Causal Black body Radiation is quantized in time and frequency

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Abstract

This paper introduces a causal framework for black body radiation, demonstrating that the imposition of causality leads to a quantized spectrum in both time and frequency. Unlike the continuous spectrum predicted by classical thermodynamics, the discrete nature of radiation becomes evident at low temperatures, as observed in black holes and Josephson junctions. Applying Laplace frequency transformations to Planck's law reveals in frequencies, providing a causal basis for line emission in quantum systems. Furthermore, this quantized spectrum eliminates the ultraviolet catastrophe and aligns with the characteristic frequencies of gravitational waves detected by LIGO. Our findings establish a deep connection between black hole radiation, quantum thermodynamics, and solid-state quantum devices, highlighting the fundamental role of causality in thermal emission processes.

Introduction

When it is in thermal equilibrium, a black body absorbs and reemits all incident radiation. Wien (1896) first determined the maximum radiation by a surface in thermal equilibrium as a function of temperature [3]. Due to its equilibrium condition, no surface emits more energy than a black body. Four years later, Planck (1900) [1] developed the first analysis of the black body, based on a hollow sphere having a small observation hole. Planks' model layed the groundwork for the future of Quantum Physics [1, 2].

According to Planck, the spectral density for black body radiation is given by

$$S(\nu) = \frac{\nu^3}{e^{h\nu/kT} - 1}\,, \tag{1}$$

where k [J/K°] is the Boltzmann constant, h [J/Hz] is the Planck constant, T [K°] is the temperature and ν [Hz] is the frequency of the emitted radiation.

The wavelength λ_{max} for which Planck's distribution reaches a maximum is given by Wien's law [3] as

$$\lambda_{max} = \frac{1}{4.965}\,\frac{hc}{kT}\ \ [m] \tag{2}$$

corresponding to a frequency $\nu_{max} = c/\lambda_{max}$, where c is the speed of light.

The Stefan-Boltzmann equation relates the total intensity $I(T) = \sigma T^4$ $[W/m^2]$ of emitted radiation to the black body temperature, where σ is Stefan's constant. Based on Eq.(1), the energy density of radiation diverges at high frequencies, as demonstrated by Rayleigh and Jeans [4], causing what is known as the *ultraviolet catastrophe*. They reported that the power density of a black body is proportional to $I(\lambda) = ckT/\lambda^4$ $[W/m^3]$.

Classical electrodynamics assumes an accelerating electric charge radiates continuously. As a result, the electron circling a proton would continue to radiate and eventually collide

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with the proton, in contrast to observation. Atomic emissions, such as for Hydrogen, is known to be due to a "quantum jump" of the electron when the atom receives energy. However such radiation for Hydrogen is not observed, since Hydrogen has a very long lifetime.

In conflict with classical electrodynamics, radiation exists at all termperatures. Examples include black holes and the Josephson junction [5]. Unlike a black body at high temperatures, which exhibits a continuous spectrum, we will show that at at low termperatues, these emissions occure at discrete frequencies, in disagreement with Eq. (1).

The power radiated by an accelerating charge [6], is

$$P_L = \frac{2}{3\pi\varepsilon_0 c^3}\, e^2 a^2 [W/m^2]\,, \tag{3}$$

where a is the radial component of the electron acceleration, ε_o is the vacuum permittivity, e is the electron charge, and c is the speed of light.

Quantum field theory predicts that an *accelerating observer* in a vacuum sees radiation *not seen by an inertial observer*, as first reported by Unruh [7]

$$kT = \frac{\hbar a}{2\pi c}\ . \tag{4}$$

According to gravitational theory, a similar effect is suggested for radiation emitted by a black hole. Hawking [8] asserts that black holes emit as a black body. The temperature of the thermal radiation emitted by a black hole is given by

$$kT = \frac{\hbar\, g}{2\pi c}\ , \qquad\qquad g = \frac{c^4}{4GM}\,, \tag{5}$$

where $g[m/s^2]$ is the surface acceleration of the black hole and $M[kg]$ is its mass.

1 Results

If the Planck formula is assumed to be causal, by expressing the frequency in Eq.(1) as the Laplace frequency $s = \sigma + j2\pi\nu$, one obtains an infinity of poles, forcing the magnitude spectrum to be discrete. An average of this across frequency yields Planck's continuous magnitude spectrum $S(\nu)$. The magnitude spectrum is real (not complex analytic), thus ν has only one singularity at $\nu = 0$. This formula is known to match the experimental data of the smoothed (non-analytic) black body magnitude spectrum [2].

1.1 Analytic continuation of Planck's formula

Allen [9, 10] showed that there is are discrete frequencies associated with the spectrum, and revealed a interesting relation to the Bohr radius. This seems consistent with the discrete eigen frequencies of hydrogen, thus is consistent with the quantum nature of the radiation by a black body.

To see this we introduce the Laplace frequency $s = \sigma + j2\pi\nu$ into Eq.(1), to obtain complex eigen-frequencies s_n

$$\exp\left(-\jmath\frac{\hbar s_n}{kT}\right) = 1\,, \qquad\qquad j = \sqrt{-1}\ , \tag{6}$$

which results in eigen-mode frequencies (poles)

$$\nu_n = n\frac{kT}{h} \qquad\qquad n = \pm 1, \pm 2, \cdots , \tag{7}$$

which are proportional to temperature T. In other words, assuming causal radiation results in discrete line emission, having frequencies ν_n. Causality results in discrete eigen frequencies for black-body radiation.

Systems exhibiting causal discrete radiation at low temperatures include Josephson junctions and black holes. This seems to be consisent with the view that a cold black hole must have discrete quantum states. Its radiation will display a discrete spectral features at low temperatures, superimposed on a continuous black-body background spectrum as the temperature rises. This seems to be a novel observation.

A typical frequency associated with temperatures at 1 [K°] falls within the [GHz] range. This closely corresponds to the 21 [cm] line emission of neutral hydrogen, observed at a frequency of 1.42 [GHz] [16, 17]. This emission arises from the hyper-fine transition between the parallel and anti-parallel spin states of the electron and proton. The 21 [cm] line is one of the significant spectral lines for studying cold neutral hydrogen in galaxies, providing valuable insights into their structure, dynamics, and mass distribution.

Allen [9, 10] also demonstrated that radiation emission experiences a characteristic delay, and in the quantized spectrum, the ultraviolet catastrophe is eliminated. This time delay is given by

$$\tau_0 = \frac{\hbar}{kT} \, . \tag{8}$$

The time-temperature relation in Eq.(8) can be compared with that in Eq.(2) which applies to a contiguous spectrum. They differ by a numerical factor of order 1. At $T \sim 1[K]$, $\tau_0 \sim 10^{-11}[sec]$. A typical wavelength associated with τ_0 at $T \sim 1[K]$ is that $\lambda_0 \sim \frac{\pi}{20}$ (0.1571 [Å]), which is similar to the Bohr radius (0.529177[Å] = 0.1 [nm]). The ratio is $\frac{0.529177}{0.1571} \approx \pi$. Perhaps there is a factor of π error in the calculation.

1.2 The role of causality

The causality requirement for a black body emission results in the discrete nature of the radiation. The delay time for the Sun with $T \sim 10^3[K]$ is $\tau_o \sim 10^{-14}[sec]$. This process is analogous to the photoelectric effect, where the delay between the absorption of a photon by a metal surface and the subsequent emission of an electron is incredibly brief. Both experimental and theoretical studies indicate that this delay occurs on the order of attoseconds ($10^{-18}[sec]$). As $\tau_0 \to 0$, the emission becomes continuous. If the emission of this radiation is linked to vacuum fluctuations, the acceleration associated with the time delay can be derived using Eqs. (4) and (8). This leads to the following expression

$$a_0 = \frac{2\pi c}{\tau_o} \, , \tag{9}$$

that amounts to $a_0 \sim 10^{20}[m/s^2]$, for $T \sim 1[K]$. this corresponds to the typical frequency at which the junction operates. The Sun has a radiating temperature of 5.772×10^3 [K°]. According to Eq.(8), it corresponds to $\tau_o \sim 10^{-3}[sec]$. It is believed that black hole radiation arises from the creation of particle-antiparticle pairs in the vicinity of the black hole. [8]. One of the particle-antiparticle pairs (with negative energy) is absorbed by the

black hole, while the particle with positive energy is ejected. As the radiation from a black hole follows black body radiation, the emission produces a discrete spectrum.

Using Eqs.(9) and (5), the characteristic frequency for a black hole will be

$$\nu_o = \frac{c^3}{8\pi GM}\,, \qquad \nu_o = \frac{c}{4\pi R_s}\,, \qquad R_s = \frac{2GM}{c^2}\,, \tag{10}$$

where G is Newton's constant and R_s is the Schwarzschild radius of a black hole. The emission from a solar black hole can be considered as a random process. Owing to Eq.(10), the frequency of this emission will be $\nu_o \sim 10^3[Hz]$. The gravitational waves recorded by LIGO reveal a frequency of this order of magnitude [11]. According to the forgoing analysis, the black hole radiates discretely, separated by an average time given by τ_o. This could be connected with the time to create a particle and its antiparticle.
Applying Eq.(9) in Eq.(3) yields

$$P_0 = \frac{8\pi e^2 \nu_o^2}{3\varepsilon_0 c}\,[W]\,, \qquad \nu_0 = \frac{1}{\tau_o}\,[Hz]\,, \tag{11}$$

which may account for the quantum thermal power of accelerating electrons inside matter. This can be compared with the quantum power recently derived by [12]

$$P_A = \frac{\hbar}{2c^2}\,a^2\,[W]\,. \tag{12}$$

Based on Eqs.(12), (4) and (8), the radiation power of Unruh radiation becomes

$$P_U = 2\pi^2\hbar\left(\frac{kT}{\hbar}\right)^2\,, \qquad P_U = 2\pi^2\hbar\,\nu_o^2\,[W]. \tag{13}$$

This formula establishes a relationship between the radiation powers of Unruh, Hawking, and Josephson effects by connecting them to their respective characteristic frequencies. Katayama *et al.* proposed a quantum circuit that functions as a black hole laser, offering a laboratory-based equivalent of a black hole [13]. Since the magnitude spectrum $S(\nu)$ of a black hole in 1+1 dimensions is described by Katayama *et al.* as

$$P_1 = \frac{\pi\hbar}{12}\left(\frac{kT}{\hbar}\right)^2\,[W], \tag{14}$$

in agreement with the quantum power Eq.(13).

Upon application of a voltage (V), the current in the Josephson junction oscillates with a frequency given by $\nu_{_J} = 2eV/h$ [5]. The working frequency in Josephson junction for $V = 0.1$ [mV] is ~ 100 [GHz] resulting in a power $P_U \sim 10^{-11}$ [W]. This agrees with the findings of Finnegan and Wahlsten [14] who observed coherent microwave radiation emitted by coupled Josephson junctions. A similar experiment demonstrated the same power output, attributed to the self-radiation of Josephson junction arrays [15]. The Josephson junction emits radiation resembling that of a black body, but with a distinct signature of quantization.

2 Concluding remarks

Possibly for the first time, we explore the intrinsic relationship between black body radiation and causality. Radiation emitted by black holes and Josephson junctions reveals the discrete

nature of black body radiation at very low temperatures. The visible light emitted by the Sun exhibits a distinct pattern characteristic of its thermal emission. Furthermore, the cold emission from black holes aligns with the frequency of gravitational waves detected by the LIGO experiment.

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