agree with the gravitational waves frequency recorded by LIGO experiment.

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